

1 INTRODUCTION

Purpose of IS code :-

- 1) ensure adequate structural safety by specify minimum requirements (like minimum reinforcement)
- 2) provide simple design tables and charts
- 3) ensure consistency amongst different designers
- 4) Some legal validity

Is 456 : 2000 (with amendment no. 4 May 2013)
SP 16, SP 24, SP 34 (SP → special publication)

plane and reinforced concrete :-

It is a Mixture of cement, FA, CA and Water.
that results in solid mass. Sometimes admixtures are also used.

↓ ↓
sand gravel

A concrete with no reinforcement is called as **PCC**. PCC is generally used where significant tensile stress does not develop. Like in the construction of Dam, levelling course of foundation etc.

Concrete is very strong in compression but weak in Tension.

It's tensile strength is approx $\frac{1}{10}$ th of compressive strength.

Nominal $\rightarrow$ expected

* sea water causes corrosion

NOTE For potable water shall be used in concrete mix and pH shall not be less than 6
sea water shall not be used

For most concrete work, Nominal Max. size of CA is 20mm.

Reinforced concrete :-

A concrete with reinforcement embedded

The embedded reinforcement makes the section capable of resisting higher tension.

All of the tension is assumed to be taken of by the reinforcement (in crack section analysis) without separating from concrete.

The Bond b/w steel and surrounding concrete ensure strain compatibility, that is the strain in steel is equal to the strain in surrounding concrete.

Reinforcement also imparts Ductility to the concrete which o/w is a brittle material.

Ductility means large deflection and this is due to yielding of steel

Ductile Members give prior warning before impending collapse

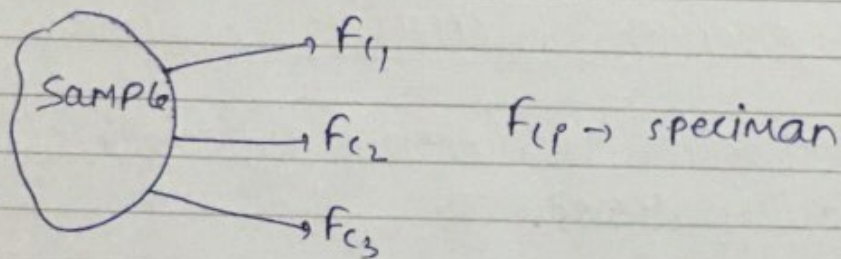
NOTE We generally design crack section in RCC to use the higher permissible stress of steel (Also the section size required will be less in crack section compare to uncracked section)
However crack width shall not be high to avoid corrosion of reinforcement, limit 4/0/3 mm permissible crack width

- 1) in General ≤ 0.3 mm
- 2) structure expose to Moisture or in contact with soil or ground water ≤ 0.2 mm
- 3) very severe and extreme weather condition ≤ 0.1 mm

Compressive strength of concrete :-

- The most important property of concrete and can be easily tested.
- Many other properties like tensile strength, Bond strength, shear strength, impermeability, durability modulus of Elasticity can be inferred from compressive strength.
- strength of concrete in uniaxial compression is determined by loading the standard test cube (150 mm size) to failure in compression testing machine.
- The Test specimen is generally tested 28 days after casting and continuous curing.

- Cube is always tested on sides ; that is faces in touch with mould are in contact with the platen (small plate) of machine
- 3 specimen of a sample are taken to report the strength and the compressive strength is the average of the three specimen.
- The individual variation shall not be more than $\pm 15\%$ of the average. if the variation is more the test result of the sample are invalid.



$$\text{sample strength } (f_{c, \text{avg}}) = \frac{f_{c1} + f_{c2} + f_{c3}}{3}$$

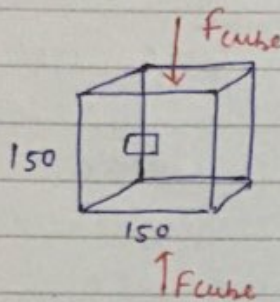
$$\frac{f_{c, \text{avg}} - f_{ci}}{f_{c, \text{avg}}} \times 100 = \pm 15$$

$$f_{c, \text{avg}} - f_{ci} = \pm 0.15 f_{c, \text{avg}}$$

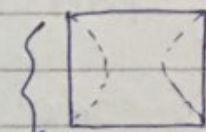
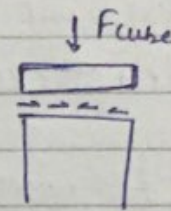
$$0.85 f_{\text{sample}} \leq f_{\text{specimen}} \leq 1.15 f_{\text{sample}}$$

Comparison of strength of cube and cylinder :-

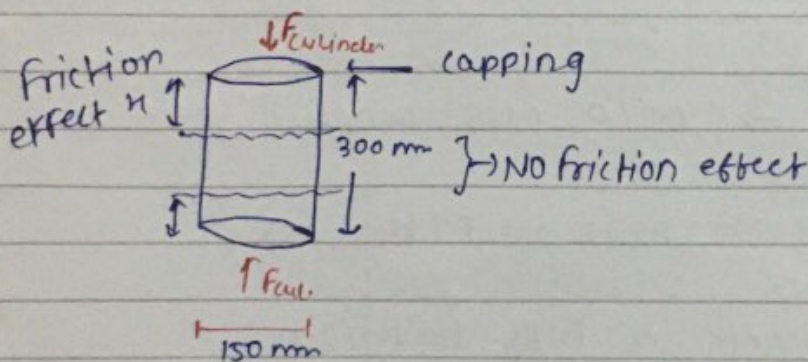
- standard cube size $\rightarrow 150 \text{ mm}$
cylinder size $\rightarrow 150 \text{ mm dia} \times 300 \text{ mm height}$
- cube tested without capping. (neat cement paste) and cylinder tested with capping.



$$\sigma_{\text{cube}} = \frac{F_{\text{cube}}}{(150)^2}$$



friction effect



$$\sigma_{\text{cyl.}} = \frac{F_{\text{cyl.}}}{\frac{\pi}{4} (150)^2}$$

- The restraining effect of the platens (because of friction) of the testing machine extends over the entire height of a cube but leads unaffected a part of Test cylinder.
- Due to restraining effect, the strength differs and the cylinder fails at a early stress than the cube and hence at the verge of failure $\sigma_{\text{cube}} > \sigma_{\text{cylinder}}$

- Cylinder strength is closer to true uniaxial compressive strength of concrete

$$F_{\text{cylinder}} = 0.8 F_{\text{cube}}$$

- IS code uses cube compressive strength.
- Same weight of cube and cylinder, cube will fail first because of edges.

Grade of concrete :-

as per IS code -

ordinary concrete $\rightarrow$ M10, M15, M20

standard concrete $\rightarrow$ M25 $\rightarrow$ M60

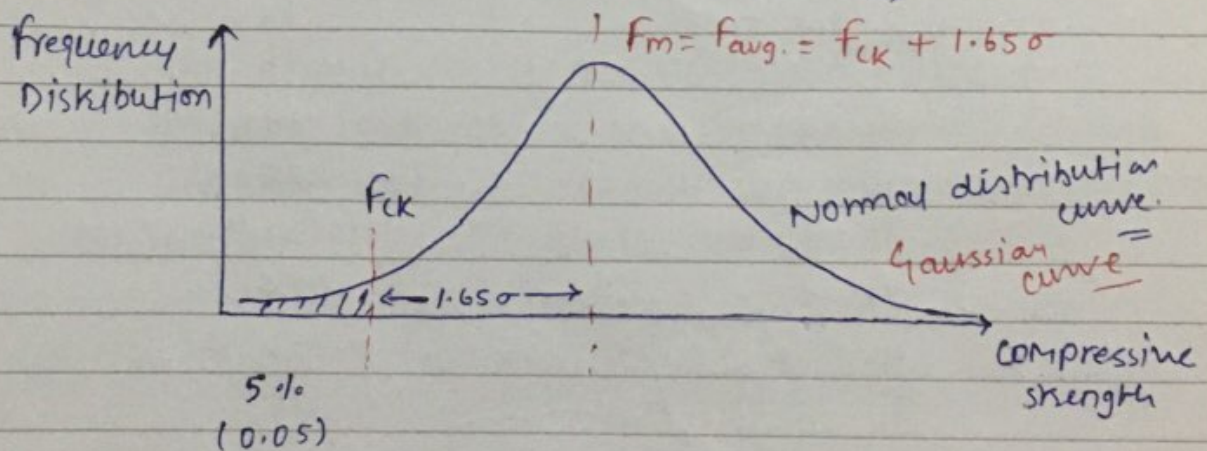
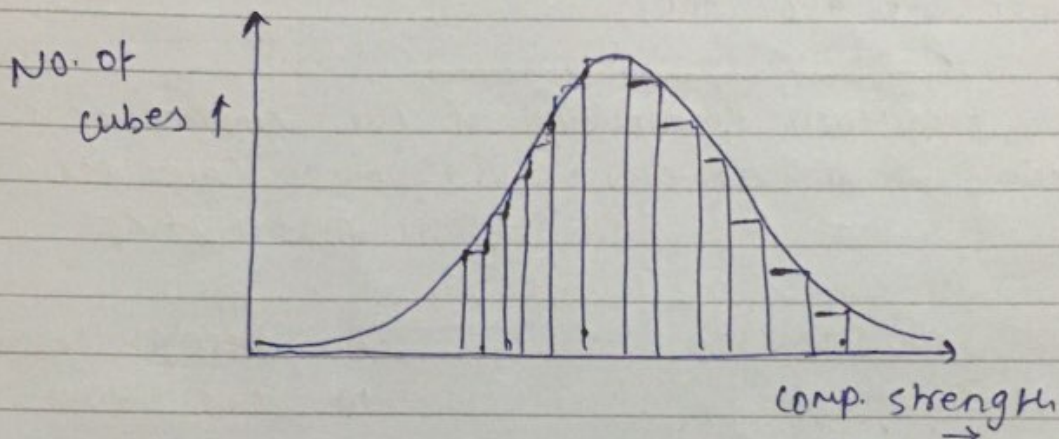
high strength concrete $\rightarrow$ M65 to M100

NOTE M30 means characteristic compressive strength of 150 mm size cube @ 28 days is 30 N/mm^2

- IS 456:2000 may not be applicable for concrete grade above M60

Characteristic strength of concrete :-

- When a large number of test result of concrete cube are plotted it follows a Normal distribution curve
- Characteristic strength is the strength below which not more than 5% results are expected to fall



$\sigma \rightarrow$ standard deviation

Minimum sample required for standard deviation calculation is 30 and

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (f_i - f_m)^2}{N-1}}$$

7 days $\rightarrow 0.7 f_{cu}$
28 days $\rightarrow f_{cu}$
6 months $\rightarrow 1.2 f_{cu}$

- concrete is designated by characteristic cube strength @ 28 days

NOTE cement hydrates and gain strength over a long period of time and hence strength of concrete increases with time.

However this increased in strength is not considered as per IS 456:2000

Que. The frequency distribution of the compressive strength of 100 sample is as follows. Calculate f_{ck} , f_m , σ and K ? where $f_m = f_{ck} + K\sigma$

NO. of samples	comp. strength (N/mm^2)
2	10
3	14
9	16
11	18
14	20
15	22
21	24
11	26
9	28
3	30
2	32

$$f_{ck} = 15 \text{ MPa}$$

$$f_m = f_{ck} + K\sigma$$

$$f_m = \frac{2 \times 10 + 3 \times 14 + \dots + 2 \times 32}{100} = 22.10 \text{ N/mm}^2$$

$$22.10 = 15 + K \times \sigma$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (f_i - f_m)^2}{N-1}}$$

$$\sigma = \sqrt{\frac{(10-22.1)^2 \times 2 + (14-22.1)^2 \times 3 + \dots + (32-22.1)^2 \times 2}{100-1}}$$

$$\sigma = 4.5$$

$$f_m = f_{ck} + K \sigma$$

$$22.1 = 15 + K \times 4.5$$

$$K = 1.57$$

Compressive strength of concrete in structure :-

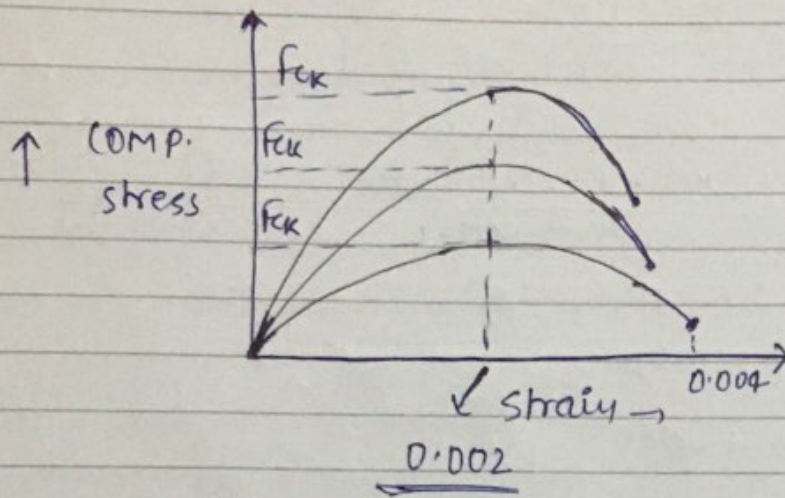
It is not equal to the strength obtained from the compression test of cube due to many factors like Duration of loading, size of member (size effect) Multiaxial state of stress and strain gradient

- Strength decreases with increase in size
- However after a certain value, it is almost constant
- Compressive strength in concrete in structure

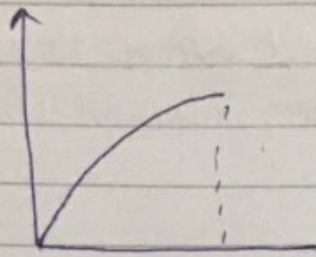
$$= 0.85 f_{cylinder}$$

$$= 0.85 \times 0.8 f_{cube} \approx 0.67 f_{ck}$$

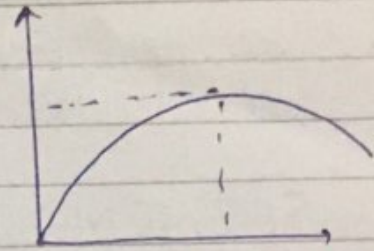
Stress-strain curve of concrete in compression:-



- Curve is generally linear upto $(\frac{1}{3} - \frac{1}{2})$ of peak stress
- Maximum compressive stress reached at a strain of 0.002
- Stress at point 0.002 (0.2%), strain is called compressive strain of concrete
- The strain at failure is in b/w 0.003 - 0.005 for usual grade of concrete.
- The higher the concrete grade the steeper is the initial portion and the sharper the peak of the curve, not flat at top.
- Low grade concrete is more ductile than high grade concrete. However in RCC structure Ductility of concrete is not considered.



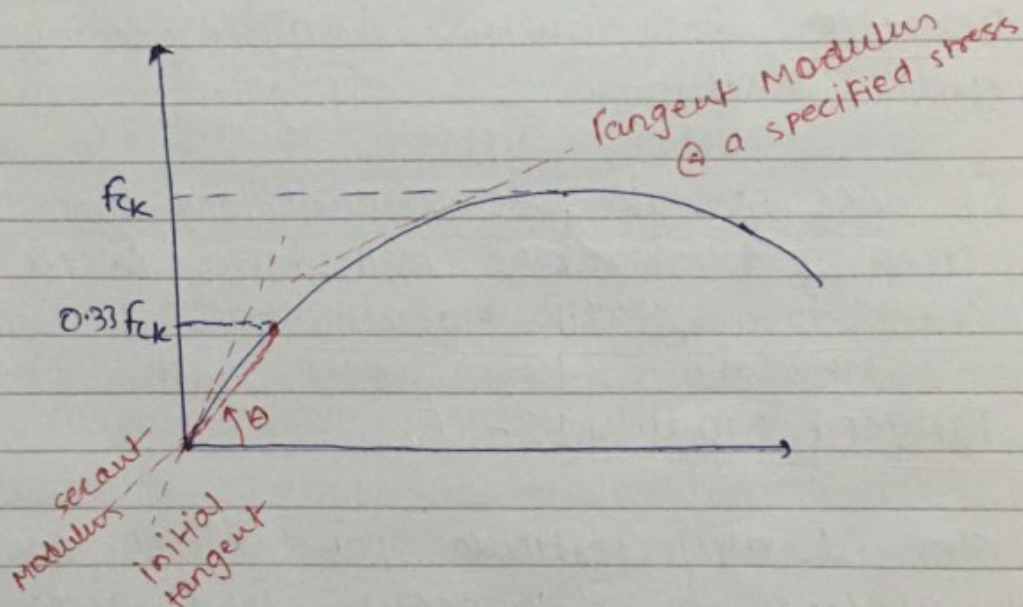
stress controlled



strain controlled

To obtain the descending part of curve, the applied load shall be strain controlled.

MODULUS OF ELASTICITY & POISSON'S RATIO :-



3 types of Modulus

① initial tangent modulus / dynamic modulus (E_d)

- Slope of tangent @ origin of σ - ϵ curve (corresponds to a small instantaneous strain)
- Can be used for dynamic load cases (EQ, wind) or very short period loading like impact loading. in which long term effect of creep & shrinkage can be neglected.

- Can be obtained by NDP (Non-Destructive Electrodynamical Method) / Resonance frequency Method)

[2] SECANT MODULUS :- (E_c)

Slop of line drawn from origin to a specified stress level, Here it is $0.33 f_{ck}$ as per

IS code $E_c = 5000 \sqrt{f_{ck}}$ ($\pm 20\%$ Variation)

$\downarrow$
MPa

$\downarrow$
MPa

- E_c is used to calculate instantaneous elastic Deflection for structure Analysed using linear static Analysis.
- E_c does not take into account long term effect of creep and shrinkage and hence called as Short term static Modulus

[3] Tangent Modulus :- (E_t)

Slop at any particular point on the curve, it applies to changes in load from a particular load level.

NOTE $E_c \uparrow$, Grade of concrete $\uparrow$

- IS code uses secant modulus / short-term static modulus.
- Generally dynamic modulus $>$ short term static modulus
- Modulus of Elasticity is primarily influenced by the elastic properties of aggregate and to a lesser

E_c mostly depends ON CA

by extent ~~of~~ curing, age of concrete, the mix proportion and type of cement.

Poisson's Ratio

0.2 for design
but can vary b/w 0.1 - 0.3

Tensile strength of concrete:-

Concrete is very weak in Tension and direct Tensile strength is about 10% (7-15%) of compressive strength.

Tensile strength of concrete generally not considered in RCC Design.

Cracking in concrete is as a result of tensile strength being more than permissible tensile stress.

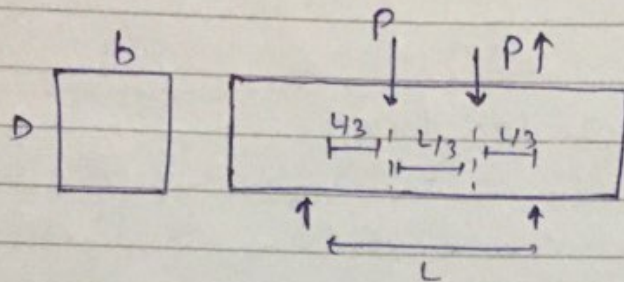
- σ - ϵ curve in Tension is always linear from origin to failure point and Modulus of Elasticity is same as that in compression.

Difficult to perform direct tension test and hence indirect test are done.

Stress $\rightarrow$ Applied load

permissible stress $\rightarrow$ strength of material

Tensile strength / Flexural Tensile strength (Modulus of Rupture) :-



$$\sigma = \frac{M}{Z} = \frac{6M}{bd^2}$$

$$M = \sigma Z$$

$$\sigma = f_{cr} = \text{Modulus of Rupture}$$

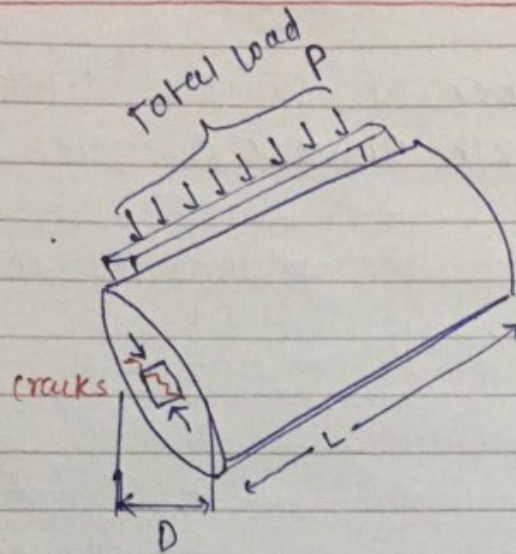
$$f_{cr} = 0.7 \sqrt{f_{ck}}$$

$$M_{cr} = f_{cr} \cdot Z$$

$M_{cr} \rightarrow$ cracking Moment

- pcc Beam is simply supported and 3rd point loading is applied
- Linear stress distribution is assumed across the c/s
- Theoretical Max. Tensile stress in the extreme tensile fibre is called as Modulus of Rupture or Flexural Tensile strength (f_{cr})
- f_{cr} is used to calculate Cracking Moment

Splitting tensile strength (f_{ct})



$$f_{ct} = \frac{2P}{\pi DL}$$

standard plane cylinder loaded on compression on its side along a diametral plane

- splitting tensile strength $f_{ct} = 0.66 f_{cr}$
- Direct tensile strength $= (0.50 - 0.625) f_{cr}$

$$f_{cr} (\text{flexure}) > f_{ct} (\text{split}) > f_t (\text{direct})$$

Acceptance criteria :-

- Due to difficulty of testing higher number of samples @ site, this criteria is adopted.
- Can be for compressive strength or flexural strength
- Mainly acceptance criteria for compressive strength is used

(i) Acceptance criteria for compressive strength

Consider the Number of sample depending on quantity of concrete volume as given below

Concrete (m^3)	Number of samples
1-5	1
6-15	2
16-30	3
31-50	4
≥ 51	4 + one additional for each 50 m^3 or part there of

1 Sample $\rightarrow$ 1 result

1 Sample = 3 specimen

• check the validity of test result of specimen and find out strength of each sample.

• The concrete shall be accepted for compressive strength only when the following conditions are satisfied.

Grade of concrete	individual Sample	Mean of 4 non overlapping consecutive test results (N/mm^2)
M15 or above	$\geq f_{ck} - 3$	$\geq \text{Max.} \begin{cases} f_{ck} + 0.025\sigma \\ f_{ck} + 3 \end{cases}$

$$f_{\text{sample 1}} \begin{cases} a_1 \\ b_1 \\ c_1 \end{cases} \quad \left(\frac{a_1 + b_1 + c_1}{3} \right)$$

$$f_{\text{sample 2}} \begin{cases} a_2 \\ b_2 \\ c_2 \end{cases}$$

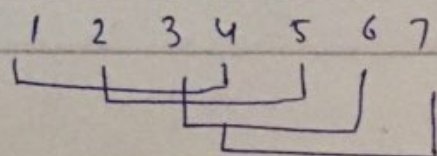
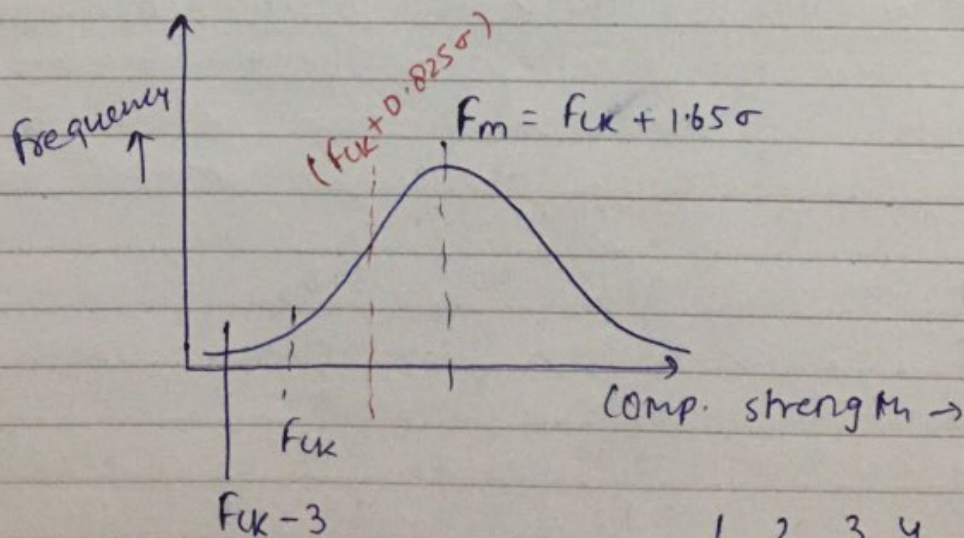
$$f_{\text{sample 3}} \begin{cases} a_3 \\ b_3 \\ c_3 \end{cases}$$

$$f_{\text{sample 4}} \begin{cases} a_4 \\ b_4 \\ c_4 \end{cases}$$

Must satisfy conditions.

* $f_{\text{sample}} > f_{ck} - 3$

* $\frac{f_{\text{sample 1}} + f_{\text{sample 2}} + f_{\text{sample 3}} + f_{\text{sample 4}}}{4} \geq \text{Max.} \begin{cases} f_{ck} + \\ 0.825 \sigma \\ f_{ck} + 3 \end{cases}$



Assume σ as per IS code, when NO data is available that is when Number of sample is ≤ 30

Grade of concrete	σ (N/mm ²)
M10, M15	3.5
M20, M25	4.0
M30 to M60	4.5 5.0

The above σ is valid for good quality control of materials. If quality control is not good then increase each σ by 1 N/mm²

*

comp. strength fails $\rightarrow$ NDT
 NDT fails $\rightarrow$ load test
 load test fails $\rightarrow$ demolish

Que For a concrete wall of 60 m³, 5 samples are taken the compressive strength of each specimen is given below, check the acceptance criteria of concrete for compressive strength, grade $\rightarrow$ M30

$\sigma = 5$

Sample NO.	specimen (N/mm ²)		
1	37.6	40.3	39.6
2	29.3	31.2	32.1
3	30.4	29.7	30.5
4	30.0	33.2	29.8
5	39.2	37.1	36.3

Solⁿ.

Sample strength (N/mm ²)	0.85 f _{sample} (N/mm ²)	1.15 f _{sample} (N/mm ²)	Remark
39.16	33.29	45.04	Valid
30.86	26.23	35.49	Valid
30.20	25.60	34.73	Valid
31.00	26.35	35.65	Valid
37.53	28 31.90	43.16	Valid

Individual

Sample $\geq f_{ck} - 3 = 27$

0.K

0.K

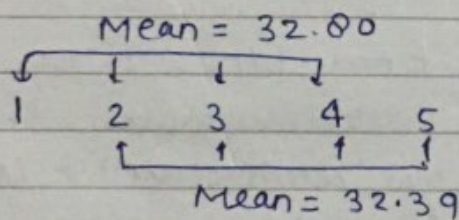
0.K

0.K

0.K

Mean of 4 $\geq$ max. $\left\{ \begin{array}{l} = 34.125 \\ f_{ck} + 0.825 \sigma \\ f_{ck} + 3 = 33 \end{array} \right.$

not $\geq$ 34.125 N/mm²



As the Mean of 4 consulative Sample result is less than 34.125 MPa, so the concrete is not accepted for compressive strength.

M₂₀ M60 → impermeable.
More
permeable.

Exposure, cover and minimum grade of concrete :-

Exposure :-

General environment to which concrete will be exposed during its working life.

- In the same structure, different members may be subjected to different exposure.

Cover :- (Excludes plasters and ~~work~~ finishers) ⇒

It is required to

- 1) protect reinforcement from corrosion (Durability)
- 2) protect reinforcement from fire
- 3) ensure proper bond b/w reinforcement & concrete without spalling of concrete.

Clear cover :-

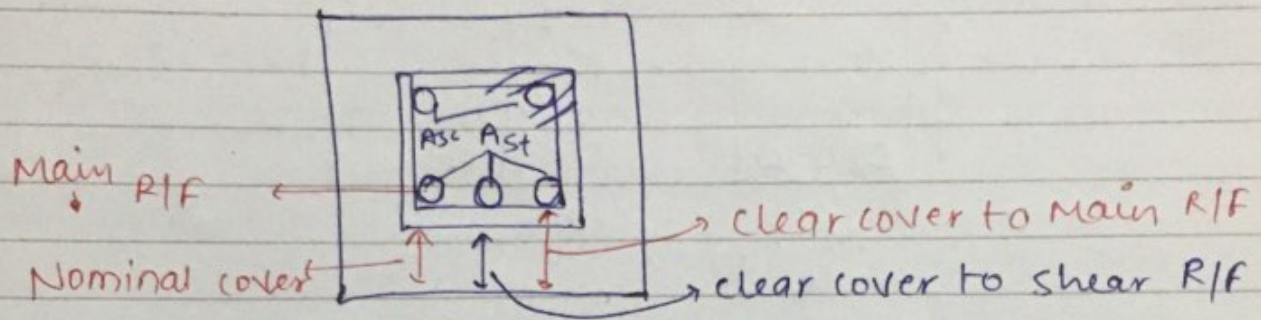
Shortest distance b/w concrete surface to the nearest surface of reinforcement.

Nominal cover :-

Design depth of concrete cover to all steel reinforcement, including links

It is the clear cover to the outer reinforcement (Stirrups, lateral ties, etc)

Nominal cover shall not be less than dia of Bar to develop proper bond.

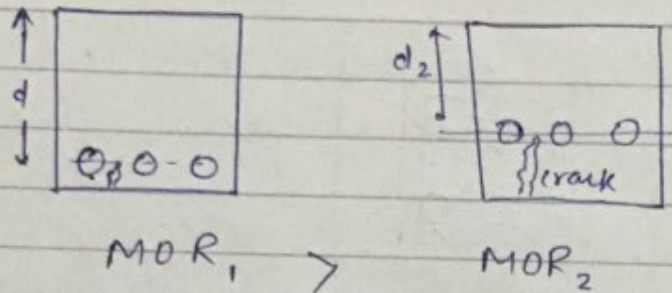


Minimum Grade of concrete and Nominal cover Based on Durability only (That is excluding fire resistance and spalling criteria for different exposure condition is as follows:-

Exposure	Nominal cover (mm)	Minimum Grade RCC	Minimum Grade PCC
Mild	20	M20	A
Moderate	30	M25	M15
Severe	45	M30	M20
Very severe	50	M35	M20
Extreme severe	75	M40	M25

NOTE • permeability of higher concrete grade is less
 • Nominal cover not applicable for pcc
 • For main R/F upto 12 mm dia Bar for mild exposure, Nominal cover may be reduced by 5mm
 Generally used in slab design with mild exposure

- Actual concrete cover shall not deviate from the required cover by +10 mm and -0 mm
- Too large cover reduces the moment capacity and increases the crack width (by reducing lever arm)



- for severe and very severe, 5 mm reduction in cover is permissible if M35 and above is used
- Completely immersed in sea water → severe
Exposed to sea water spray → very severe
coastal Environment → severe
- for very severe exposure condition the min. grade is M35. so if we provide M35 concrete then we can not reduce the required cover (However if we provide M40 or above then impermeability will increase and here we can reduce the cover by 5 mm) so we can conclude for severe exposure reduce the cover by 5 mm when M35 and above is provided and for very severe reduce the cover by 5 mm when M40 or above provided.

CONCRETE MIX DESIGN :-

Involves economical selection of relative proportion of various ingredients of concrete.

Concrete apart from meeting the strength criteria must also be workable (ease to work) in fresh state / green state and impermeable & durable in the solid state.

Normally classified as

- 1) Nominal Mix concrete
- 2) Design Mix concrete

① Nominal Mix concrete :-

- permitted ~~of two~~ upto M20 grade concrete only.
- Design Mix is not only preferred over Nominal mix but also economical compare to Nominal mix
- Nominal mix generally contains higher cement volume compare to that in same grade of design mix.
- proportioning of Nominal mix as per IS code is as follows-

Grade	Mass of (FA+CA) in 50 kg cement	FA: CA by mass	Max ^m water (in kg)
M5	800	Generally	60
M15	330	1:2 but can	32
M20	250	Vary b/w 1:1.5 to 1.25	30

For M15

Cement $\rightarrow 50$ kg

FA + CA $\rightarrow 330$ kg

Ratio assume 1:2

FA = 110, CA = 220

C : FA : CA

50 : 110 : 220

1 : 2.2 : 4.4

W = 32

$$W/C = \frac{32}{50} = 0.64$$

M5	$\rightarrow$	1:5:10
M7.5	$\rightarrow$	1:4:8
M10	$\rightarrow$	1:3:6
M15	$\rightarrow$	1:2:4
M20	$\rightarrow$	1:1.5:3

NOTE

• 1 bag of cement = 50 kg

and packed volume = 34.5 litres

• Water content can vary based on workability, Durability etc.

DESIGN MEX CONCRETE :-

Various steps as per IS 10262 : 2009

Step 1 : Determine target mean strength of concrete

Step 2 : select max. free W/C ratio from the Actual relationship if available based on the type of material used (like type of aggregate). or take it from durability consideration (IS 456) depending on Grade and exposure condition.

Exposure condition	Min grade R.C.C	Min cement in kg / m^3 of concrete	Max. free w/c
Mild	M20	300	0.55
Moderate	M25	300	0.50
Severe	M30	320	0.45
Very severe	M35	340	0.45
Extreme severe	M40	360	0.40

NOTE

- Free Water Means water excluding that absorbed by aggregate
- For purely chemical react i.e for complete hydration of cement w/c nearly equal to 0.25
- Cement content should not exceed $450 \text{ kg}/m^3$ of concrete to limit cracking due to drying shrinkage, reduce early thermal cracking or risk of damage due to Alkali silica reaction.

Step-3 select the water content based on workability requirement.

Nominal Max ^m size of C.A (mm)	Max ^m water content (litre)
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10

208

40

165

NOTE Normally the water content is $180 - 200 \text{ litre}/m^3$ of concrete.

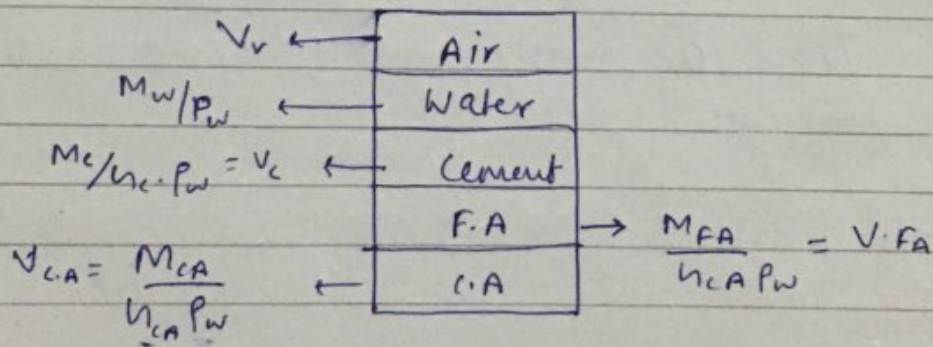
Step-4. Calculate cementitious material based on $f_{be}/w/c$ ratio (step-2) and Water content (step-3). The cement provided shall satisfy the Mini. and Max^m cement criteria

Step-5. Estimate CA proportion in total mix by volume using following table.

Nominal Size of CA	Volume of CA Volume of (CA+FA)			
	Zone IV F.A	Zone III F.A	Zone II F.A	Zone I F.A
10 mm	0.50	0.48	0.46	0.44
40 mm	0.75	0.73	0.71	0.69

NOTE Zone I Sand is coarser than Zone IV

Step-6 Estimate the quantities of CA and FA by absolute volume principle.



$$\frac{M_{CA}}{w_{CA} \gamma_w} + \frac{M_{FA}}{w_{FA} \gamma_w} + \frac{M_s}{w_s p_w} + \frac{M_w}{p_w} + V_v = 1$$

$$\rho_{\text{absolute}} = \rho_{\text{bulk}} (1+e)$$

$$\begin{aligned} \rho_{\text{bulk}} &= \frac{M}{V_{\text{total}}} = \frac{M}{V_{\text{solid}} + V_{\text{voids}}} = \frac{M / V_{\text{solid}}}{\frac{V_{\text{solid}} + V_{\text{voids}}}{V_{\text{solid}}}} \\ &= \frac{\rho_{\text{abs}}}{1+e} \Rightarrow \rho_{\text{absolute}} = \rho_{\text{bulk}} (1+e) \end{aligned}$$

NOTE :- As per IS 10262 : 2009, NO air / voids shall be considered for normal concrete.

- When nothing is mentioned take C:F:A:L:A by mass
- If by volume is given then it should be by bulk vol.
- W/c is generally given by mass.

Que. Data of material for making cement concrete is—

- 1) $W/c = 0.46$ by mass
- 2) entrained Air = 2.1.

3) Material	specific gravity	Bulk density (kg/m^3)	Proportion in mix by volume
Cement	3.15	1500	1
F.A	2.61	1620	1.359
C.A	2.70	1530	2.79

Determine the following

- a) absolute volume of fully compacted fresh concrete (ignoring Air content) produced by 1 bag of cement of 50 kg.
- b) Quantity of Materials to make 1 m^3 of concrete

a) $M_{\text{cement}} = 50 \text{ kg}$

$$\text{Volume}_{\text{cement}} = \frac{50}{1500} \text{ m}^3$$

$$\text{Vol. F.A} = \frac{50}{1500} \times 1.359 \text{ m}^3$$

$$\begin{aligned} \text{Mass F.A} &= \frac{50}{1500} \times 1.359 \times 1620 \text{ kg} \\ &= 73.38 \text{ kg.} \end{aligned}$$

$$M_{\text{CA}} = \frac{50}{1500} \times 2.79 \times 1530 = 142.2 \text{ kg.}$$

$$M_w = 0.46 \times M_{\text{cement}} = 0.46 \times 50 = 23 \text{ kg}$$

$$\frac{M_c}{\gamma_c \gamma_w} + \frac{M_{\text{FA}}}{\gamma_{\text{FA}} \gamma_w} + \frac{M_{\text{CA}}}{\gamma_{\text{CA}} \gamma_w} + \frac{M_w}{\rho_w} = 0.1196 \text{ m}^3 \quad \underline{A}$$

$$\text{b) } \frac{M_c}{\gamma_c \gamma_w} + \frac{M_{\text{FA}}}{\gamma_{\text{FA}} \gamma_w} + \frac{M_{\text{CA}}}{\gamma_{\text{CA}} \gamma_w} + \frac{M_w}{\rho_w} + \boxed{V_a} = 1$$

0.02

$$\frac{M_c}{\gamma_c \gamma_w} + \frac{M_{\text{FA}}}{\gamma_{\text{FA}} \gamma_w} + \frac{M_{\text{CA}}}{\gamma_{\text{CA}} \gamma_w} + \frac{M_w}{\rho_w} = 0.98 \text{ m}^3$$

$$0.1196 \text{ m}^3 \text{ of concrete} \rightarrow 50 \text{ kg}$$

$$0.98 \text{ m}^3 \text{ of concrete} \rightarrow \frac{50}{0.1196} \times 0.98$$

$$= 409.69 \text{ kg.}$$

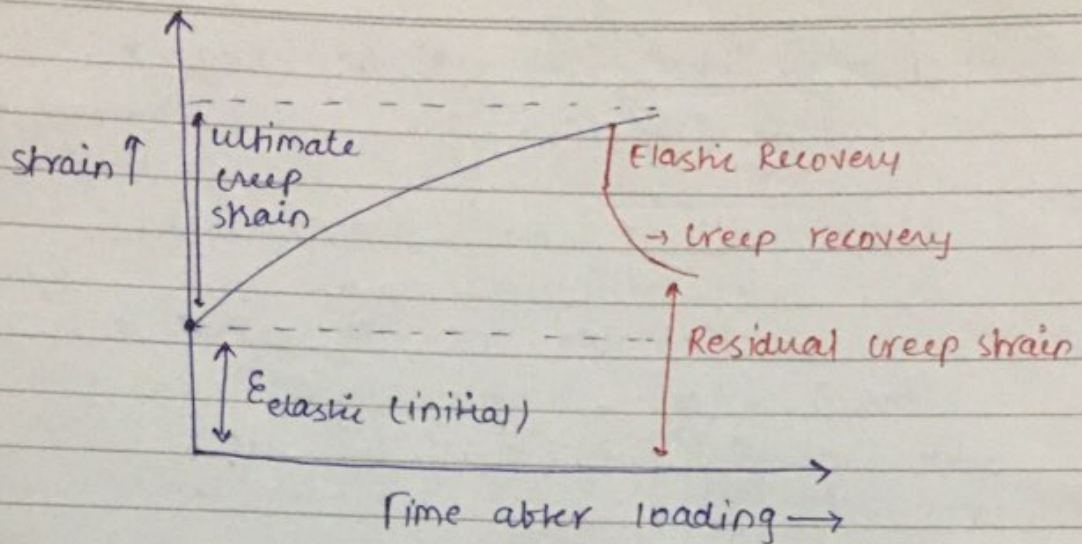
$$M_{FA} = \frac{73.38}{0.1196} \times 0.98 = 601.33 \text{ kg.}$$

$$M_{CA} = \frac{142.29}{0.1196} \times 0.98 = 1165.92 \text{ kg}$$

$$M_w = \frac{23}{0.1196} \times 0.98 = 188.46 \text{ Kg.}$$

CREEP OF CONCRETE :-

- Under sustained compressive loading (can be tensile also) deformation in concrete increases with time even though the applied stress level is not changed
- The time dependent component of strain is called creep
- creep is generally due to dead load + permanent LL
- creep depends on applied stress level i.e. higher the applied stress, higher is the creep.
- Exact Mechanism of creep is not clear. But it is generally attributed to
 - 1) internal movement of adsorbed water
 - 2) sliding b/w gel particles (C-S-H gel)
 - 3) Moisture loss
 - 4) Growth in microcracks.



$$\text{Creep coefficient } (C_t) = \frac{\text{Creep strain at any time } t}{\text{Elastic strain } (\epsilon_i)}$$

$$\begin{aligned} \text{Total strain at any time 't'} &= \epsilon_i + \epsilon_i \times C_t \\ &= (1 + C_t) \epsilon_i \end{aligned}$$

$$\text{Ultimate creep coefficient } (C_\infty) = \frac{\text{Ultimate creep strain}}{\text{Elastic strain}}$$

$\epsilon_i > \epsilon_r \rightarrow$ for concrete

Age $\uparrow$ $f_{ck} \uparrow$

$$\delta_i = \frac{5}{384} \frac{w l^4}{EI} \quad ; \quad E = 5000 \sqrt{f_{ck}} \quad (10 \text{ day})$$

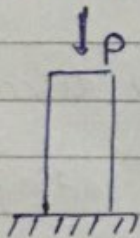
$$\delta_i \downarrow = \frac{5}{384} \frac{w l^4}{EI} \quad , \quad E = 5000 \sqrt{f_{ck}} \uparrow \quad (50 \text{ years})$$

NOTE

• Due to $E \uparrow$ of concrete with time Recovery (Elastic) will be less than deformation at 0 days.

• As long as the stress in concrete doesn't exceed $\frac{1}{3}$ rd of its characteristic compressive strength (that is generally in service load condition) creep may be assumed to be proportional to the stress. This enables us to calculate the total deflection (initial + creep) by the linear elastic principle with reduced modulus.

Que Calculate the total deflection i.e. initial + creep for the below given structure assuming ultimate creep coefficient as θ and linear elastic material for the given load.



$$\delta_{\text{total}} = \delta_{\text{initial}} + \delta_{\text{creep}}$$

$$\theta = \frac{\text{ultimate creep strain}}{\text{Elastic strain}}$$

$$\delta_{\text{initial}} = \frac{PL}{AE} = \delta_{\text{elastic}}$$

$$\epsilon_{\text{elastic}} = \frac{P}{AE}, \quad \epsilon_{\text{creep}} = \frac{P}{AE} \theta$$

$$\delta_{\text{creep}} = \frac{PL\theta}{AE}$$

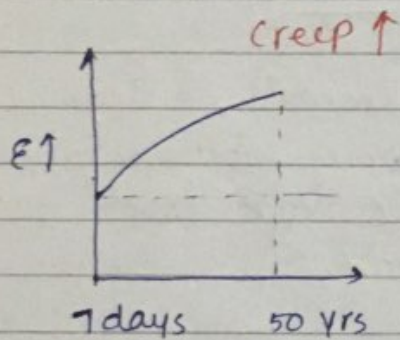
$$\delta_{\text{total}} = \frac{PL}{AE} + \frac{PL\theta}{AE} = \frac{PL}{AE} (1 + \theta) = \boxed{\frac{PL}{A(E/1 + \theta)}}$$

- * long term modulus of Elasticity including the effect of creep is E_{cc} and $E_{cc} = \frac{E_c}{1+\theta}$

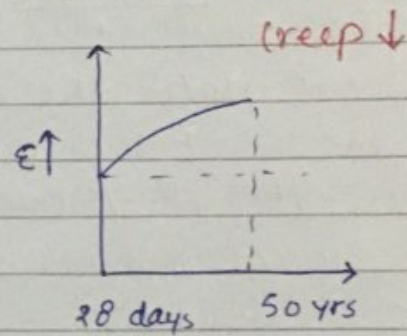
$E_c \rightarrow$ short term modulus

$\theta \rightarrow$ ultimate creep coefficient

- E_{cc} is not the actual property of concrete, it is the reduced or effective modulus of Elasticity taking into account long term effect of creep.



↓ time after loading →
Age @ loading



↓ time after loading →
Age @ loading

Age @ loading

θ

7 days

2.2

28 days

1.6

1 year

1.1

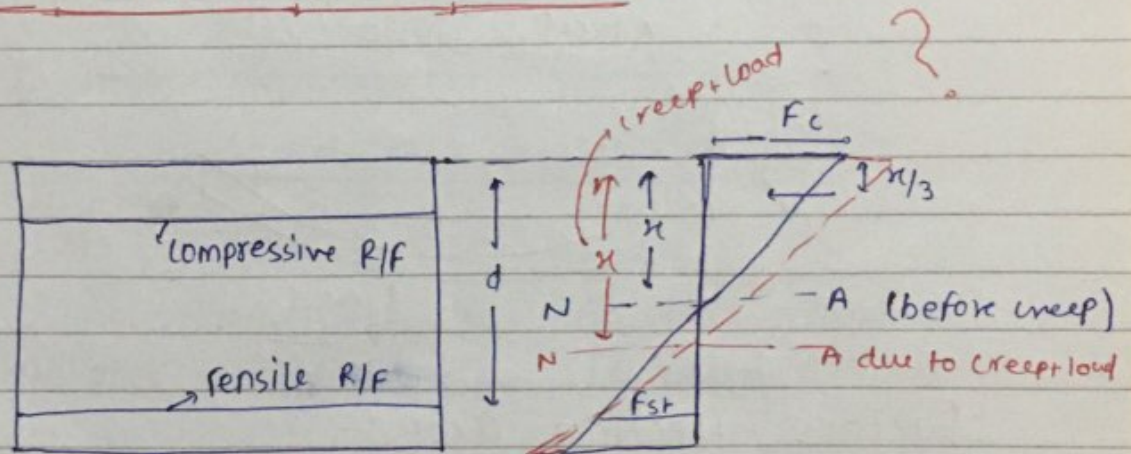
- for a different age of loading θ can be obtained by interpolation assuming that $\theta \downarrow$ linearly with the logarithmic of time in case days
(interpolate b/w 2 range)

EFFECTS OF CREEP:-

• The detrimental effects are -

- 1) Increased deflection of Beams and slabs
- 2) Increased deflection of cylinder column
- 3) loss of prestress
- 4) Gradual transfer of load from concrete to r/f steel in compression member

Effects of creep in Flexure Member :-



$$C = 0.5 F_c \times b, \text{ MOR} = 0.5 F_c \times b (d - x/3) = B.M$$

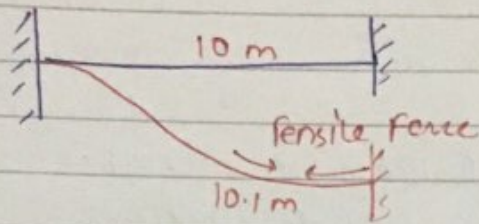
$$T = F_{st} \cdot A_{st}$$

$$C = T$$

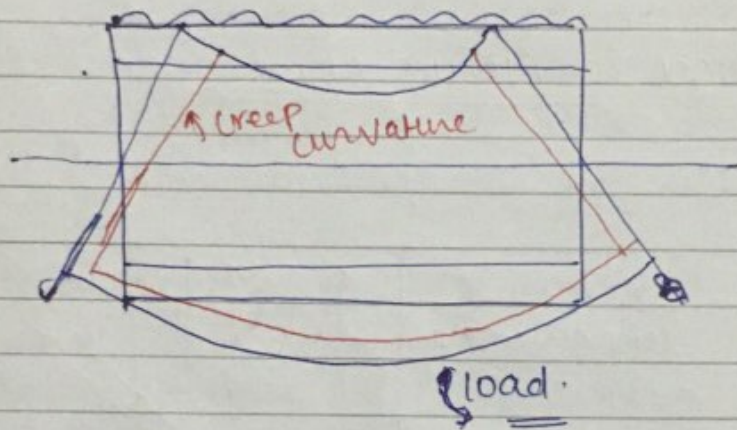
NOTE:- • Due to creep, compressive strain in compression steel $\uparrow$, and tensile strain in tensile steel $\uparrow$

- presence of compression r/f reduces creep effect
- some creep effects are beneficial like reduction of stress due to support movement.

- Creep deflection and Elastic deflection are additive



Due to tensile creep the length of member will increase and is beneficial for support & Member



Factors affecting creep :-

In General creep $\uparrow$ When

- 1) Cement content is high
- 2) Aggregate content is low
- 3) W/c Ratio is high
- 4) High Volume of Air
- 5) RH is low
- 6) Temp. is high
- 7) size / Thickness of Member is small
- 8) loading occurs @ an early age
- 9) loading is sustained over a long period of time

Shrinkage in concrete :-

- It results from volume change of concrete
- Like creep, shrinkage introduces time dependent strain in concrete, unlike creep and shrinkage strain are independent of applied stress
- Generally 5 types of shrinkage
 - 1) chemical shrinkage
 - 2) plastic shrinkage
 - 3) Autogeneous shrinkage
 - 4) drying shrinkage
 - 5) carbonation shrinkage

1) chemical shrinkage :-

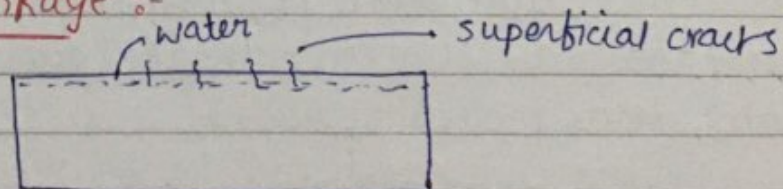
Absolute volume of
unhydrated materials



Absolute volume of
hydrated materials

- Due to chemical reaction i.e. hydration of cement
- in early stage → volume reduction
in later stage → voids creation.

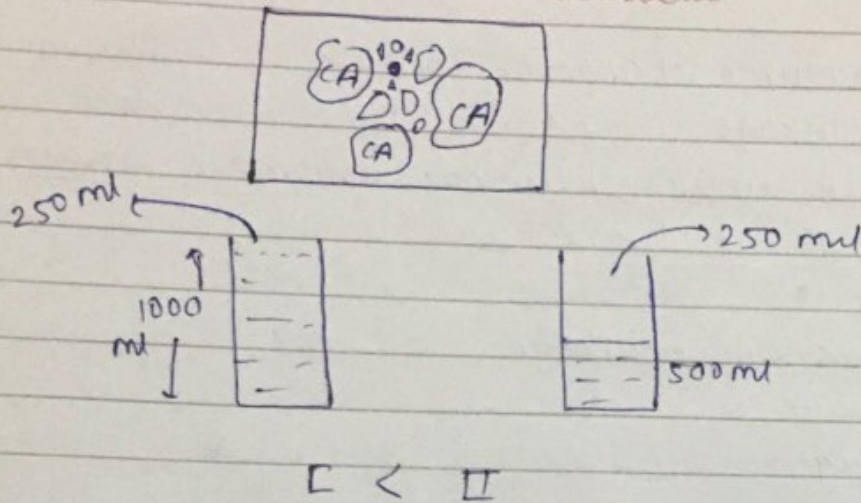
2) plastic shrinkage :-



Due to loss of moisture from top surface short term process -

cracks develop are generally on the top surface
shrinkage before concrete gets harden.

iii) Autogenous shrinkage :-



- Volume reduction if no moisture transfer outside
- Mainly due to self desiccation of cement (Extreme state of dryness)
- Resulting in rise in capillary pressure
- Increases as the grade of concrete increases
- Generally occurs in early days.
- ~~drying~~ short term process.

4) Drying shrinkage :-

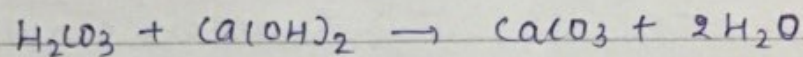
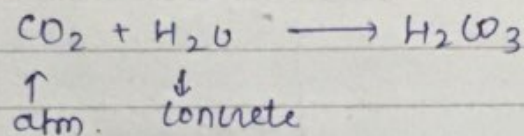
- Contraction of the hardened concrete due to loss of water from the pores.
- The Most imp. type of shrinkage
- decreases as the grade of concrete $\uparrow$ *
- long term process

5) Carbonation shrinkage :-

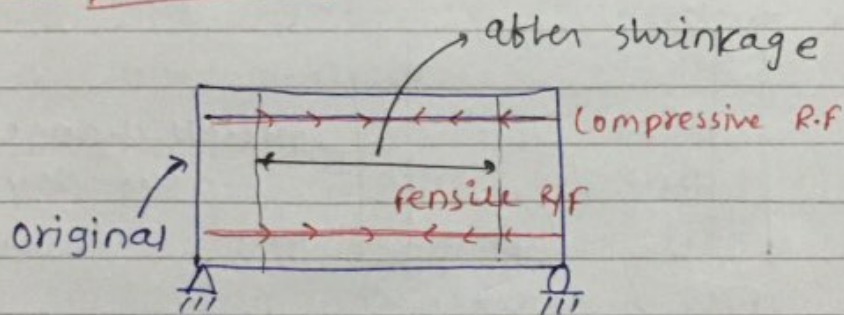
As the Ca(OH)_2 from concrete $\downarrow$, its pH $\downarrow$ and chances of corrosion $\uparrow$

Major factors that affects carbonation shrinkage are permeability of concrete, Moisture content, RH and amount of CO_2 in the air

The rate of carbonation shrinkage is very slow.

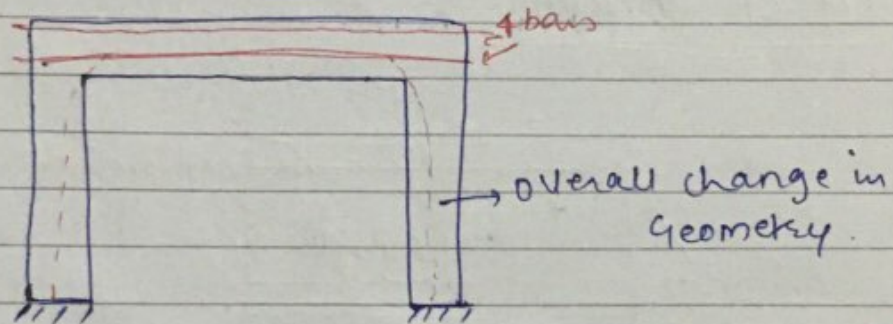
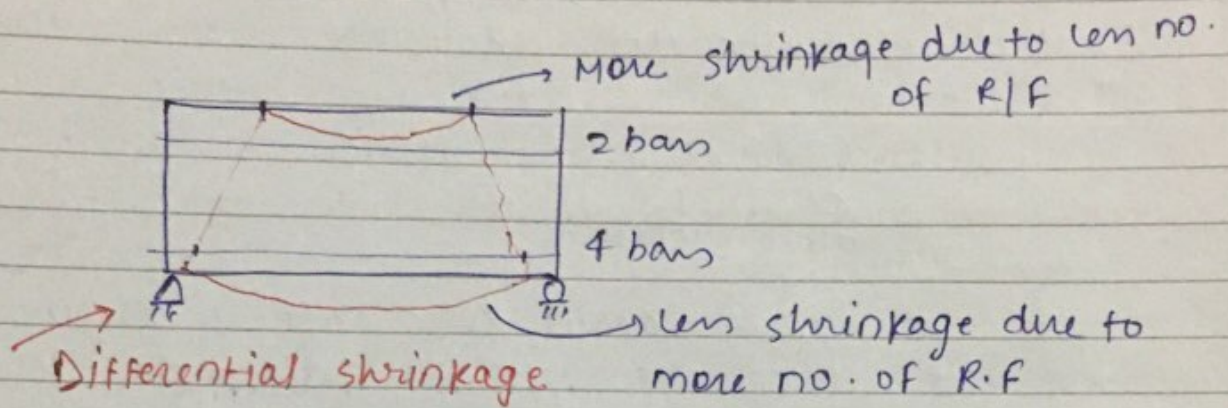
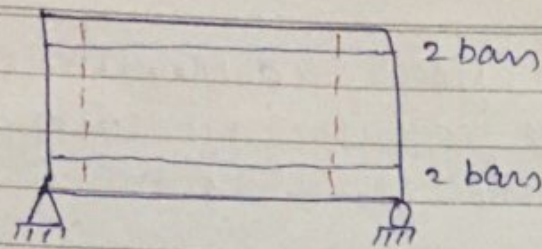


Effect of shrinkage :-

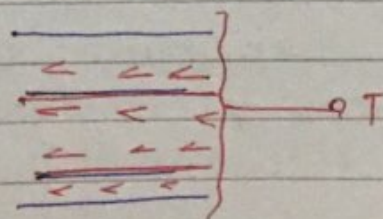
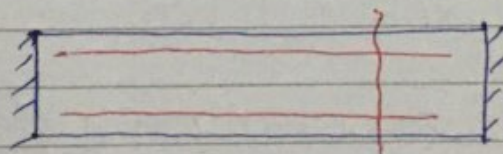


In unstrained RCC member, shrinkage of concrete results in shortening of the member. However the embedded R/F resist the shortening and as a result compressive stress is developed in both compressive R/F and Tensile R/F

- For symmetrically placed R/F (that is equal compressive and Tensile R/F), shrinkage does not result in any curvature of the member for determinate structure. However in indeterminate structure shrinkage results in overall change of geometry.

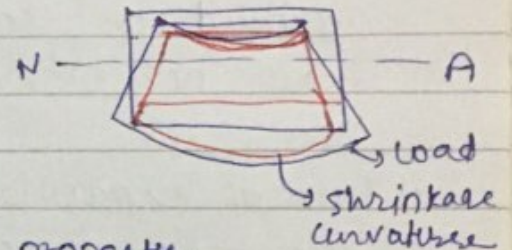


- In restrained RCC member shrinkage develop ~~the~~ tensile stress in concrete and compressive stress in both Tensile & compressive R/F



- For unsymmetrically placed RIF (line in the flexural member) curvature results due to differential shrinkage

- Both flexural curvature and shrinkage curvature are additive hence the long term deflection increases due to shrinkage.



Factors affecting shrinkage:-

All factors related to Material property, Composition, of Mix, curing and environmental condⁿ. Member size and age that affect creep also effect shrinkage

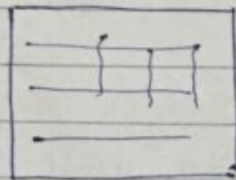
- Also shrinkage is reversible to great extent (i.e. alternative volume (ch. dry & wet condⁿ) will cause alternating vol. change in concrete.

Shrinkage for design:-

- For Normal concrete generally drying shrinkage dominates
- Generally half of the total shrinkage occurs in first month and $\frac{3}{4}$ in first 6 months.
- In the absence of test data the absolute value of shrinkage strain for design shall be taken as 0.0003 (3×10^{-4})

Temperature Effect:-

- Thermal contraction produces similar effect as shrinkage
- Some Nominal reinforcement (Temp. & shrinkage RF) provided in concrete to reduce cracks due to temp. and shrinkage.
- Thermal expansion coefficient of Steel and concrete are nearly equal and generally No Relative movement b/w the steel & concrete



REINFORCEMENT:-

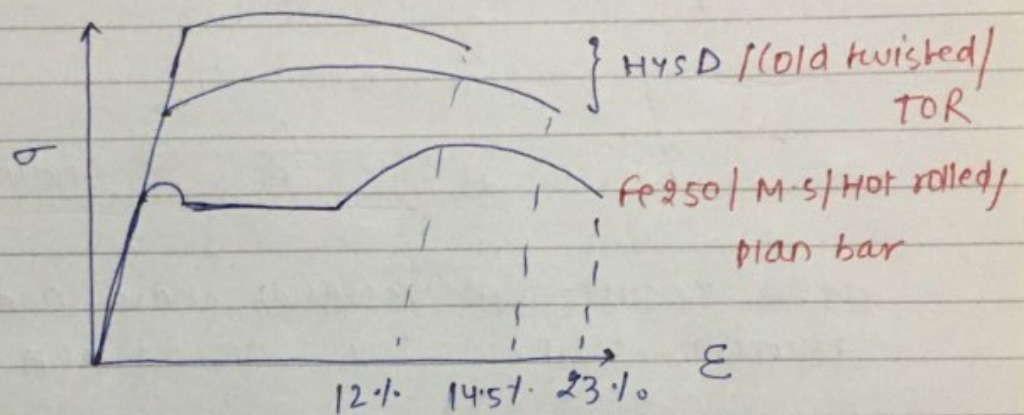
reinforcement (Generally steel) is provided to account for negligible tensile strength of concrete. Also provides ductility to concrete

Grades and type of steel :-

- RF Generally used are Fe250, Fe415, Fe500
- Can be Mild steel (Plane bar) Fe250 or HYSD (High yield strength deform bars) Fe415, 500
- HYSD Bars can be produced from CTD (Cold Twisted deform) or TMT (Thermo Mechanically treated)

M.S and HYSD (CTD)

Grade	yield stress	ultimate stress	% elongation at rupture
Fe 250	250	412	23 %
Fe 415	415	485	14.5 %
Fe 500	500	545	12 %

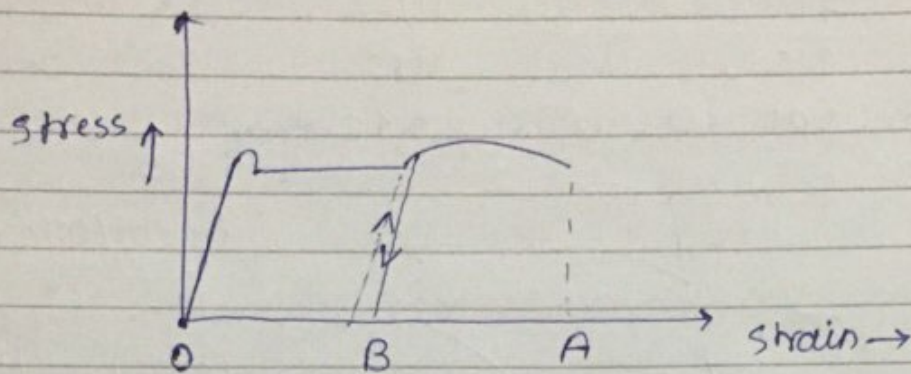


- The yield strength describe above is the Minimum Guaranteed value. Actual strength can be higher
- The specified yield strength of steel is taken as The characteristic strength of steel
- for all Grade of steel, Modulus of elasticity both in tension and compression is taken as 2×10^5 mpa

Cold Working (CTD Process)

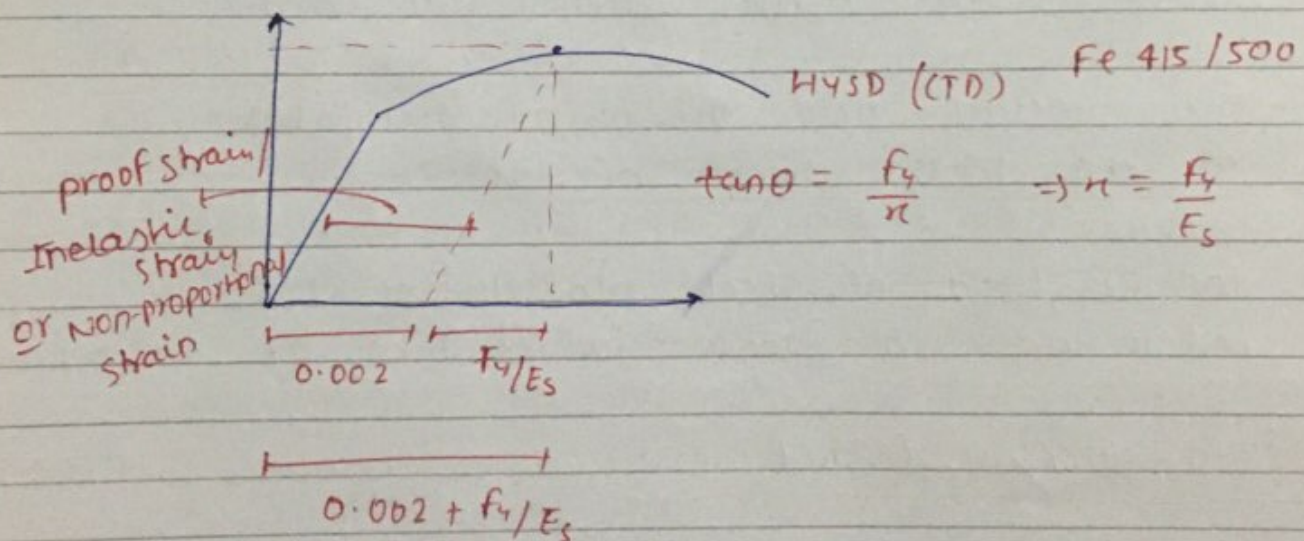
stretching and twisting of higher grade mild steel beyond the plastic zone and subsequently releasing the load.

It eliminates the yield plateau, (However Decreases the margin b/w ultimate & yield strength)



Also results in residual strain and lowers the ductility. that is from OA to BA

In HYSD (TD) Bars yield strength is taken as 0.2% proof stress



proof stress is the stress @ which a non-proportional strain of 0.002 is there or it is the stress which on unloading results in a residual strain of 0.002

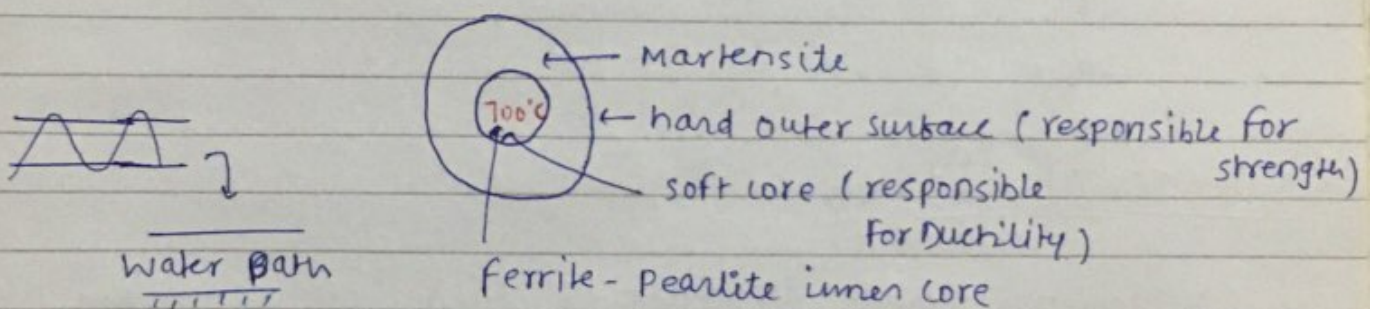
indue stress $\rightarrow$ during formation

Disadvantage OF HYSD CTD :-

- low ductility and difficulty in Bending
- Corrossion due to indue stress
- low fire resistance, low weldability.
- Not good for stress reverstion (dynamic loading)

TMT BAR :-

- Hot rolled Hysd BARS which are rapidly cooled by quenching process (Water Bar)
- σ -E curve of TMT BAR is similar to that of M.S Bar



Advantages :-

- Higher Ductility (Almost like M.S BAR)
- less corrosion
- Good fire resistance & weldability
- Good for stress reverse

size OF R/F :-

Reinforcing bars (rebars) are available in Nominal diameter from 5mm to 50 mm

✓ $\gamma_s \rightarrow 7850 \text{ kg/m}^3$

- Nominal dia of a bar is dia of a plane round bar Having the same mass per meter length as the deform bar.

• Most commonly used bar and their Area :-

Bar dia (mm)	Area	
6	28	—X
* 8	50	
10	78.5	
* 12	113	
* 16	201	
20	314	
25	490	
28	615	—X

Design Method :-

- 3 design methods in order of their evolution are
- 1) Working Stress Method (WSM)
 - 2) Ultimate Load Method (ULM)
 - 3) Limit State Method (LSM)

2] WSM

Introduction:-

- This was the traditional method of design
- Structural Materials are assumed to behave in linear elastic manner i.e. $\sigma = \epsilon E$
- Induced stresses in materials are kept below the permissible stresses due to working load for safety.
- No load factor considered
- Structure analysed at working load / service load
- Also called as Modulus ratio Method

Permissible stress:-

$$\text{permissible stress} = \frac{\text{strength of material}}{\text{factor of safety}}$$

permissible stress in concrete:-

Grade of concrete	Bending Compression $\approx \frac{F_{ck}}{3}$ MPa σ_{cbc}	Direct compression $\approx \frac{F_{ck}}{4}$ MPa σ_{cc}
M20	7	5
M25	8.5	6
M30	10	8

NOTE In case of Bending / Flexure, permissible stress (WSM) / permissible strain (LSM) for concrete is higher compare to direct stress as chances of redistribution is more in flexure

Permissible stress in steel :-

Type of stress	Fe250 (MPa)	Fe415 (MPa)
Tensile ($\phi \leq 20\text{mm}$)	140 $\rightarrow 0.55 f_y$	230
Tensile ($\phi > 20\text{mm}$)	130	230
Compressive	130	190

Due to better strain distribution across the c/s in smaller dia bar, higher permissible stress can be taken.

The above permissible stresses in steel are not applicable for Water tank design with WSM

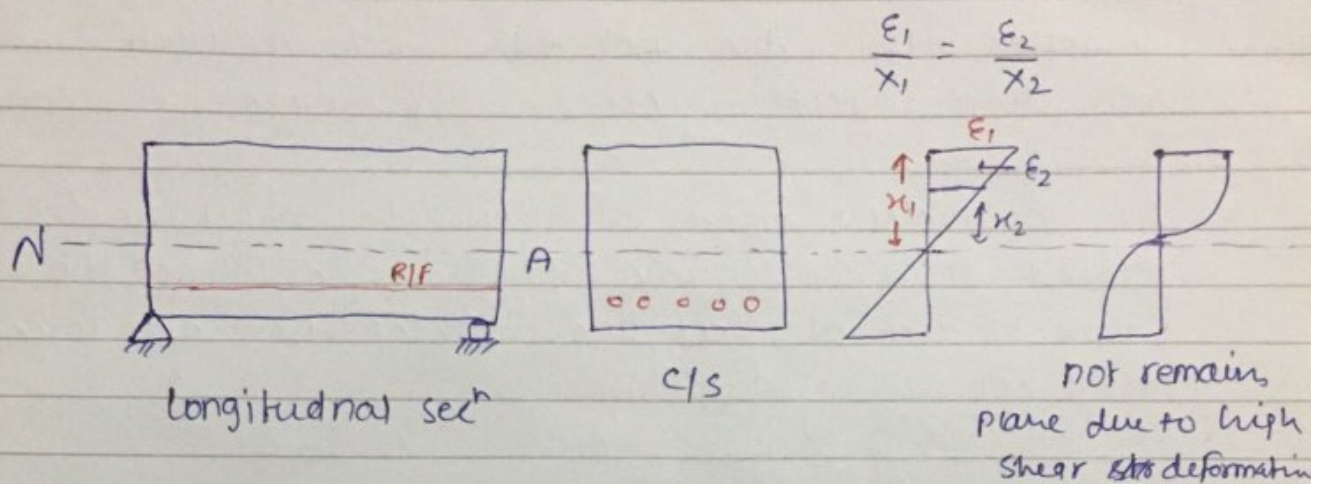
$\sigma_{sr} \rightarrow$ permissible stress in steel

NOTE In extreme loading condition like DL+LL+EO/WL the permissible stress in material is increased by $33\frac{1}{3}\%$ as the probability of occurrence of the extreme loading is very less

Assumption in Flexure :-

- 1) plane section before bending remains plane after bending i.e strain varies linearly over the depth of section

- Here shear deformations are neglected.



- strain compatibility is assumed i.e. that strain in Reinforcing steel is assumed to be equal to that in the adjoining concrete to which it is bonded.
- for both concrete & steel stress $\propto$ strain (linearly)
- All tensile stresses are resisted by the R/F and none by concrete (except in the uncracked phase)
- Modular Ratio is considered as $m = \frac{280}{3\sigma_{cbc}}$
- stress

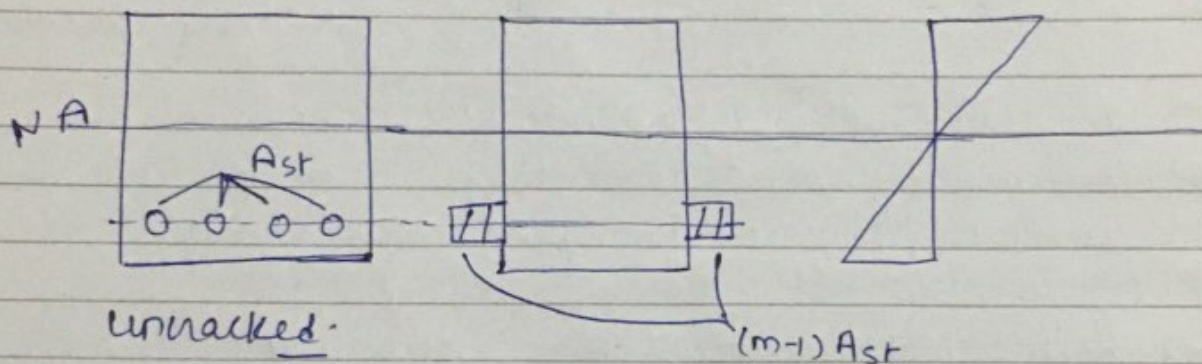
Modular ratio and transformed sections:-

In elastic theory, the composite secⁿ is transformed into a single secⁿ i.e. in this case equivalent concrete section.

- short term Modulus ratio $m = \frac{E_s}{E_c}$

However this does not take into account long term effect of creep & shrinkage.

- The equivalent area of concrete shall be located at the centroid of steel, developed the same strain at steel and develop the same force on steel.



$$\text{Force in steel} = f_{st} \cdot A_{st}$$

$$= E_s \epsilon_s \cdot A_{st}$$

$$\Rightarrow A_{con-eg} = \frac{E_s}{E_c} \cdot A_{st}$$

$$E_s \epsilon_s A_{st} = f_{chp} A_{con-eg}$$

$$E_s \epsilon_s A_{st} = E_c \epsilon_c A_{con-eg}$$

$$A_{st} = \frac{E_c}{E_s} A_{con-eg}$$

$$A_{st} = m A_{con-eg}$$

$$m = \frac{E_s}{E_c} \rightarrow \text{short term}$$

$$m = \frac{200}{3\sigma_{chc}} \rightarrow \text{long term}$$

M25

$$M_{\text{short}} = \frac{E_s}{E_c} \rightarrow \frac{2 \times 10^5}{5000 \sqrt{f_{ck}}} = 8$$

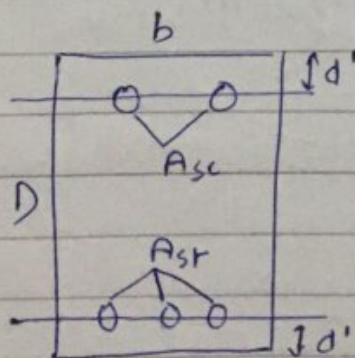
$$m_{\text{long}} = \frac{280}{3 \sigma_{cbc}} = 10.98$$

$$m_{\text{long}} > m_{\text{short}} =$$

• partially taking into account long term effect such as creep, the long term Modulus ratio $m = \frac{280}{3 \sigma_{cbc}}$

~~•~~ • Modulus ratio $m = \frac{280}{3 \sigma_{cbc}}$ is applicable for

flexure R/F only. for compression R/F Modulus ratio $m' = 1.5m$. This is because long term effect of creep & shrinkage of concrete and non-linearity at higher strain in concrete results in much larger compressive stress in compressive R/F than that calculated using linear elastic theory and Modulus ratio $m = \frac{280}{3 \sigma_{cbc}}$.

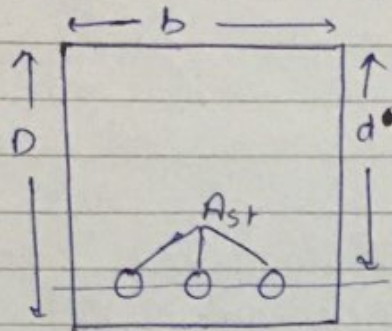


$$= bD + (m-1)A_{st} + (1.5m-1)A_{sc}$$

Uncracked

Uncracked and cracked phases :-

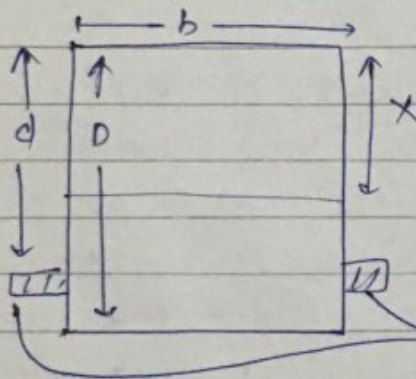
Concrete in the extreme tensile fiber of a beam will crack in flexure for the first time when stresses due to load greater than modulus of rupture.



$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow M_{cr} = \frac{\sigma \cdot I}{y} = \frac{f_{cr} \cdot I_{gr}}{y}$$

$$\hookrightarrow \text{just crack} = 0.7 \sqrt{f_{ck}} = f_{cr}$$



$$x = \frac{bD \cdot \frac{D}{2} + (m-1)A_{st} \cdot d}{bD + (m-1)A_{st}}$$

$$bx \cdot \frac{x}{2} = b(D-x) \left(\frac{D-x}{2} \right) + (m-1)A_{st} (d-x)$$

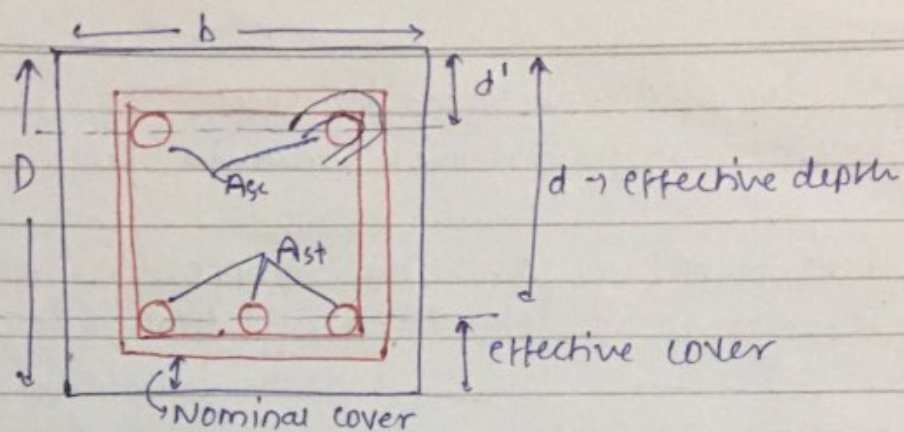
$$I_{gr} = \frac{bx^3}{3} + \frac{b(D-x)^3}{3} + (d-x)^2 (m-1)A_{st}$$

$I_{gr} \rightarrow$ gross moment of inertia

$$y = (D-x)$$

if applied BM is greater than M_{cr} , section is crack or uncrack

NOTE



D → overall depth / Gross depth / Total depth

b → width of section

d → effective depth → distance b/w extreme compressive fibre to centroid of tensile R/F

effective cover → distance from extreme tensile fibre to centroid of tensile R/F

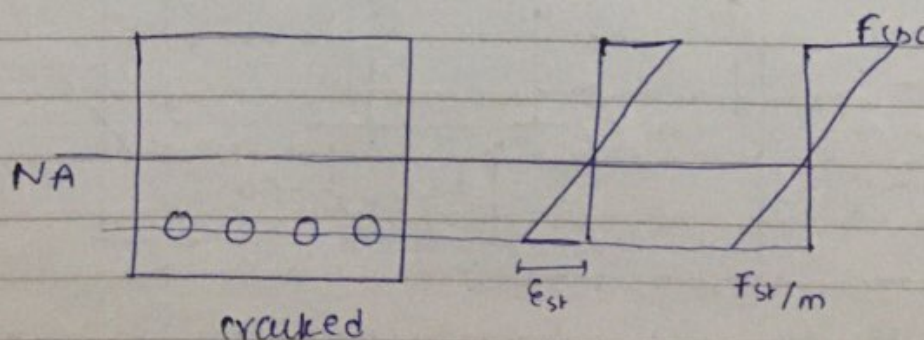
d' → distance from extreme compressive fibre to centroid of compressive R/F (also called as effective cover for compression steel)

$$\text{effective cover} = \text{Nominal cover} + \phi_{\text{shear}} + \frac{\phi_m}{2}$$

Analysis & Design :-

Analysis → Calculation of σ & MOR

Design → ~~Cal~~ Design of C/S and R/F



$f_{st} \rightarrow$ Actual stress in steel

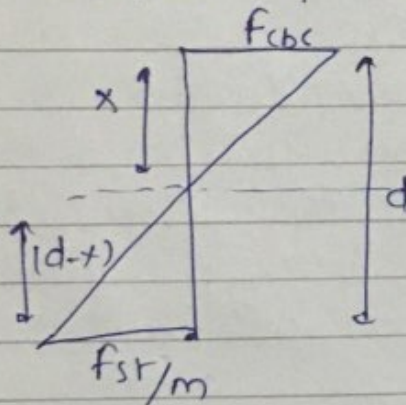
$\sigma_{st} \rightarrow$ permissible stress in steel

$f_{bc} \rightarrow$ Actual bending compression stress in concrete

$\sigma_{bc} \rightarrow$ permissible bending compression stress in concrete

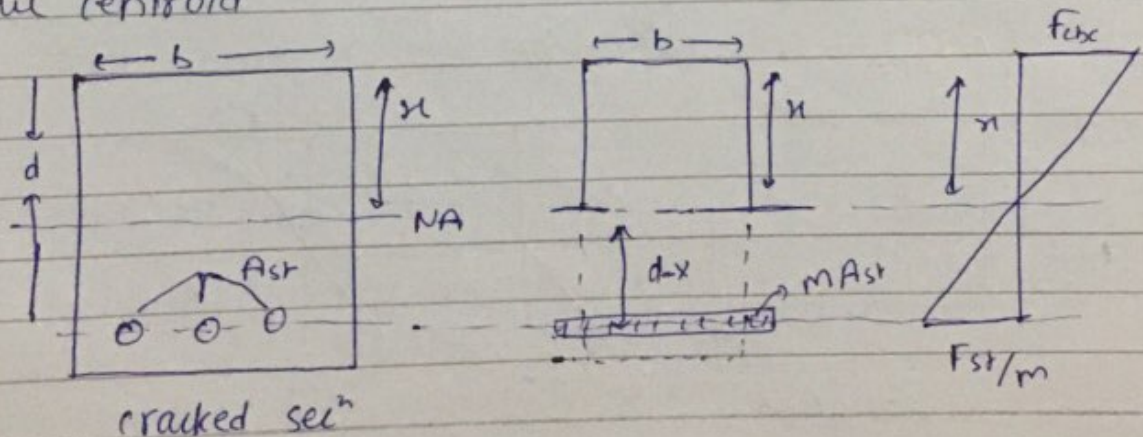
Depth of NA:-

- * In linear elastic method NA coincides with the centroidal axis when no axial force is there
- When stresses in steel & concrete are given NA can be calculated by similar triangle



$$\frac{f_{bc}}{x} = \frac{f_{st}}{m(d-x)}$$

When section dimension and A_{st} are given, NA can be calculated by $C = T$ or by calculating the centroid



$$c = T$$

$$0.5 f_{cbc} \times b = m A_{st} \frac{f_{st}}{m} = f_{st} A_{st}$$

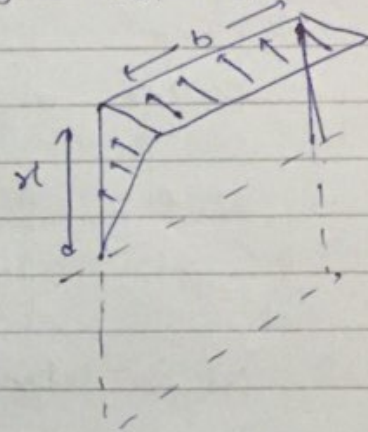
$$\frac{f_{cbc}}{n} = \frac{f_{st}}{m(d-n)}$$

$$\Rightarrow 0.5 \frac{f_{st} n}{m(d-n)} \times b = f_{st} A_{st}$$

$$0.5 b n^2 = m A_{st} (d-n)$$

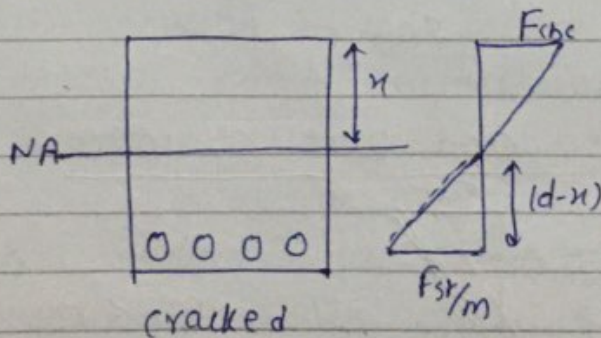
Or from ^{First} MOMENT Area

$$b n \cdot \frac{n}{2} = m A_{st} (d-n)$$



Stresses in steel and concrete :-

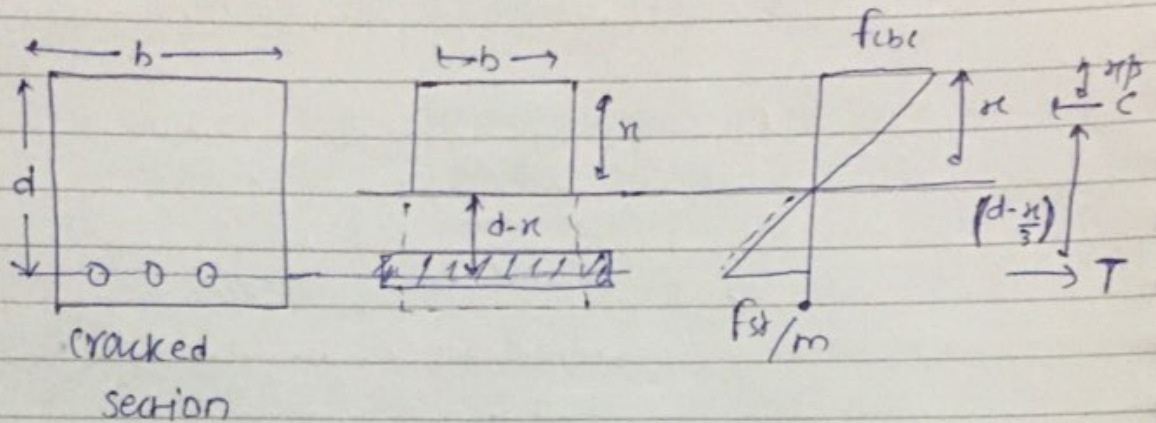
If BM is given, stresses can be calculated by using flexure formula in linear elastic method.



$$f_{cbc} = \frac{M}{I} \cdot n \quad \left[\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R} \right]$$

$$f_{st} = \frac{M}{I} (d-n) m$$

Calculation of MOR :-



$$MOR = \frac{1}{2} \times f_{cbc} \times x \times b \left(d - \frac{x}{3} \right) \quad \left(\begin{array}{l} \text{about} \\ \text{along the} \\ \text{tension} \\ \text{force} \end{array} \right)$$

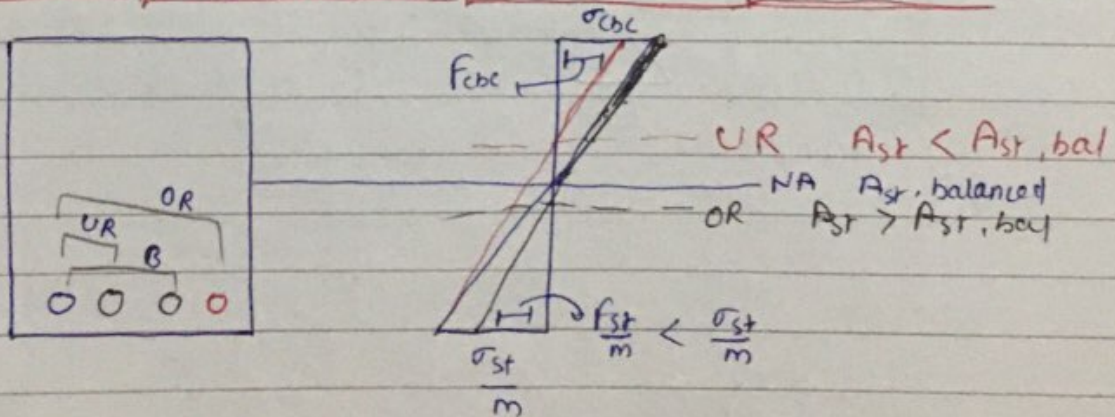
$$MOR = f_{st} \cdot A_{st} \left(d - \frac{x}{3} \right) \quad \text{about compressive force}$$

$$MOR = C \times \frac{2x}{3} + T (d - x) = C \left[\frac{2x}{3} + d - x \right]$$

$$= C \left(d - \frac{x}{3} \right)$$

about NA

Balanced, under reinforced and over reinforced :-



UR	Balanced	OR
$A_{st} < A_{st, bal}$	$A_{st, balanced}$	$A_{st} > A_{st, bal}$
$x < x_b$	x_b	$x > x_b$
$f_{st} < \sigma_{st}$	σ_{st}	$f_{st} > \sigma_{st}$
$f_{cbc} < \sigma_{cbc}$	σ_{cbc}	$f_{cbc} = \sigma_{cbc}$
$MOR < MOR_{balanced}$	$MOR_{balanced}$	$MOR > MOR_{balanced}$
Failure (steel)	simultaneously	Failure (concrete)
Ductile	Ductile	Brittle

Balance section is a section in which both steel and concrete reaches their Max. permissible stresses simultaneously.

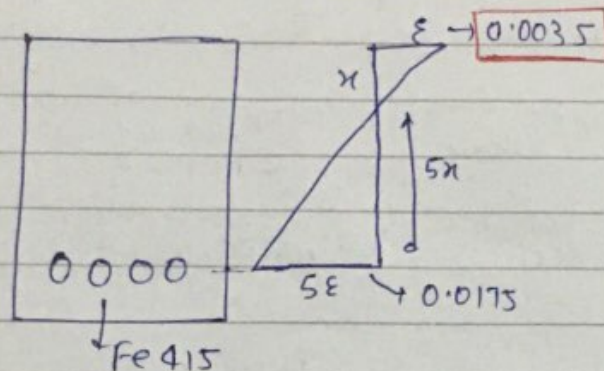
UR section is a section in which tension steel reaches Max. permissible stress ~~high~~ prior to concrete

OR section is a section in which compression concrete reaches Max. permissible stress prior to steel.

NOTE ① For safety stresses shall not be more than permissible Value (If stresses are more then section will fail)

As stresses govern the failure, failure in WSM is based on principle stress theory.

- ② Ductile failure gives sufficient warning before collapse and hence advisable (OR section are not permitted by IS code due to Brittle failure)



MOR for balanced section

$$= 0.5 \sigma_{cbc} x_b b \left(d - \frac{x_b}{3} \right)$$

$$= \sigma_{st} A_{st} \left(d - \frac{x_b}{3} \right)$$

MOR for UR section

$$= 0.5 f_{cbc} x b \left(d - \frac{x}{3} \right)$$

$$= \sigma_{st} A_{st} \left(d - \frac{x}{3} \right)$$

$$\frac{f_{cbc}}{x} = \frac{\sigma_{st}}{m(d-x)}$$

MOR for OR section

$$= 0.5 \sigma_{cbc} x b \left(d - \frac{x}{3} \right)$$

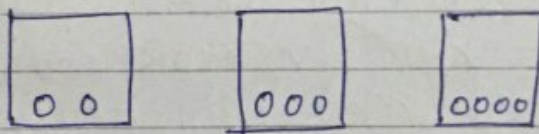
$$= f_{st} A_{st} \left(d - \frac{x}{3} \right)$$

properties of balance section :-

$A_{st, balance}$ is the amount of R/F required in a section which will make the section balance and in this case $x = x_b$ and $MOR = MOR_{balance}$. If we provide less reinforcement then $A_{st, balance}$ section will be UR and more R/F then $A_{st, balance}$, section will be OR

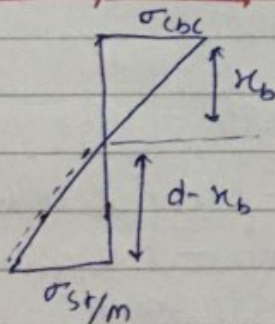
For same size C/S, Grade of steel and grade of concrete balance amount of R/F is same

for ex:



$A_{st, balance}$ of the above three secⁿ will be same

Balance depth of NA :-



$$\frac{x_b}{\sigma_{chc}} = \frac{d - x_b}{\sigma_{st/m}}$$

$$\Rightarrow \frac{x_b}{\sigma_{chc}} = \frac{d - x_b}{\frac{\sigma_{st} \cdot 3 \sigma_{chc}}{280}}$$

$$\Rightarrow x_b = \frac{d - x_b}{3 \sigma_{st} / 280}$$

$$\Rightarrow \frac{d - x_b}{x_b} = \frac{3 \sigma_{st}}{280}$$

$$\frac{d}{x_b} - 1 = \frac{3 \sigma_{st}}{280}$$

$$\frac{d}{x_b} = 1 + \frac{3 \sigma_{st}}{280}$$

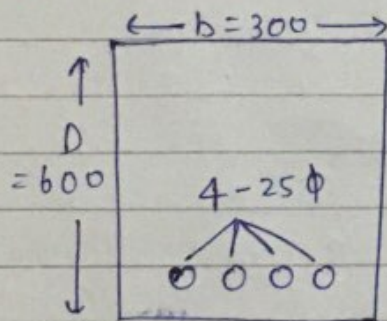
$$\Rightarrow x_b = \left[\frac{1}{1 + \frac{3 \sigma_{st}}{280}} \right] d \quad *$$

NOTE • NA coefficient that is $\left[\frac{1}{1 + \frac{3\sigma_{st}}{280}} \right]$ in

balanced condition depends only on permissible stress in steel (Tension)

- The above formula is valid for all type of cross-section i.e. rectangular, Trapezoidal, circular, Flange (T-section)

Que A RF concrete Beam of rectangular sectⁿ has the c/s dimension as shown below. Take M20 & Fe415, calculate the i) cracking Moment ii) Ast, balance iii) MOR, balance, iv) stress due to BM of 50 kNm v) MOR vi) stress due to BM of 100 kNm vii) stress due to BM of 140 kNm



M20, Fe 415

$$m = \frac{200}{3\sigma_{abc}} = 13.33$$

$$A_{st} = 4 \times 490 = 1960 \text{ mm}^2$$

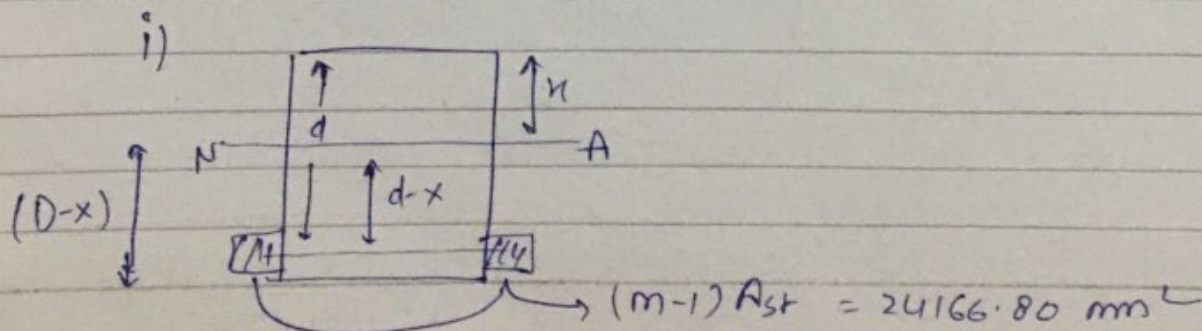
$$d = D - \text{effective cover}$$

$$= 600 - 50$$

$$= 550 \text{ mm}$$

$$\sigma_{cbc} \rightarrow 7 \text{ MPa} = f_{ck}/3$$

$$\sigma_{st} = 230 \text{ MPa}$$



$$b \cdot x \cdot \frac{x}{2} = b \cdot (D - x) \left(\frac{D - x}{2} \right) + (m - 1) A_{st} (d - x)$$

$$300 \cdot \frac{x^2}{2} = 300 (600 - x) \left(\frac{600 - x}{2} \right) + (11.33 - 1) 1960 (550 - x)$$

$$x = 329.59 \text{ mm}$$

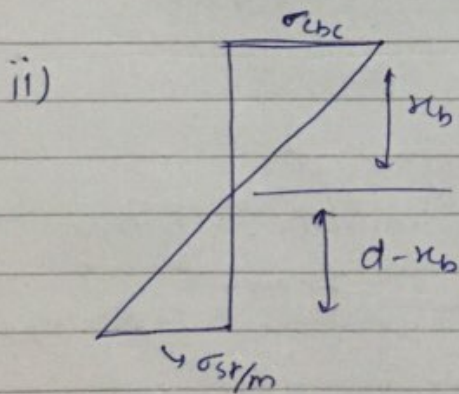
$$I = \frac{bx^3}{3} + \frac{b(D-x)^3}{3} + (m-1)A_{st}(d-x)^2$$

$$I = 6.73 \times 10^9 \text{ mm}^4$$

$$M_{cr} = \frac{\sigma}{y} \cdot I_{gr}$$

$$= \frac{0.7 \sqrt{f_{ck}}}{600 - 329.59} \times 6.73 \times 10^9$$

$$M_{cr} = 77.9 \text{ kNm}$$



From \$C = T\$

$$0.5 x_b b \sigma_{cbc} = \sigma_{st} A_{st, bal.}$$

$$x_b = \left[\frac{1}{1 + \frac{3 \sigma_{st}}{200}} \right] d$$

$$= 158.763 \text{ mm}$$

$$\Rightarrow A_{st, bal} = 724.79 \text{ mm}^2$$

$$\text{iii)} \quad M O R_b = 0.5 \sigma_{cbc} x_b b \left(d - \frac{x_b}{3} \right)$$

$$= 0.5 \times 7 \times 158.763 \times 300 \times \left(550 - \frac{158.76}{3} \right)$$

$$= 82.86 \text{ kNm}$$

or

$$\sigma_{st} \cdot A_{st} \left(d - \frac{x_b}{3} \right)$$

$$= 230 \times 724.79 \left(550 - \frac{158.76}{3} \right)$$

$$= 82.86 \text{ kNm}$$

$$\text{(iv)} \quad B.M = 50 \text{ kNm} < M_{ur}$$

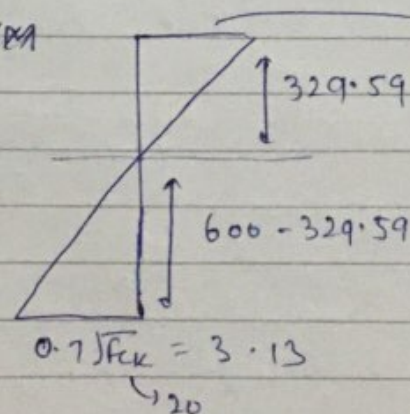
$\Rightarrow$ section is uncracked

$$f_{cbc} = \frac{M}{I} \times y$$

$$= \frac{50 \times 10^6}{6.73 \times 10^9} \times 329.59$$

$$= 2.45 \text{ MPa}$$

~~W~~ B.M

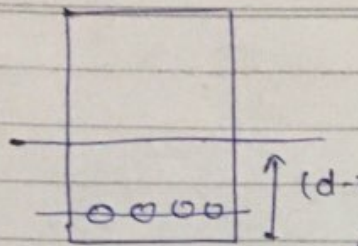


by similar triangle

$$f_{cbc} = 3.81 \text{ MPa}$$

$$77.9 \longrightarrow 3.81$$

$$50 \longrightarrow \frac{3.81}{77.9} \times 50 = 2.45 \text{ MPa}$$

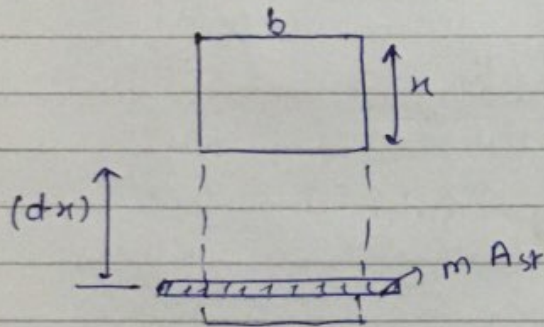


$$f_{st} = \frac{M}{I} \cdot (d-x)$$

$M \rightarrow 50 \times 10^6$
 $I \rightarrow 6.73 \times 10^9$
 $(d-x) \rightarrow 550$
 $\rightarrow 329.59$
 $\rightarrow 13.33$

$$f_{st} = 21.84 \text{ MPa}$$

(v) MOR of the crack section will be always higher than that of uncrack section.



$$b \cdot x \cdot \gamma/2 = m A_{st} (d-x)$$

$$x = 234.44 \text{ mm}$$

OR

$$\text{MOR} = 0.5 \sigma_{cbc} \cdot x \cdot b \left(d - \frac{x}{3} \right)$$

$$= 116.15 \text{ KNm}$$

OR. $\text{MOR} = f_{st} A_{st} \left(d - \frac{x}{3} \right)$

$$\text{MOR} = 116.15 \text{ KNm}$$

$$\left\{ \frac{\sigma_{cbc}}{x} = \frac{f_{st}}{m(d-x)} \right.$$

$$f_{st} = 125.6 \text{ MPa}$$

vi) $B.M = 100$

$M_{cr} < B.M < M_{OR}$

$$f_{cbc} = \frac{7 \times 100}{116.15} = 6.02 \text{ MPa}$$

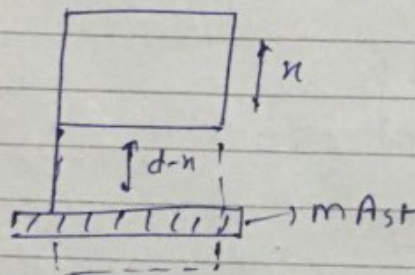
$$f_{st} = \frac{125.6}{116.15} \times 100 = 108.13 \text{ MPa}$$

or

$$f_{cbc} = \frac{M}{I} \cdot y$$

$\nearrow 100 \times 10^6$
 $\searrow 234.4$
 $\nwarrow 3.89 \times 10^9$

$I =$



$$I = \frac{bn^3}{3} + m A_{st} (d-n)^2$$

$$= \frac{300(234.4)^3}{3} + 13.33 \times 1960 (550 - 234.4)^2$$

$$= 3.89 \times 10^9 \text{ mm}^4$$

$$f_{cbc} = 6.02 \text{ MPa}$$

vii)

$B.M = 140 \text{ kNm}$

$B.M > M_{OR}$

$$f_{cbc} = \frac{6.02}{100} \times 140 = 8.42 \text{ MPa} < 77 \text{ MPa}$$

$$f_{st} = \frac{108.13}{100} \times 140 = 151.39 \text{ MPa} < 230 \text{ MPa}$$

Section should have been failed.

Que.
Design

$$b = 350 \text{ mm}$$

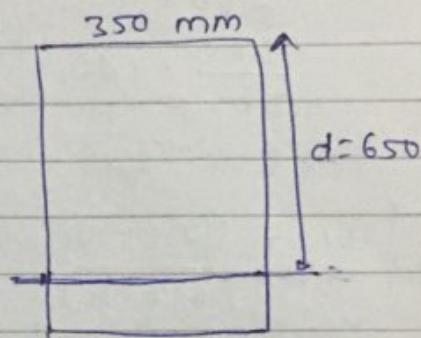
$$D = 700 \text{ mm}$$

$$d = 650 \text{ mm}$$

$$M20, \text{ Fe 415}$$

$$\text{i) } BM = 100 \text{ kNm}$$

$$\text{ii) } BM = 160 \text{ kNm}$$



$$\sigma_{cbc} = 20/3 = 7 \text{ MPa}$$

$$\sigma_{st} = 230 \text{ MPa}$$

$$x_b = \left[\frac{1}{1 + \frac{3\sigma_{st}}{2\sigma_{cbc}}} \right] d = 187.63 \text{ mm}$$

$$MOR_{balance} = 0.5 \sigma_{cbc} x_b b \left(d - \frac{x_b}{3} \right)$$

$$MOR_b = 135.02 \text{ kNm}$$

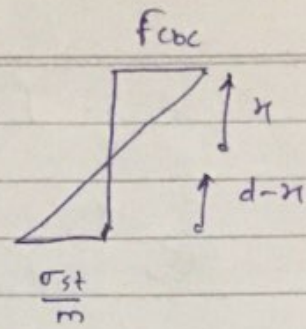
$$\text{i) } BM = 100 \text{ kNm}$$

design a UP section

$$0.5 \underline{f_{cbc}} x \underline{b} \left(d - \underline{x/3} \right) = M \quad \text{two unknowns}$$

$$\sigma_{st} A_{st} \left(d - \underline{x/3} \right) = M \quad \text{one unknown}$$

$$\frac{f_{cbc}}{x} = \frac{\sigma_{st}}{m(d-x)}$$



$$f_{cbc} = \frac{\sigma_{st} \cdot x}{m(d-x)}$$

$$= \frac{230 \times x}{\frac{280}{3\sigma_{cbc}} \times (d-x)}$$

7

$$m = 13.33$$

$$f_{cbc} = \frac{17.25 x}{(650-x)}$$

put this in eqn ①

$$0.5 \left(\frac{17.25 x}{650-x} \right) \times x \times b \left(d - \frac{x}{3} \right) = 100 \times 10^6$$

$$0.5 \left(\frac{17.25 x}{650-x} \right) \times x \times 350 \left(650 - \frac{x}{3} \right) = 100 \times 10^6$$

$$x = 164.37 \text{ mm.}$$

$$(x < x_b) \quad \checkmark \quad \text{keep checking.}$$

$$230 \leftarrow \sigma_{st} \cdot A_{st} \left(d - \frac{x}{3} \right) = 100 \times 10^6$$

$$A_{st} = 730.46 \text{ mm}^2$$

ii) $BM = 160 \text{ kNm} > M_{OR_b}$

OR Design.

$$0.5 \sigma_{cbc} x \cdot b \left(d - \frac{x}{3} \right) = M$$

$$x = 227.47 \text{ mm}$$

$$x > x_b \cong$$

$$f_{st} \cdot A_{st} \left(d - \frac{x}{3} \right) = M$$

$$\frac{\sigma_{cbc}}{x} = \frac{f_{st}}{m(d-x)}$$

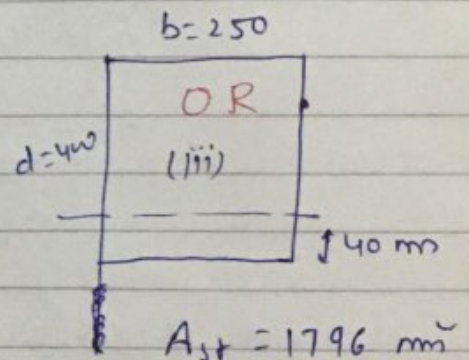
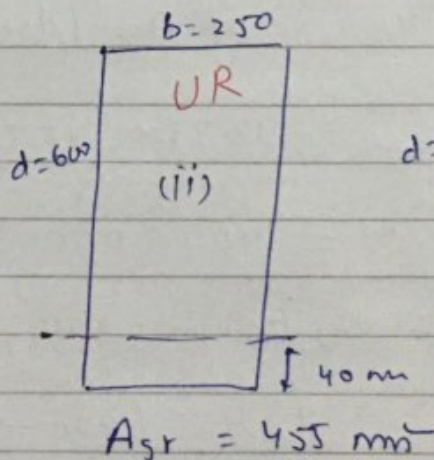
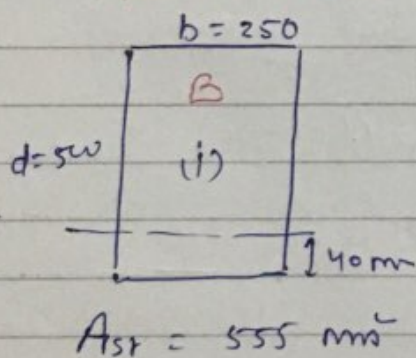
$$\frac{7}{227.47} = \frac{f_{st}}{13.33(650 - 227.47)}$$

$$f_{st} = 173.33 \text{ MPa}$$

$$173.33 A_{st} \left(650 - \frac{227.47}{3} \right) = 160 \times 10^6$$

$$A_{st} = 1607.68 \text{ mm}^2$$

Que. For the below given ~~three~~ ^{Three} sections calculate the MOR and comment on the type of section
M20 & Fe 415



MOR

$$\sigma_{cbc} = 7, \quad \sigma_{st} = 230 \text{ MPa}$$

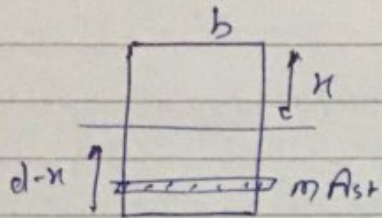
$$m = 13.33$$

$$x_b = \left[\frac{1}{1 + \frac{230 \times 13.33}{280}} \right] d$$

$$= 0.289 d$$

$$(i) \quad x_b = 0.289 d = 0.289 \times 500 = 144.33 \text{ mm}$$

$$MOR_b = 0.5 \sigma_{cbc} x_b b (d - x_b/3) = \underline{57.06}$$



$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

$$\frac{250 \cdot x^2}{2} = 13.33 \times 555 (500 - x)$$

$$x = 144.95 \text{ mm}$$

Balance section

$$MOR = \sigma_{st} A_{st} (d - \frac{x_b}{3}) = 230 \times 555 (500 - \frac{144.33}{3})$$

Balanced

$$= 857.65 \text{ MPa}$$

$$(ii) \quad x_b = 0.289 \times 600 = 173.4 \text{ mm}$$

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

$$\frac{250 \cdot x^2}{2} = 13.33 \times 455 \times (600 - x)$$

$$x = 148.08 \text{ mm} < x_b \quad \underline{\text{UR}}$$

$$MOR = \sigma_{st} A_{st} (d - x_b/3)$$

$$= 230 \times 455 (600 - \frac{148.08}{3})$$

$$= 57.62 \text{ kNm}$$

$$(iii) \quad x_b = 0.289 \times 400 = 115.6 \text{ mm}$$

$$b \cdot x \cdot \frac{x}{2} = m A_{st} (d - x)$$

$$\frac{250 \cdot x^2}{2} = 13.33 \times 1796 (400 - x)$$

$$x = 197.02 \text{ mm} > 0R$$

$$MOR \neq \sigma_{st} A_{st} (d - \frac{Mx}{3})$$

$$= 2130$$

$$MOR = 0.5 \sigma_{cebc} x b (d - \frac{x}{3})$$

$$= 0.5 \times 7 \times 197.12 \times 250 (400 - \frac{197.12}{3})$$

$$= 57.65 \text{ kNm}.$$

*

For a particular BM, keeping width same

$$d_{UR} > d_b > d_{OR}$$

Total stiffness of UR will be Max.

Generally UR will deflect less in service stage
Due to higher stiffness

UR will undergo Higher deflection and give more time during failure due to higher ductility

Generally OR will deflect more in service stage
due to lesser stiffness

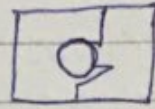
OR will ~~and~~ undergo lesser deflection during
failure due to lesser Ductility.

For same c/s size, f_{ck} , f_y $MOR_{(OR)} \uparrow > MOR_b > MOR_{UR}$

All these are valid in
LSM also.

Deficiency of WSM:-

Because of long term effect of creep & shrinkage stress concⁿ and other secondary effect it may not be possible to keep the stresses in Material within the permissible limit as assumed



increase in σ due to conⁿ.

- The FOS has NO sound theoretical basis
- The Actual Margin of safety cannot be determined due to non-linearity of stress-strain curve upto collapse.
- Fails to discriminate b/w different types of loads that acts simultaneously but have different degrees of uncertainty.
- results in larger c/s and hence uneconomical. However due to its simplicity it is still used for some structure and serviceability criteria

Ultimate Load Method / ultimate strength Method :-

- To improve on WSM, ULM was developed
- safety was introduced by load factors
- Load Factor = $\frac{\text{ultimate load}}{\text{working load}}$

- stress condition at the state of impending failure at the section i.e. (ultimate strength) is analysed and non-linear curve of both steel and concrete are considered. No FOS is used in Material strength.
- Generally the stress at ultimate load is obtained by load factor $\times$ stress @ service load. However this is illogical and incorrect because significant in elastic behaviour and redistribution of stresses take place as the loading is increased from service load to ultimate load.
- This Method results in cylinder section and economical design, However this doesn't guarantee satisfactory serviceability condition and hence it is no more in used.

3 LSM

- In a structure there are uncertainties in loading, material properties and dimension of member also some uncertainties are there due to simplified assumptions made in the analysis and design.
- To account for above uncertainties FOS was used in WSM. However (there was no theoretical basis for the FOS) To overcome this reliability based analysis (Reliability level I) was performed and FOSs were established for both load & material strength (These FOS are called partial safety factors and the method based on this concept is called LSM).

$$\begin{array}{ccc} \text{Strength of structure} & & \text{Load effects considering} \\ \text{considering reduced} & & \text{higher load} \\ \text{material strength} & \geq & \\ (R_d) & & (R_s) \end{array}$$

Limit states:-

Limit state is the state beyond which structure becomes unfit for use, in terms of either safety or serviceability. that is either collapses or becomes unserviceable.
Two Types of limit states-

- 1) Limit state of collapse
- 2) Limit state of serviceability

1) Limit state of collapse (ultimate limit state)

- It includes strength, Flexure, Compression, Shear, Torsion, ϕ , overturning, sliding, buckling, Fatigue fracture etc.

Adequate Margin of safety for normal overloads (ultimate load)

2) Limit state of serviceability :-

- It includes deflection, crack width, vibration, leakage, loss of durability, Noise etc.

Satisfactory performance under service load.

NOTE • Ultimate limit state are conditioned to be avoided and serviceability conditions are not desirable.

- If a structure has just reached before the limit state of serviceability and loads are removed, the structure regains its original shape. However a structure reaching the collapse state can not recover its original shape.

The structure shall be designed for most critical limit states (can be collapse or serviceability) and shall be checked for other limit state

for ex.

slabs are critical in debtection

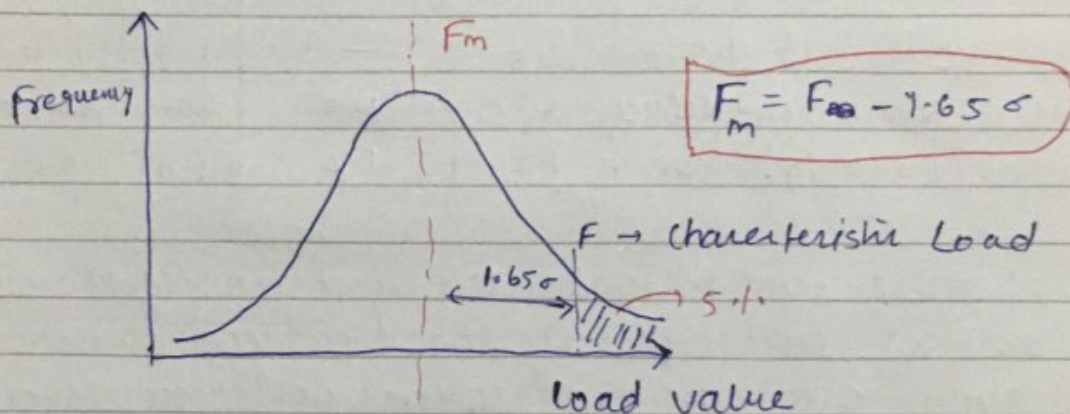
Design value for loads :-

$$f_D = F \cdot \gamma_F$$

f_D → Design load

F → characteristic load

γ_F → partial safety factor depending on limit state being considered and load combination



Characteristic load is the load that has a 95% probability to not being exceeded during the life of the structure.

If actual load data is not available, the characteristic load can be considered from IS code

(part-1)

IS 875 → Dead load

live load → IS 875 (part-2)

wind load → IS 875 (part-3)

snow load → IS 875 (part-4)

EQ load → IS 1893 (part-1)

If the dead load of concrete is not given then take $\gamma_c = 25$ (RCC)

$$\gamma_c = 24 \text{ (PCC)}$$

Value of partial safety factor for loads (γ_f):-

Load combination	Limit state of collapse			Limit state of serviceability		
	D.L	L.L	E θ /WL	D.L	D.L	W.L/EL
D.L + L.L	1.5	1.5	—	1.0	1.0	—
D.L + E θ /WL	1.5 or 0.9	—	1.5	1.0	—	1.0
D.L + L.L + E θ /WL	1.2	1.2	1.2	1.0	0.8	0.8

E θ and Wind loads are not considered together. 0.9 factor shall be considered when stability against overturning or sliding or stress reversal is critical, underestimating the counter acting effect of DL in DL + E θ /WL results in Greater safety.

The 1.2 factor is considered to account for reduced probability of all loads acting at their peak values. (In WSM permissible stresses are increased)

Design Value for material strength:-

$$f_d = \frac{f}{\gamma_m}$$

f_d $\rightarrow$ design strength

γ_m $\rightarrow$ partial safety factor for material

$$0.67 f_{cu} \text{ for } \delta_c$$

$f \rightarrow$ characteristic strength however apply γ_m
On ~~to~~ $0.67 f_{cu}$ for concrete

for limit state of collapse

$$\begin{aligned}\gamma_m &= 1.15 \text{ for steel} \\ &= 1.5 \text{ for concrete}\end{aligned}$$

for serviceability limit state, Material properties like Modulus of Elasticity shall be considered as those corresponding to characteristic strength of materials so we can say $\gamma_m = 1$ for both concrete & steel.

NOTE Higher partial safety factor is considered in concrete as control on the quality of concrete is not as good as that of steel and also higher variability is associated with concrete.

probability of failure% - (IN LSM)

failure = concrete fails $\times$ load O.K. +
probability concrete fails $\times$ load exceed +
concrete O.K. $\times$ load exceed

$$= 0.05 \times 0.95 + 0.05 \times 0.05 + 0.95 \times 0.05$$

$$= 0.0975 = 9.75 \times 10^{-2} \text{ order of failure}$$

here, order of Failure.

$$\begin{aligned}\text{Failure} &= 1 - \text{Safety} \\ &= 1 - 0.95 \times 0.95 \\ &= 9.75 \times 10^{-2}\end{aligned}$$

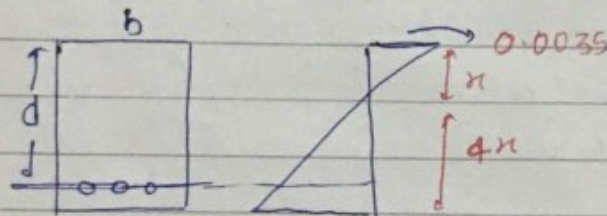
Limit state of collapse : Flexure $\Rightarrow$

It includes the members such as Beams, slabs etc.

Assumptions :-

- 1) plane section before bending remains plane after bending
- 2) Max. compressive strain in concrete in extreme compressive fibre is taken as 0.0035 in Bending. (In WSM stress was limited)

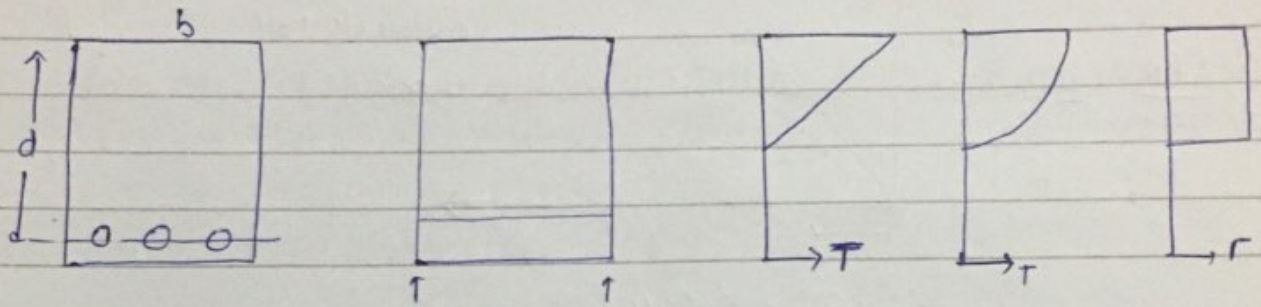
NOTE.



$$4 \times 0.0035 = 1.4\% \leq \text{ductility of steel.}$$

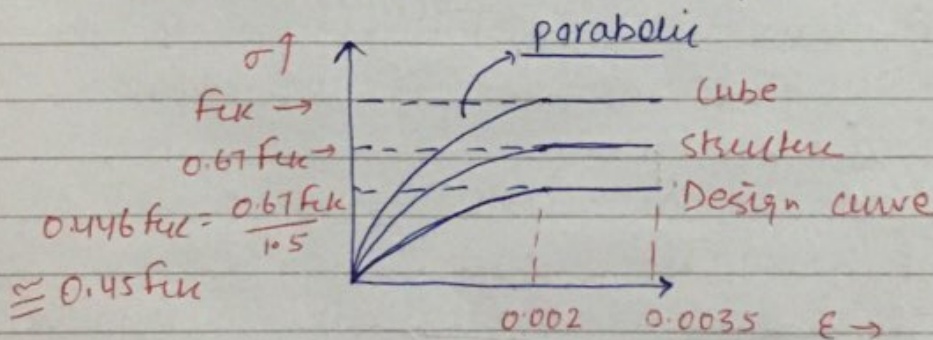
- As elongation of Reinforcement that is ductility is very high compare to concrete failure strain, failure of RCC flexural member always occurs due to crashing of concrete on compression side and very rarely occurs due to rupture / snapping] breaking apart suddenly of reinforcement, so for all c/s that is UR, Balance or OR collapse invariably occurs by crashing of concrete on, compression side.

- 3) Relationship b/w compressive stress & strain in concrete may be assumed to be rectangular, trapezoidal, parabolic or any other shape which results in prediction of strain in substantial agreement with the test result.



if $MOR_{calculate} = MOR_{test} \Rightarrow$ Can adopt the curve.

→ An acceptable σ - ϵ curve may be taken as follows
(In WSM curve is linear)

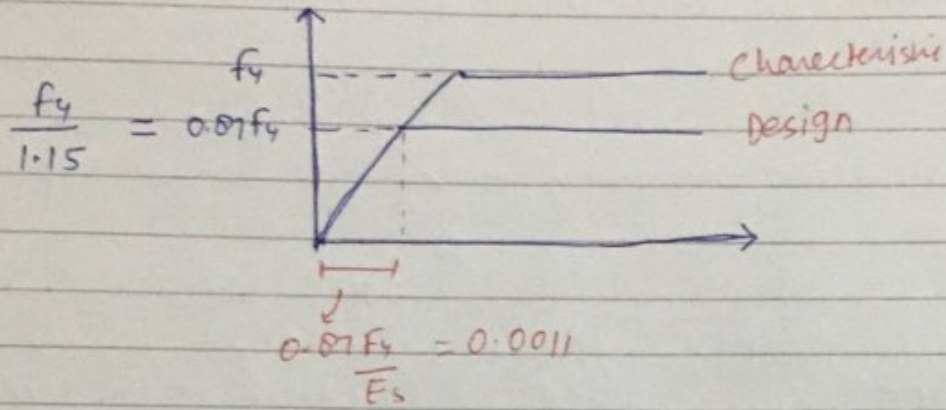


NOTE: permissible strain in Bending compression is taken higher than permissible strain in direct compression. Failure in LSM is based on principle strain theory

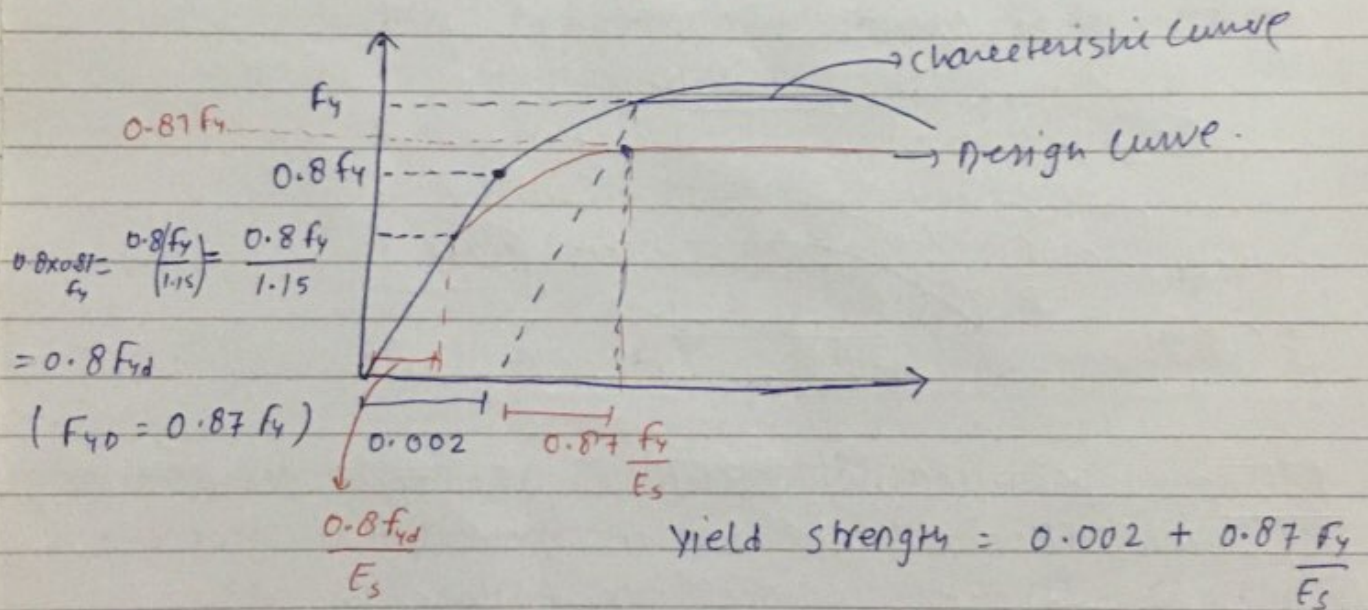
④ The Tensile strength of concrete is ignored.

⑤ stress in R/F is derived from the representative σ - ϵ curve of steel

MS / hot rolled / Fe 250



Gold worked / HYS.D / TOR / CTD / Fe415 / 500



Grade	Linear upto $0.8 f_{yk} / E_s$	Design strain $0.002 + 0.87 f_k / E_s$
Fe 415	0.00144	0.00380
Fe 500	0.00174	0.0042

NOTE

For HYSD CTD Bars, The transition from Linear elastic behaviour to Non-Linear behaviour is assume to occur at $0.8 f_y$ in characteristic curve and $0.8 f_{yd}$ in design curve, Where $f_{yd} \rightarrow$ design yield strength $= 0.87 f_y$

In case of concrete $\rightarrow$ partial safety factor @ all stress level
In case of steel $\rightarrow$ partial safety factor only for inelastic zone

The design σ - ϵ table For Fe415 is -

Strain	0.00144	0.00163	0.00192	0.00241	0.00276	0.00380
Stress	$0.8 f_{yd}$	$0.85 f_{yd}$	$0.90 f_{yd}$	$0.95 f_{yd}$	$0.975 f_{yd}$	f_{yd}

- ⑥ The Max. strain in the tensile R/F (that is at the centroid) at ultimate limit state should not be less than $0.002 + \frac{0.87 f_y}{E_s}$

This assumption will ensure design stress of $0.87 f_y$ @ ultimate limit state leading to efficient utilization of steel and ductile failure

NOTE In actual the yield strain of Fe250 is $0.0011 \left(\frac{0.87 f_y}{E_s} \right)$
However code has not specified design yield strain for Fe250 separately. Taking higher design strain in Fe250 will ensure ductile failure providing ample warning of the impending collapse.

Grade of steel

design yield strain

$$0.002 + 0.07 \frac{f_y}{E_s}$$

Fe 250

0.0031

Fe 415

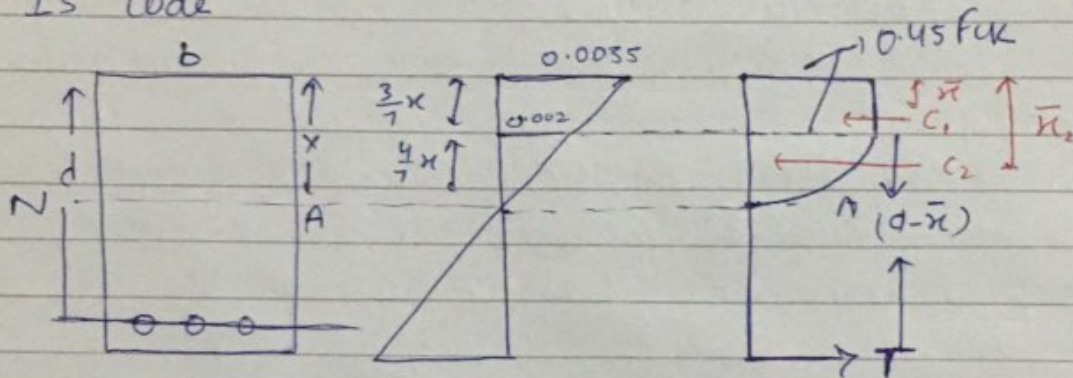
0.0038

Fe 500

0.0042

Stress-Block parameters:-

Considering the σ - ϵ curve of concrete given in IS code



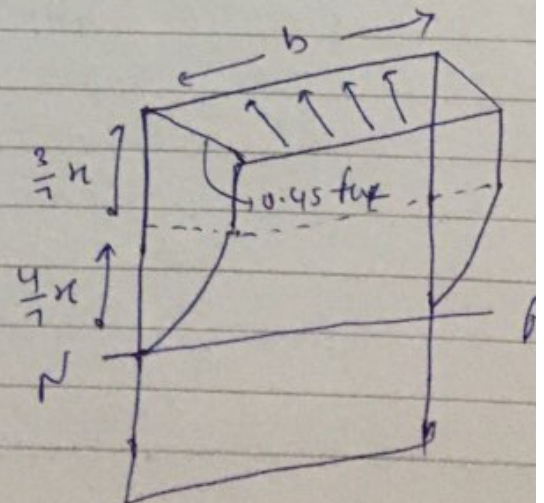
$$C = C_1 + C_2$$

$$\bar{x} = \frac{C_1 x_1 + C_2 x_2}{C_1 + C_2}$$

$$MOR = C (d - \bar{x})$$

$$C_1 = 0.45 f_{ck} \frac{3}{7} n \cdot b$$

$$= 0.193 f_{ck} b \cdot n$$



$$X_1 = \frac{1}{2} \left(\frac{3}{7} x \right) = \frac{3}{14} x$$

$$C_2 = \frac{2}{3} \times \frac{4}{7} x \times 0.45 f_{ck} \times b$$

$$= 0.171 f_{ck} b x$$

$$X_2 = \frac{3}{7} x + \frac{3}{8} \left(\frac{4}{7} x \right)$$

$$= \frac{9}{14} x$$

$$C_1 + C_2 = 0.36 f_{ck} b x$$

$$\bar{X} = \left(\frac{0.193 \times \frac{3}{14} + 0.171 \times \frac{9}{14}}{0.36} \right) x = 0.42 x$$

$$\boxed{C = 0.36 f_{ck} b x}$$

$$l.A = d - 0.42 x$$

* Valid for singly R/F Rectangular secⁿ with IS code Curve.

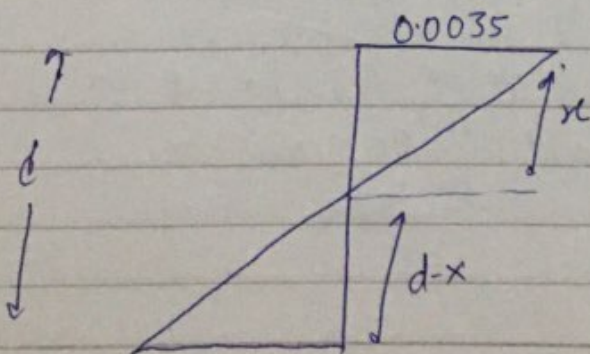
Limiting depth of NA:-

In order to achieve min. strain in tension steel

$$x < x_{lim}$$

$x \rightarrow$ Actual depth

$x_{lim} \rightarrow$ Limiting depth



$$\epsilon_{st} = \frac{0.0035 (d-x)}{x}$$

$$= 0.0035 \left(\frac{d}{x} - 1 \right)$$

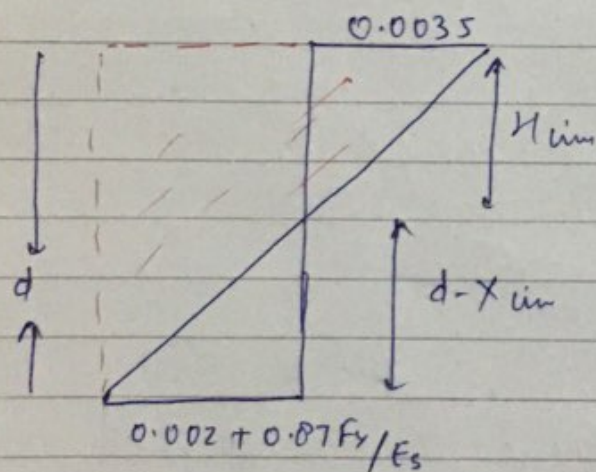
$$\epsilon_{st} \geq 0.002 + 0.87 \frac{f_y}{E_s}$$

$$0.0035 \left(\frac{d}{x} - 1 \right) \geq 0.002 + 0.87 \frac{f_y}{E_s}$$

$$\Rightarrow 0.0035 \frac{d}{x} \geq 0.0055 + 0.87 \frac{f_y}{E_s}$$

$$x \leq \left[\frac{0.0035}{0.0055 + 0.87 \frac{f_y}{E_s}} \right] d$$

$$x_{lim} = \left[\frac{0.0035}{0.0055 + 0.87 \frac{f_y}{E_s}} \right] d \quad *$$



$$\frac{x_{lim}}{d} = \frac{0.0035}{0.0055 + 0.002 + 0.87 \frac{f_y}{E_s}}$$

$$\therefore x_{lim} = \left[\frac{0.0035}{0.0055 + 0.87 \frac{f_y}{E_s}} \right] d$$

NOTE : • Limiting depth of NA only depend on Grade of Steel σ_{lim}

• x_{lim} is independent of type of c/s i.e it is same for singly R/f, doubly R/f, Or any other type of section.

Grade of steel

X_{lim}/d

Fe 250

0.53

Fe 415 *

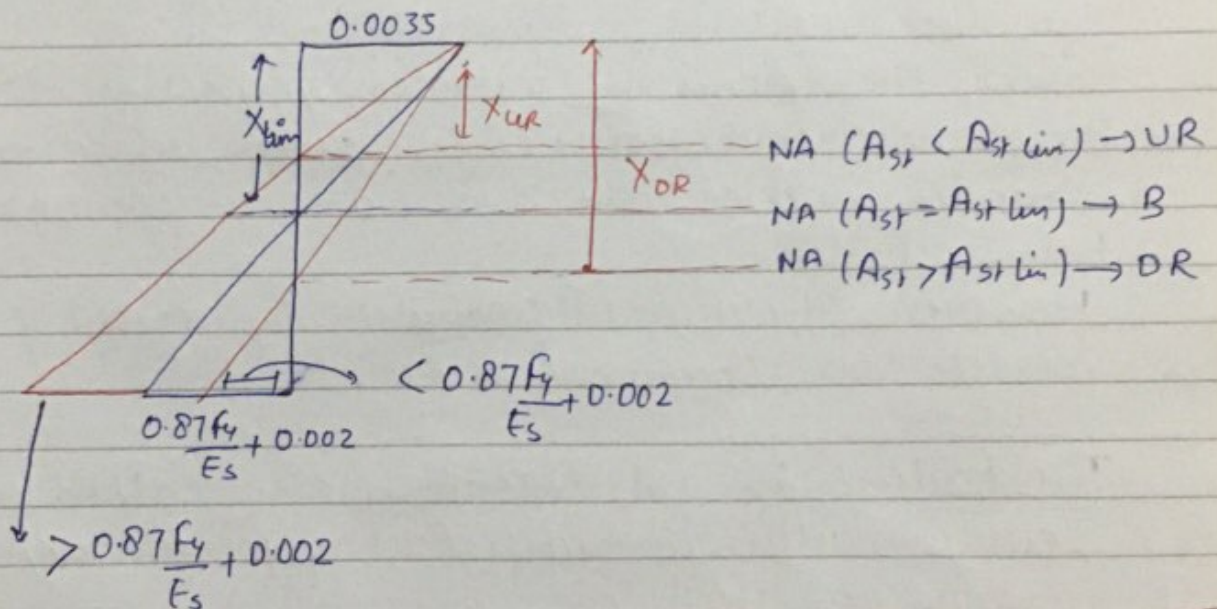
0.48 *

Fe 500

0.46

Types of sections:-

Depending upon amount of R/F section can be UR, balance or OR



UR

B

OR

$$X < X_{lim}$$

$$X = X_{lim}$$

$$X > X_{lim}$$

$$A_{st} < A_{st,lim}$$

$$A_{st} = A_{st,lim}$$

$$A_{st} > A_{st,lim}$$

$$e_c < 0.0035$$

$$e_c = 0.0035$$

$$e_c > 0.0035$$

$$e_{st} > 0.002 + \frac{0.87f_y}{E_s}$$

$$e_{st} = 0.002 + \frac{0.87f_y}{E_s}$$

$$e_{st} < 0.002 + \frac{0.87f_y}{E_s}$$

$$f_{st} = 0.87f_y$$

$$f_{st} = 0.87f_y$$

$$f_{st} < 0.87f_y$$

$$MOR < M_{lim}$$

$$MOR = M_{lim}$$

$$MOR > M_{lim}$$

① BALANCE SECTION:-

Section that is expected to result in Balance failure i.e simultaneous attainment of limiting compressive strain of 0.0035 in concrete and Minimum specified tensile strain of $0.002 + \frac{0.87f_y}{E_s}$ in steel at ultimate limit state ⁱⁿ flexure.

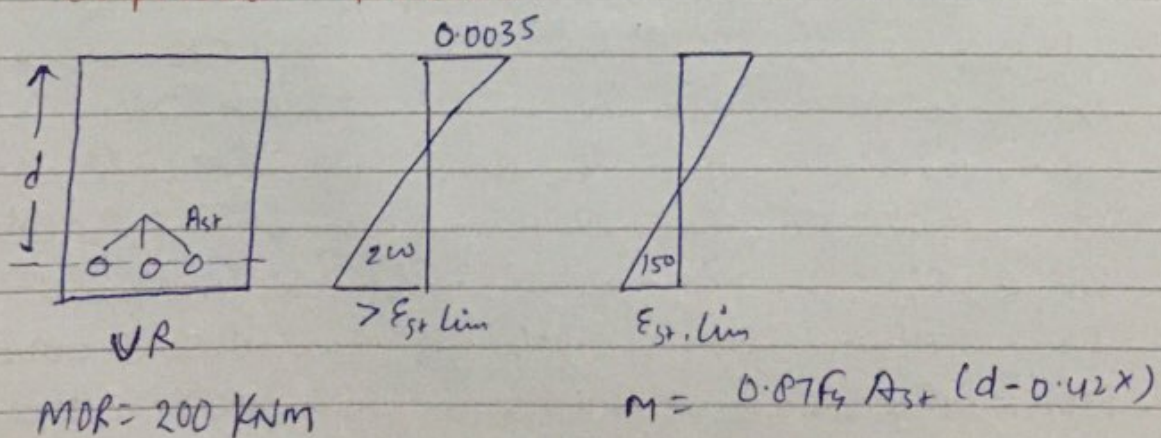
② UR

Section in which steel yield before collapse that is at collapse strain in steel $> 0.002 + \frac{0.87f_y}{E_s}$

Since the failure is initiated by yielding of steel, the section undergoes large scale deflection, extensive cracking and gives ample warning before failure.

However the final collapse is by crushing of concrete on compression side.

The failure in UR section is also called as Tensile Failure / Ductile Failure / secondary Compression failure.



In a UR section, as the applied BM approaches the MOR, the NA shifts upward (compression fibre) valid in LSM only because once the steel has yielded, tension force is constant and hence in order to maintain moment equilibrium the lever arm has to increase.

How?

OR.

Section in which crushing of concrete occurs prior to yielding of steel, the curvature, deflection and crack width remain relatively low and failure occurs without any sign of warning. For this reason OR Flexure Member are not permitted by IS Code.

The failure is called as Brittle failure / compression failure / primary compression failure

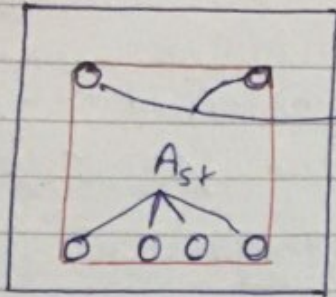
The tension steel may remain in the elastic zone upto collapse.

As the applied BM approaches MOR, in order to maintain equilibrium compression are increases and NA shifts downward.*

How?

[4] Singly R/F Rectangular Section

- R/F only in the tensile side



Hanger Bar (strength not considered in design)

ANALYSIS:-

Analysis implies the determination of ultimate Moment of resistance for a given section dimension, A_{st} , f_{ck} , f_y

$$MOR = 0.36 f_{ck} b x (d - 0.42 x)$$

Calculation of Depth of NA:-

by force equilibrium

$$C = T$$

$$0.36 f_{ck} x b \quad \cancel{f_{ck} b (d - 0.42 x)} = 0.87 f_y A_{st}$$

$$x = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$X_{lim} = \left[\frac{0.0035}{0.0055 + 0.87 \frac{f_y}{E_s}} \right] d$$

if $x = X_{lim} \Rightarrow$ Balanced

$x < X_{lim} \Rightarrow$ UR

$x > X_{lim} \Rightarrow$ OR

NOTE: The X obtained in case of OR section by taking $x = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$ is not the correct value of NA as in OR $f_{st} < 0.87 f_y$

The correct NA depth can be obtained by trial and error procedure and ~~X_{lin}~~ ~~$X_{correct}$~~

$$X_{lin} < X_{correct} < \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

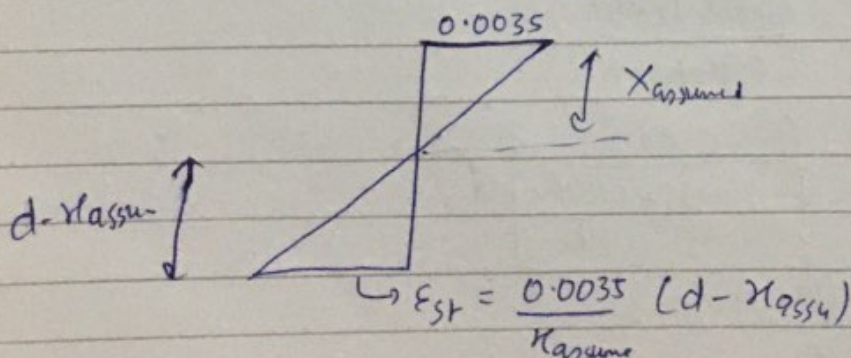
• stress for calculation of correct depth in case of OR

STEP-1 assume some value of x , generally take

$$X_{assumed} = \frac{X_{lin} + X_{calculated}}{2}$$

$$X_{calc.} = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

STEP-2. Find strain in steel corresponding to $X_{assumed}$ and the respective stress from the stress-strain table



Step-3

Taking the above f_{st} value and using $c=T$ calculate X_{new}

$$c = T$$

$$0.36 f_{ck} b X_{new} = f_{st} A_{st}$$

$$X_{new} = \frac{f_{st} A_{st}}{0.36 f_{ck} b}$$

if X_{assume} of step 1 $\overset{is}{\approx}$ nearly $= X_{new}$ then the calculated value can be taken as the correct depth of NA o/w repeat the steps.

Determining MOR

$$MOR = 0.36 f_{ck} b x (d - 0.42 x)$$

? compression side

Valid for all section

Tension side

$$MOR = 0.87 F_y A_{st} (d - 0.42 x)$$

Valid for UR & B only.

$$MOR = f_{st} A_{st} (d - 0.42 x) \text{ valid for all}$$

$$f_{st} \geq 0.87 F_y \quad (\text{for B \& UR})$$

$$f_{st} \leq 0.87 F_y \quad (\text{for OR})$$

NOTE

The MOR of OR in actual is Greater than MOR_{lim} However we limit or Report MOR of a OR section as MOR_{lim} as per IS Code

$$C \quad MOR = 0.36 f_{ck} b x (d - 0.42x) \\ = 0.36 \frac{x}{d} \left(1 - 0.42 \frac{x}{d}\right) f_{ck} b d^2 \quad \rightarrow R$$

$$M = R b d^2$$

$$T \quad M = 0.87 f_y A_{st} (d - 0.42x) \quad \{UR, B\}$$

$$0.36 f_{ck} b x = 0.87 f_y A_{st}$$

$$\Rightarrow x = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \quad (\text{Valid for UR, B})$$

$$M = 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right] \quad \text{Valid for UR, B}$$

Limiting MOR (M_{lim})

$$M_{lim} = 0.36 \frac{x_{lim}}{d} \left(1 - 0.42 \frac{x_{lim}}{d}\right) f_{ck} b d^2 \quad \rightarrow R_{lim}$$

$$M_{lim} = R_{lim} b d^2$$

for a given grade of concrete and steel
 M_{lim} depends on C/s

Limiting percentage of R/F :-

From $C = T$

$$0.87 f_y A_{st, \text{lim}} = 0.36 f_{ck} b x_{\text{lim}}$$

$$\frac{A_{st, \text{lim}}}{bd} = \frac{0.36 f_{ck} b x_{\text{lim}}}{0.87 f_y bd} \times 10^2 = 41.38 \frac{f_{ck} x_{\text{lim}}}{f_y d}$$

$$P_t = \frac{A_{st}}{bd} \times 100$$

or

$$P_{t, \text{lim}} = 41.38 \frac{f_{ck}}{f_y} \times \frac{x_{\text{lim}}}{d}$$

Consider d in P_t for strength criteria
and D for criteria like Temp. R/F, shrinkage R/F etc

Grade	x_{lim}	R_{lim}	$P_{t, \text{lim}}$	
Fe 250	$0.53 d$	$0.148 f_{ck}$	$0.088 f_{ck}$	
Fe 415	$0.48 d$	$0.138 f_{ck}$	$0.048 f_{ck}$	Rat level!
Fe 500	$0.46 d$	$0.133 f_{ck}$	$0.038 f_{ck}$	

$\Downarrow$
Valid for all

only singly rectangular!

Ques Obtain MOR for the following different amount of A_{st} provided, also compare the rate of increase in MOR w.r.to percentage increase in tensile steel in different %s and give observation
Take M20, Fe415

a) calculate $A_{st, \text{lin}}$ & MOR_{lin}

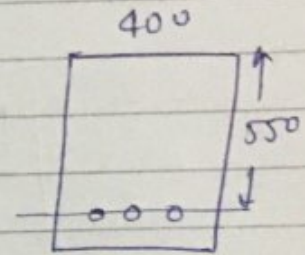
b) MOR when $A_{st} = 4 - 20 \phi$

$A_{st} = 6 - 20 \phi$

$A_{st} = 7 - 20 \phi$

$A_{st} = 10 - 20 \phi$

$b = 400 \text{ mm}$, $d = 550 \text{ mm}$.



Stress	$0.8 f_{yd}$	$0.85 f_{yd}$	$0.90 f_{yd}$	$0.95 f_{yd}$	$0.975 f_{yd}$	f_{yd}
Strain	0.00144	0.00163	0.00192	0.00241	0.00276	0.00380

$$a) \frac{P_{t, \text{lin}}}{100} b d = A_{st, \text{lin}}$$

$$= \frac{0.048 \times 400 \times 550 \times 20}{100} = 2112.00 \text{ mm}^2$$

OR

$$C = T$$

$$0.36 f_{ck} b x_{\text{lin}} = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 400 \times 0.48 \times 550 = 0.87 \times 415 A_{st}$$

$$A_{st, \text{lin}} = 2105.86 \text{ mm}^2$$

$$MOR_{\text{lin}} = R_{\text{lin}} b d^2$$

$$= 0.138 f_{ck} \times 400 \times 550^2$$

$$= 333.96 \text{ kNm}$$

OR. $\hookrightarrow$

$$MOR_{lin} = 0.36 \times f_{ck} \times b \times x_{lin} (d - 0.42 x_{lin})$$

$$= 0.36 \times 20 \times 400 \times 264 (550 - 0.42 \times 264)$$

$$= 333.87 \text{ kNm.}$$

T

$$MOR_{lin} = 0.87 f_y A_{st, lin} \left[d - \frac{f_y A_{st, lin}}{f_{ck} b} \right]$$

$$= 0.87 \times 415 \times 2112 \left[550 - \frac{415 \times 2112}{20 \times 400} \right]$$

$$= 335.85 \text{ kNm.}$$

$$x_{lin} = 264 \text{ mm}, A_{st, lin} = 2112 \text{ mm}^2$$

$$M_{lin} = 335.85 \text{ kNm}, \epsilon_c = 0.0035$$

$$\epsilon_{st} = 0.0038$$

(b) $A_{st} = 4 - 20 \phi = 4 \times 314 = 1256 \text{ mm}^2$
 $< A_{st, lin}$

Hence. UR

$$C = T$$

$$0.36 f_{ck} b x = 0.87 f_y A_{st}$$

$$x = 157.45 \text{ mm}$$

$$MOR = 0.36 f_{ck} b x (d - 0.42 x)$$

$$= 0.36 \times 20 \times 400 \times 157.45 (550 - 0.42 \times 157.45)$$

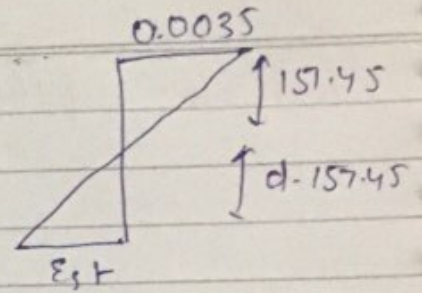
$$= 219.41 \text{ kNm}$$

OR.

$$MOR = 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right] = 219.86 \text{ kNm.}$$

By similar triangle

$$\begin{aligned} \epsilon_{st} &= \frac{0.0035}{157.45} (d - 157.45) \\ &= 0.0087 \end{aligned}$$



$$\begin{aligned} X &= 157.45 \text{ mm}, A_{st} = 1256 \text{ mm}^2 \\ M &= 219.86 \text{ kNm}, \epsilon_c = 0.0035 \\ \epsilon_{st} &= 0.0087 \end{aligned}$$

c) $A_{st} = 6 \times 314 = 1884 \text{ mm}^2$

UR

$$C = T$$

$$x = 236.18 \text{ mm}$$

$$\begin{aligned} MOR &= 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{cu} b} \right] \\ &= 307.64 \text{ kNm} \end{aligned}$$

$$\begin{aligned} X &= 236.18 \text{ mm}, A_{st} = 1884 \text{ mm}^2 \\ M &= 307.64, \epsilon_c = 0.0035 \\ \epsilon_{st} &= 0.0046 \end{aligned}$$

d) $A_{st} = 7 \times 314 = 2198 \text{ mm}^2$

OR

$$X = 275.55, X_{lim} = 264 \text{ mm}$$

$$X_{assume} = 269.77 \text{ mm}$$

$$\epsilon_{st} = \frac{0.0035}{269.77} (550 - 269.77)$$

$$\epsilon_{st} = 0.0036$$

by interpolation $f_{st} = \left(0.0975 + \left(\frac{1-0.975}{380-276} \right) (360-276) \right) \times f_{yd}$

$$= 359.31 \text{ N/mm}^2$$

$$X_{new} = \frac{F_{st} A_{st}}{0.36 f_{ck} b}$$

$$= \frac{359.31 \times 2198}{0.36 \times 20 \times 400}$$

$$= 274.22 \text{ mm} \neq X_{assumed}$$

Trial 2

$$X_{anne} = \frac{X_{new} + X_{assumed}}{2}$$

$$= \frac{274.22 + 269.77}{2}$$

$$= 272 \text{ mm}$$

$$E_{st} = 0.00357 \text{ mm}$$

$$f_{st} = 0.0975 + \left(\frac{1-0.975}{380-276} \right) \times (357-276)$$

$$f_{st} = 359.05 \text{ N/mm}^2$$

$$X_{new} = \frac{F_{st} A_{st}}{0.36 f_{ck} b} = 274.024 \text{ mm}$$

$$= X_{assumed}$$

Considering $X = 272 \text{ mm}$

$$MOR = \frac{0.87 f_{st} A_{st}}{f_{ck} b} \left(d - \frac{f_{st} A_{st}}{f_{ck} b} \right) = 343 \text{ kNm}$$

$$= 358.18 \text{ kNm from C.}$$

$$X = 272 \text{ mm}, A_{st} = 2198 \text{ mm}^2$$

$$M = 343 \text{ kNm}, E_c = 0.0035$$

$$E_{st} = 0.00357 \text{ mm}$$

$$MOR = 0.36 f_{ck} b x (d - 0.42 x)$$

$$= 341.34 \text{ kNm}$$

e) $A_{st} = 10 \times 314 = 3140 \text{ mm}^2 > A_{st, \text{lim}}$
CR

$$X = \frac{0.87 F_y A_{st}}{0.36 f_{ck} b} = \frac{0.87 \times 415 \times 3140}{0.36 f_{ck} \times b} = 393.64 \text{ mm}$$

exact depth $\Rightarrow X = 355 \text{ mm}$

MOR = 409 kNm.

Type	A_{st}	MOR
① UR	1256	219.86 kNm
② UR	1884	307.64 kNm
③ B	2112	335.85 kNm
④ OR	2198	343 kNm
⑤ OR	3140	409 kNm

① & ②

$$\% \text{ increase in steel} = \frac{1884 - 1256}{1256} \times 100 = 50\%$$

$$\% \text{ increase in MOR} = \frac{307 - 219}{219} \times 100 = 40.18\%$$

④ & ⑤

$$\% \text{ increase in steel} = \frac{3140 - 2198}{2198} = 42.85\%$$

if these rate of increase of MOR were to be same as that in UR $\Rightarrow$ % increase in MOR

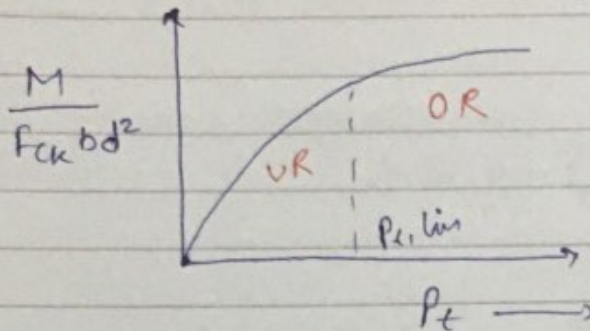
for 50% $\uparrow$ in $A_{st} \rightarrow$ MOR 40.18% $\uparrow$

$$42.85\% \uparrow \text{ in } A_{st} \Rightarrow \frac{40.18}{50} \times 42.85$$

$$= 34.43\%$$

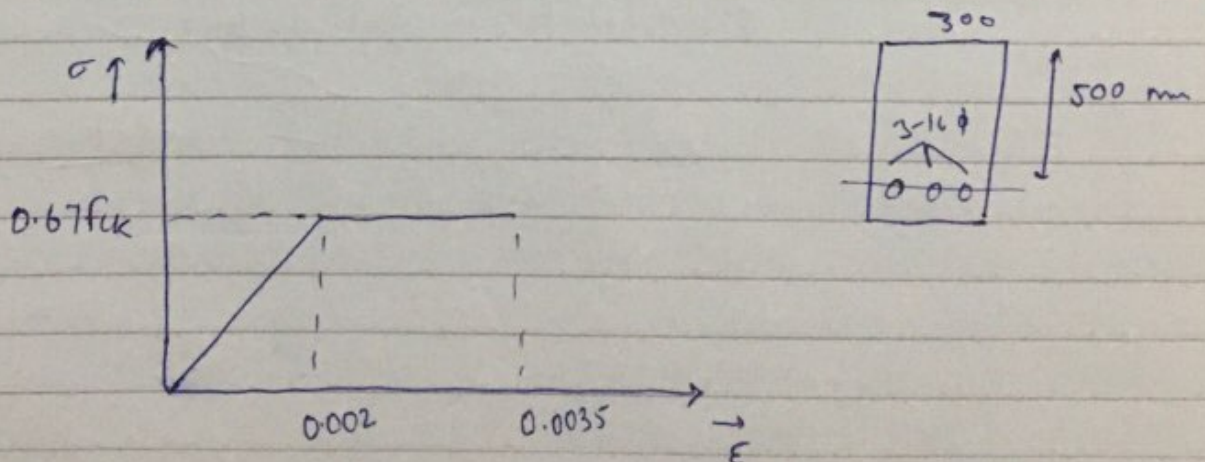
However, actual increase in MOR
from 4 to 5

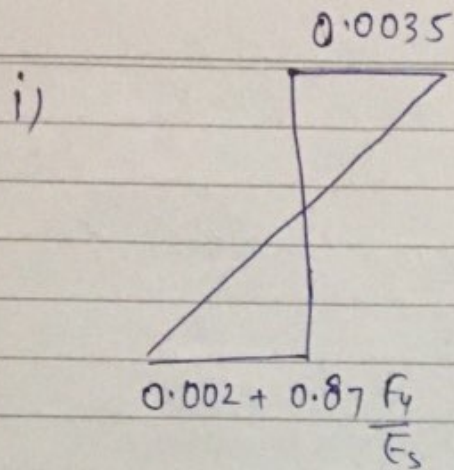
$$= \frac{409 - 393}{393} \times 100 = 19.24\%$$



NOTE The gain in MOR with increase steel Area follows a nearly linear relationship almost upto the balance point ($P_{t, lin}$). However beyond the balance point MOR is generally governed by compressive strength of concrete and the rate of increase in MOR decreases.

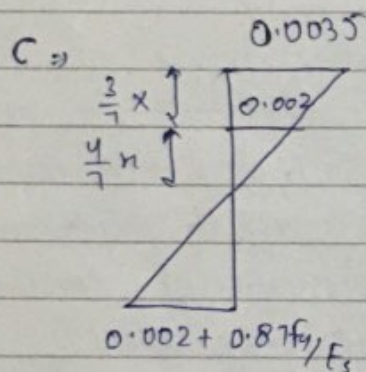
Que Considering the following stress-strain curve for concrete with partial safety factor 1, calculate
1) Limiting depth of NA
2) depth of NA for the below section, (M20, Fe 415)



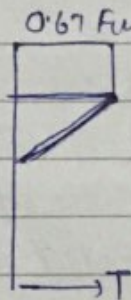


$$X_{lim} = 0.48 d. = 240 \text{ mm.}$$

ii) $C = T$



$$T = 0.87 f_y A_{st}$$



$$C = 0.67 f_{yk} \times \frac{3}{7} \pi \times b + \frac{1}{2} \times 0.67 f_{yk} \times \frac{4}{7} \pi \times b$$

$$C = 0.47 f_{yk} b \pi$$

$$\therefore 0.47 f_{yk} b \pi = 0.87 f_y A_{st}$$

$$X = 75.93 \text{ mm.}$$

UR.

DESIGN:-

Generally external loads or load effects are given and it is required to obtain a suitable c/s and R/F with adequate spacing as per IS code.

Design problem does not have a unique solⁿ.
The following steps shall be followed.

Step-1. obtain c/s if not given

- in order to calculate DL, section dimensions are required, if not given width can be considered from shuttering criteria, column size, brick wall size, Architectural consideration, or economy criteria.
- Basic size of b can be taken as 115 mm, 230, 250, 300 mm etc.
- Also $\frac{b}{d} \approx \frac{b}{D} \approx 0.5$ can be considered
- if D is not given take approx $\left(\frac{L}{10} - \frac{L}{12} \right)$

Step-2. Calculate design BM (BM ultimate)

- based on size and LL, calculate design BM with appropriate partial safety factor

NOTE • When nothing is mention / service load / working load / characteristic load / safe load $\Rightarrow$ Multiply partial safety factor

- ultimate load / factored load / design load
 $\Rightarrow$ Don't Multiply partial safety factor

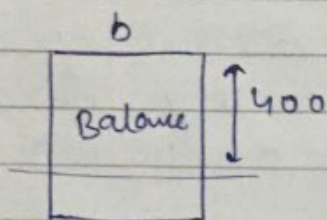
- Max. Moment doesn't necessarily mean factored BM, it can be the moment at the max. BM location.

Step-3: obtain design depth (d, D)

- The c/s shall be such that

$$M_{lim} \geq BM_u$$

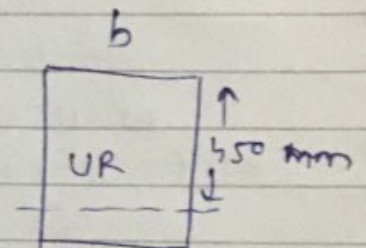
$$d \geq \sqrt{\frac{B.M_u}{R_{lim} b}}$$



$$BM_u = 200 \text{ kNm} = M_{lim}$$

$$A_{st, req.} = A_{st, lim} = \frac{P_{t, lim} \times b \times d}{100}$$

$$= 1200 \text{ mm}^2$$



$$BM_u = 200 \text{ kNm} < M_{lim}$$

$$A_{st} = 1000 \text{ mm}^2$$

$$A_{st, lim} = \frac{P_{t, lim} \times b \times d}{100}$$

$$= 1400 \text{ mm}^2$$

The d calculated from the above formula is increased by approx. 5%.

This will generally make the design c/s UR after providing R/F in the form of bars.

Obtain D based on exposure cond. and shear stirrups diameter.

NOTE: The overall depth provided shall be less than equal to overall depth assumed for calculating DL. as providing higher than assumed overall depth will increase the actual moment and the section design will be unsafe

Step-4: Calculate A_{st}

$$M = 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} b} \right]$$

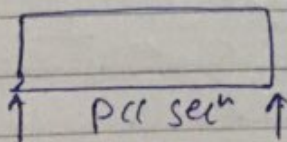
$$A_{st} = \frac{f_{ck}}{2 f_y} \left[1 - \sqrt{1 - \frac{4.6 M}{f_{ck} b d^2}} \right] b d$$

Step-5: Calculate $A_{st, min}$, $A_{st, lim}$, $A_{st, max}$.

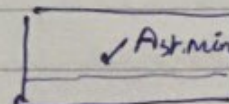
$$\frac{A_{st, min}}{b d} = \frac{0.05}{f_y}$$

$$A_{st, lim} = \frac{P_{t, lim}}{100} \times b d$$

$$A_{st, Max} = 0.04 b D$$



Uncracked moment = M_{cr}



At cracked section
= M_{cr}

$A_{st, provided} > A_{st, min}$ To avoid
a) snapping / Rupture of RIF bars
b) ensure adequate ductility @ ultimate limit state
(if both options available in exam then choose in order of priority)

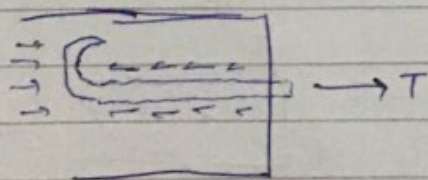
$A_{st, min}$ is the amount of RIF for which,
MOR of RCC beams with $A_{st, min}$ in cracked analysis $\approx$ MOR of RCC beam in uncracked secⁿ Analysis

- $A_{st, provided} \leq A_{st, lim}$ To ensure UR and balance design
- $A_{st, provided} \leq A_{st, Max}$ To avoid practical difficulty in placing and compaction of concrete

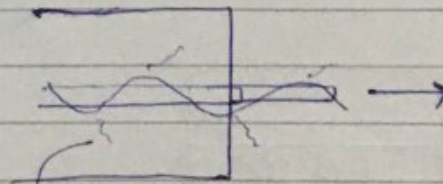
Max Area of Compression RIF = $0.04bd$

Step-6: select bar type, dia as per code criteria

- g) Generally no plane bar provided in flexure due to less bond strength, plane bars can be provided for shear RIF

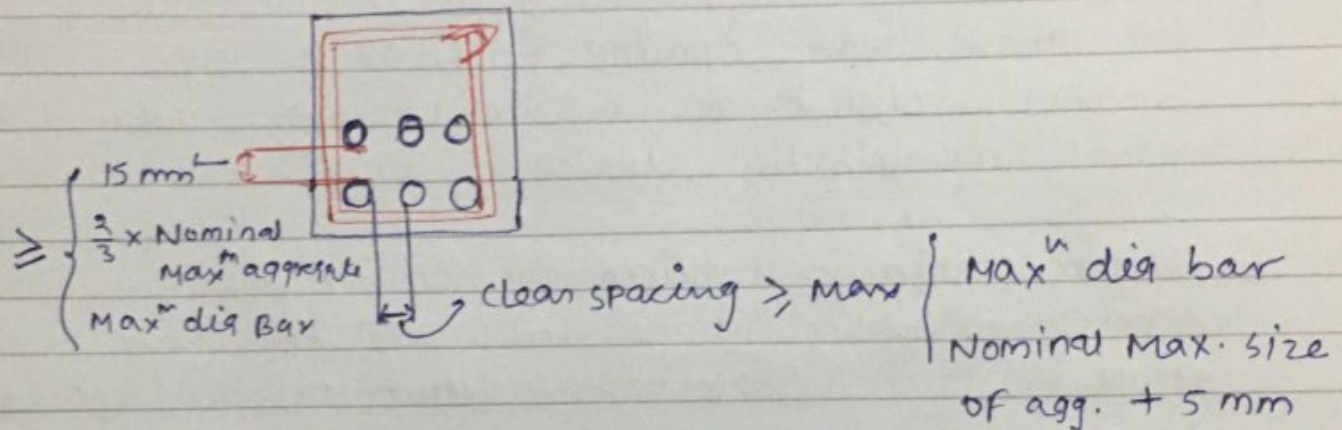


MS bar
provided for
additional safety.



HYSD
→ wedge action in
deformed bars.

- select bar dia based on spacing criteria
- R/F shall be placed symmetrically in the C/S
- Generally, same dia bars shall be used, For different dia bars, place the Max dia bars at corners



Minimum spacing criteria (clear spacing b/w bars) are based on Workability criteria.

If Bars can not be placed in a single layer then place the bars in two or more layers the bars shall be vertically in line.

Max Spacing criteria for bars in tension are based on controlling crack width and improving bond

Grade of steel

Fe 250

Fe 415

Fe 500

Max. spacing

300

180

150

only valid for beam.

NOTE:- ① Minimum spacing criteria is valid for all structure

② Max spacing criteria as per IS code based on clear spacing. However in design question we generally consider it as center to center.

③ The above Max. spacing is based on 0% moment redistribution. as per IS code, Max. moment redistribution allowed is 30%.

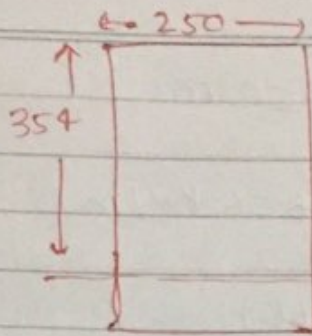
④ Higher strain is required to achieve design stress in higher grade steel and this will result in larger crack width. Hence the Max. spacing decreases as the grade of steel increases.

⑤ No limitation on Max. spacing for compression Reinforcement.

⑥ providing large number of small dia Bars is more effective than providing small number of large diameter Bars in controlling crack and improving Bond.

Que. for the below section with M20, Fe415, calculate the amount of Tensile R/F for a service load moment of 57.66 kNm including DL. Also comment on material's consume if this section is to be designed with WSM.

M20, Fe 415



$$M = 57.66 \text{ kNm}$$

$$M_u = 1.5 \times 57.66 = 86.49 \text{ kNm}$$

$$M_{lim} = R_{lim} b d^2$$

$$M_{lim} = 0.038 \times 20 \times 250 \times 354$$

$$M_{lim} = 86.47 \text{ kNm}$$

From $c = T$

Hence, BALANCED.

$$0.36 f_{ck} b x_{lim} = 0.87 f_y A_{st}$$

$$A_{st} = 847.13 \text{ mm}^2$$

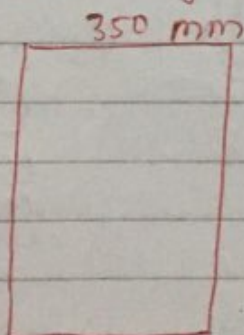
$$\} x_{lim} = 0.48d$$



A section designed with WSM will consume Higher concrete and lesser R/F than section designed with LSM

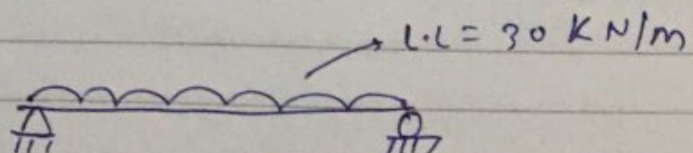
NOTE. For same BM, Balance section design will consume less concrete and higher R/F than under reinforced section.

Que: Design the c/s of a simply supported beam having eff. span of 7.5 m ($b = 350 \text{ mm}$, w load = 30 kN/m excluding self wt., M20 & Fe 415)



M20, Fe 415, $b = 350 \text{ mm}$

$$L = 7.5 \text{ m}$$



assume $B = \frac{L}{10} = \frac{7.5}{10} = 750 \text{ mm}$

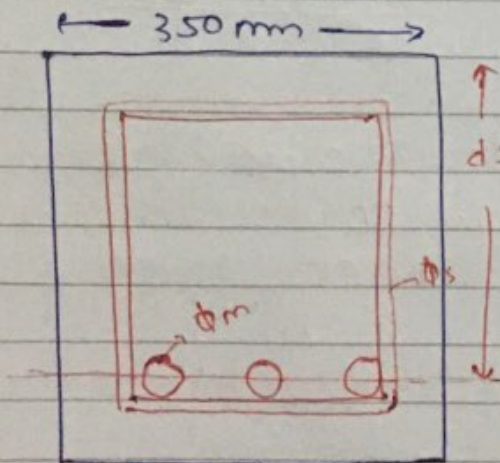
$D.L = 0.35 \times 0.75 \times 25 = 6.56 \text{ kN/m}$

factored load $= 1.5 (DL + LL)$
 $= 1.5 (6.56 + 30)$
 $= 54.84 \text{ kN/m}$

$(B.M)_{ultimate} = \frac{W_u l^2}{8} = \frac{54.84 \times 7.5^2}{8}$
 $= 385.59 \text{ kNm}$

$d \geq \sqrt{\frac{M_{ult}}{R_{lin} b}} = \sqrt{\frac{385.59}{0.138 \times 20 \times 350}} = 631.79 \text{ mm}$

$d_{provided} \text{ nearly } \approx 1.05 \times 631.79$
 $= 663.37 \text{ mm}$



$D = d + \frac{\phi_m}{2} + \phi_s + \text{Nominal cover}$
 $= 701$

$\phi_m \rightarrow 20$
 $\phi_s \rightarrow 8$
 $\text{Nominal cover} \rightarrow 20$

Mild.
M20 $\rightarrow 20 \text{ mm}$
exposure
cond.

NOTE: During the design calculation the eff. cover shall be consider slightly on a higher side than that calculated by assuming Main R/F Dia and shear Dia. to avoid nominal cover less than specified minimum in the final c/s.

$$D = 663.37 + \overset{\rightarrow 20}{\frac{\phi_m}{2}} + \overset{\rightarrow 8}{\phi_s} + 20 = 701.37 \text{ mm}$$

Take $D = 710 \text{ mm}$. $< D_{\text{assumed}}$ safe

The final D provided shall be less than D assumed for DL calculation.

$$D = 710 \text{ mm}$$

$$d = 665 \text{ mm}, \text{ effective cover} = 45 \text{ mm}$$

$$A_{st} = \frac{f_{ck}}{2 f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} b d^2}} \right] b d$$

$$A_{st} = \frac{20}{2 \times 415} \left[1 - \sqrt{1 - \frac{4.6 \times 385.59 \times 10^6}{20 \times 350 \times 665^2}} \right] 350 \times 665$$

$$\underline{A_{st, \text{req.}} = 1943.52 \text{ mm}^2}$$

$$A_{st, \text{lim}} = 2234.40 \text{ mm}^2 \left\{ \begin{array}{l} A_{st, \text{lim}} = \frac{P_{t, \text{lim}}}{100} \times b d \end{array} \right.$$

$$A_{st, \text{min}} = \frac{0.85}{f_y} b d = 476.71 \text{ mm}^2$$

$$A_{st, \text{max}} = 0.04 b D = 9940 \text{ mm}^2$$

$$i) 7-20 \phi = 2198 \text{ mm}^2$$

$$\text{spacing} = \frac{350 - 2 \times (20 + 8) - 7 \times 20}{6} = 25.66$$

$$ii) 4-25 \phi = 1960 \text{ mm}^2$$

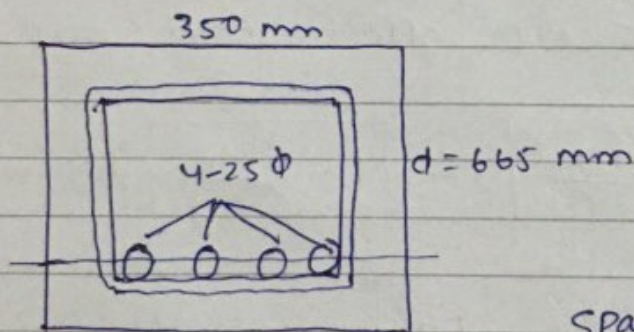
$$\text{spacing} = \frac{350 - 2 \times (20 + 8) - 4 \times 25}{3}$$

$$\text{spacing} = 64.67 \text{ mm} \rightarrow \left\{ \begin{array}{l} \text{max. bar dia} \\ \text{aggregate} + 5 \text{ mm} \\ \text{20 mm} \end{array} \right.$$

let us provide 4 numbers of 25 mm dia

$$A_{st} \text{ provided} = 4 \times 490 = 1960 \text{ mm}^2$$

$$A_{st, \min} < A_{st, \text{provided}} < A_{st, \text{lim}}$$



c-c spacing

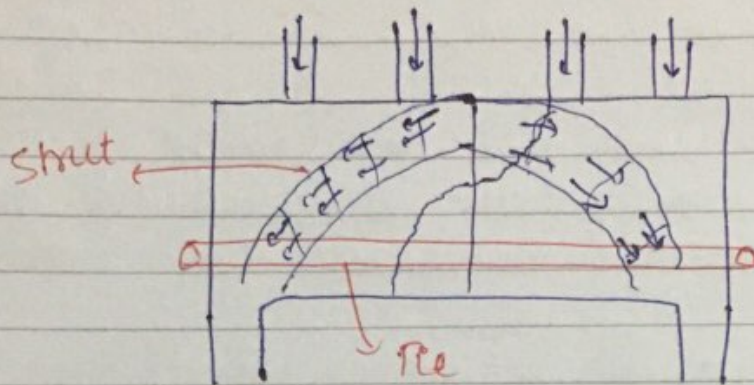
$$\text{Spacing} = 64.67 < 180 \text{ mm.}$$

$$+ 2 \times 25$$

$$\frac{\quad}{2} \approx \underline{0.k}$$

Moment is more in S.S so, we provide continuous support
Also the Number of paths to transfer the load are more

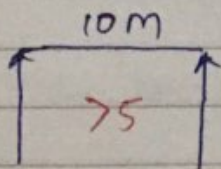
DEEP BEAMS :-



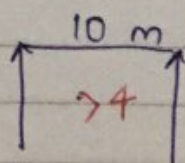
- provided when less number of columns are required like in the basement.
- Govern primarily by shear
- plane section remaining plane after bending is **NOT valid** due to significant warping of c/s
- Arch action is predominant and its design by **Strut and Tie Model**.
- As per IS code a Beam will be called Deep beam if

$$\frac{l_{eff}}{D} < 2 \rightarrow \text{Simply supported}$$

$$\frac{l_{eff}}{D} < 2.5 \rightarrow \text{for continuous support}$$



S.S



Continuous support

[5] Doubly R/F Rectangular sections

- section with both Tension steel and Compression steel

Purpose :-

When ~~we~~ b, D, F_{ck}, f_y are fixed and ~~we~~ $BM_{ultimate} > M_{lim}$ of a singly R/F section, the section design will be OR.

To avoid this, Doubly R/F section are designed. By providing R/F on compression side, the section can resist BM greater than M_{lim} for a singly R/F section without being OR.

presence of compression R/F will —

- 1) be helpful in stress reversal (dynamic loading) (in reversal $\rightarrow$ stress will be opposite)
- 2) Reduce Long term effect of creep & shrinkage
- 3) Reduce Cls size.
- 4) increase headroom clearance
- 5) ensure better lighting due to smaller depth
- 6) Also good from Aesthetic & Architectural Consideration
- 7) Hold the shear stirrup at place.

- even if NOT required from strength criteria, Doubly R/F section are generally provided)
- The compression R/F must be enclosed by stirrup to prevent buckling of Compression R/F

NOTE • In Doubly R/F section M_{lim} is neither Fixed nor restricted as we can increase A_{st} and correspondingly A_{sc} to increase M

• Generally Hooks of shear stirrups are provided on compression side to ensure higher Bond strength and thereby less chances of pull out.

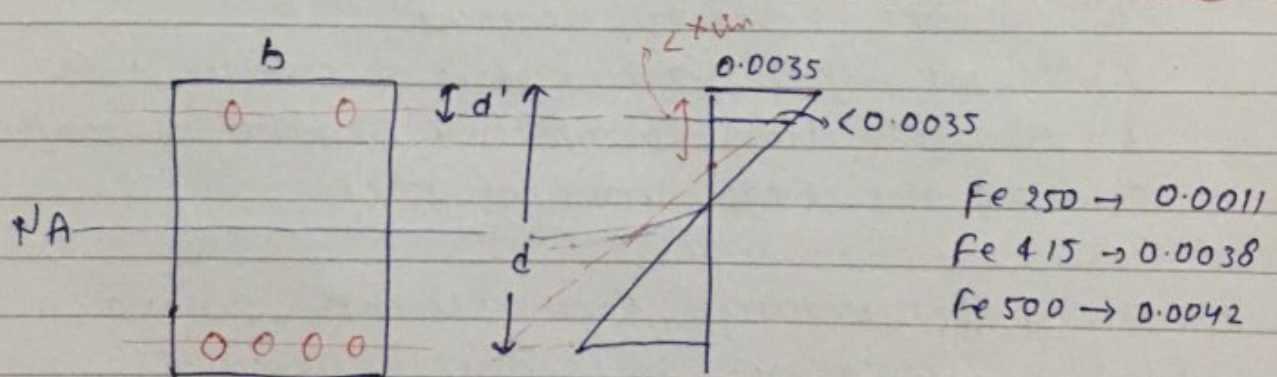
• pull out नहीं होगा compression में
because crack नहीं होगा!

Analysis

• For a given c/s , A_{sc} , A_{st} we need to calculate the MOR.

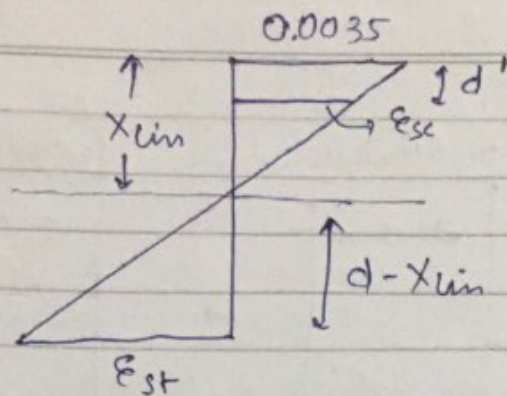
• The section given can be UR, B, OR

Strain @ level of compression steel when ($X = X_{lim}$)



• stress in compression steel will be dependent on strain @ compression steel level

• strain @ compression steel level assuming Max. permissible value of NA depth i.e X_{lim} and for $\frac{d'}{d} = 0.05$ to 0.20
(Practical Range)



$$\epsilon_{sc} = \frac{0.0035}{X_{uin}} (X_{uin} - d')$$

$$\epsilon_{sc} = 0.0035 \left(1 - \frac{d'}{X_{uin}} \right)$$

$$\epsilon_{sc} = 0.0035 \left[1 - \frac{d'}{d} \right]$$

$\hookrightarrow 0.53d$
 $0.48d$
 $0.46d$

Grade of steel	$\frac{d'}{d} = 0.05$	$\frac{d'}{d} = 0.2$
Fe 250	0.003169	0.00217
Fe 415	0.00313	0.00204
Fe 500	0.00311	0.00197

NOTE In UR Doubly RIF section, the strain in compression steel will be less than the above tabulated values.

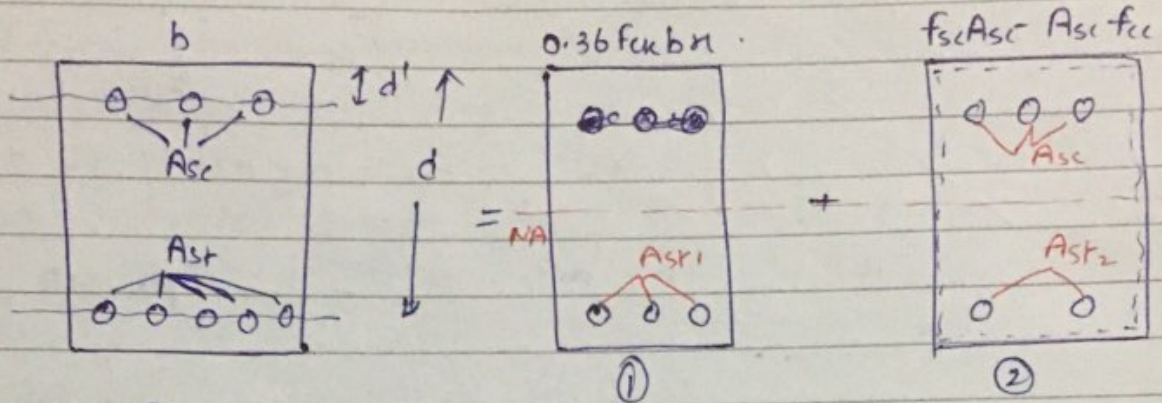
- For UR/B Doubly RIF section, stress in Tension steel = $0.87 f_y$ for all grade of steel

① Ultimate limit state.

However ^{stress} ~~strain~~ in compression steel will be $0.87 f_y$ for Fe 250 (most of the cases) and less than $0.87 f_y$ for HYSD Bars.

- So we need σ - ϵ curve or table for HYSD Bars to calculate stress in compression steel.
- Also we may require σ - ϵ curve of concrete to calculate stress in concrete at the level of compression steel. However this can be taken as $0.45 f_{ck}$

Calculation of MOR :-



f_{sc} = stress in compression steel

f_{cc} = stress in concrete at the level of compression steel.

$$C = T$$

$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} b x + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$$

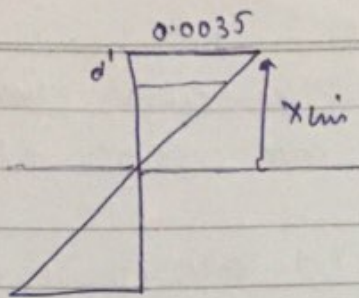
$$= 0.87 f_y A_{st}$$

* $MOR = 0.36 f_{ck} b x (d - 0.42 x) + A_{sc} (f_{sc} - f_{cc}) (d - d')$

$$MOR = (C_1 + C_2) \left[d - \frac{(C_1 \times 0.42 x + C_2 \times d')}{C_1 + C_2} \right]$$

In the above eqⁿ of MOR we require x and x is dependent on f_{sc} & f_{cc} . However f_{sc} & f_{cc} are dependent on x , and hence we can't have a direct solⁿ, we need to adopt the following Trial and error procedure.

Step-1



$$\epsilon_{sc} = \frac{0.0035}{x_{lim}} (x_{lim} - d')$$

Step-2 using ϵ_{sc} calculate f_{sc} & f_{cc} from the respective σ - ϵ table / curve

Step-3 using $c = T$, calculate x_{new}

$$0.36 f_{ck} b x_{new} + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

Step-4

Case-1 $x_{new} \approx x_{lim}$

implies x_{new} is the correct depth of NA

Hence Balance section!

Case-2 $x_{new} < x_{lim}$

x_{new} is not correct depth of NA and we need to carry out further Trial & error

However UR

Case-3 $x_{new} > x_{lim}$

x_{new} is not correct depth of NA and we need

OR, $MOR = MOR_{lim}$

$$\text{Also } f_{st} < 0.87 f_y$$

NOTE: MOR in OF Doubly RIF section for the provided amount of compression R/F

$$MOR_{in} = 0.36 f_{ck} b X_{lim} (d - 0.42 X_{lim}) + A_s (f_{sc} - f_{cc}) (d - d')$$

f_{sc} shall be calculated at $x = X_{lim}$

2) σ - ϵ curve for Fe250 is not required as f_{sc} can be taken as $0.87 f_y$

for Fe250 steel or when f_{sc} is given no need to carry out Trial and error procedure

In case of HYSD Bar $f_{sc} \cong 0.8 f_y$ can be taken if σ - ϵ table not given.

Que: Determine the MOR of a doubly RIF section having $b = 250 \text{ mm}$, $d = 460 \text{ mm}$, $d' = 40 \text{ mm}$, $f_{ck} = 20 \text{ N/mm}^2$ for the following conditions.

- 1) $A_{st} = 2000 \text{ mm}^2$, $A_{sc} = 1200 \text{ mm}^2$, Fe250
- 2) $A_{st} = 2000 \text{ mm}^2$, $A_{sc} = 1000 \text{ mm}^2$, Fe 415
- 3) $A_{st} = 2000 \text{ mm}^2$, $A_{sc} = 804 \text{ mm}^2$, Fe 415

ϵ	0.00144	0.00163	0.00192	0.00241	0.00276	0.00380
σ	$0.8 f_{yd}$	$0.85 f_{yd}$	$0.9 f_{yd}$	$0.95 f_{yd}$	$0.975 f_{yd}$	f_{yd}

1) from $C = T$

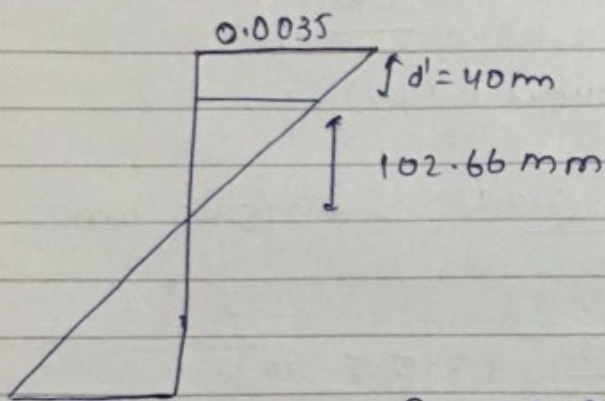
$$0.36 f_{ck} b x + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 \times 1200 (0.87 \times 250 - 0.45 \times 20) \\ = 0.87 \times 250 \times 2000$$

$$x = 102.66 \text{ mm}$$

$$x_{lim} = 0.53 \times d \\ = 0.53 \times 460 = 243.8 \text{ mm}$$

$$x < x_{lim} \quad \underline{\text{VR}}$$



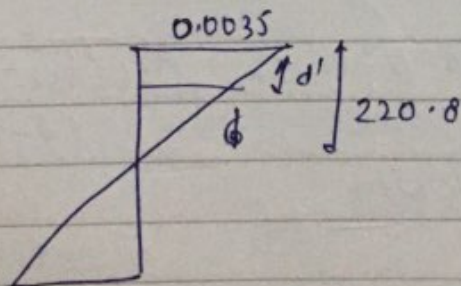
$$\epsilon_{sc} = \frac{0.0035}{102.66} (102.66 - 40) \\ = 0.0021 > 0.0011$$

$$f_{sc} = 0.87 f_y \text{ is correct}$$

$$\text{MOR} = 0.36 \times F_{ck} \times b \times x (d - 0.42 x) + A_{sc} (f_{sc} - f_{ck}) (d - d') \\ = 0.36 \times 20 \times 250 \times 102.66 (460 - 0.42 \times 102.66) + \\ 1200 (0.87 \times 415 - 20 \times 0.45) (460 - 40) \\ = 102.11 \text{ kNm}$$

2) ^{assumed} $x = x_{lim}$

$$x_{lim} = 0.48 d = 0.48 \times 460 = 220.8$$



$$\epsilon_{sc} = \frac{0.0035}{x_{lim}} (x_{lim} - d') \\ = 0.00286$$

$$f_{sc} = \left[0.975 + \frac{(1-0.975)}{(380-276)} (286-276) \right] f_{yd}$$

from $c = T$

$$0.36 f_{ck} \times b \times X_{new} + A_{sc} (f_{sc} - f_{ci}) = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 \times X_{new} + 1000 (352.75 - 0.45 \times 20) = 0.87 \times 415 \times 2000$$

$$X_{new} = 210.19 \text{ mm} \quad \underline{\text{UR.}}$$

$$X_{new} \neq X_{assumed} (X_{lim})$$

trial 2

$$X = \frac{220.8 + 210.19}{2} = 215.5 \text{ mm}$$

$$\epsilon_{sc} = \frac{0.0035}{215.50} (215.5 - 40)$$

$$= 0.00285$$

$$f_{sc} = \left[0.975 + \frac{(1-0.975)}{(280-276)} (285-276) \right] f_y \times 0.87$$

$$f_{sc} = 352.80 \text{ N/mm}^2$$

{ same आँकड़ा है
we can't iterate further! }

Consider depth of NA as 215.50 mm

$$MOR = 0.36 f_{ck} b x (d - 0.42 x) + A_{sc} (f_{sc} - f_{ci}) (d - d')$$

$$= 207.72 \text{ kNm.}$$

iii)

$$\text{At } x = x_{\text{lim}}$$

$$f_{sc} = 352.75 \text{ N/mm}^2$$

$$C = T$$

$$0.36 f_{ck} b x_{\text{new}} + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 250 \times x_{\text{new}} + 804 (352.75 - 9) = 0.87 \times 415 \times 2000$$

$$x_{\text{new}} = 247.56 \text{ mm}$$

$$x_{\text{lim}} = 220.80 \text{ mm}$$

$$x > x_{\text{lim}}$$

OR.

$$MOR = MOR_{\text{lim}}$$

$$M_{\text{lim}} = 0.36 \times f_{ck} \times b \times x_{\text{lim}} (x.d - 0.42 x_{\text{lim}}) + A_{sc} (f_{sc} - f_{cc}) (d - d')$$
$$= 148.94 \text{ kNm}$$

$$= 262.04 \text{ kNm}$$

Design :-

Two types of problems

- 1) b, d, f_{ck}, f_y & B.M Given Calculate A_{sc}, A_{st}
- 2) F_{ck}, f_y & B.M Given, Calculate section dimension and A_{sc}, A_{st} .

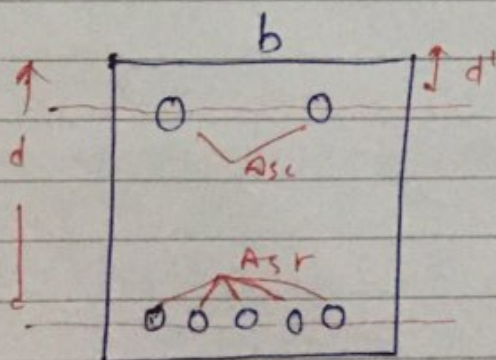
Section shall be design as Doubly R/F when $B_{M_{ult}} > M_{lim}$ of a singly R/F section

NOTE - if b, D are Not given consider the dimension such that the secⁿ can be designed as Doubly R/F
take ϕ

$$d \leq \sqrt{\frac{M}{R_{lim} b}}, \text{ o/w section}$$

will be designed as a singly R/F

(Reduce the above calculated d by 50-100 mm)

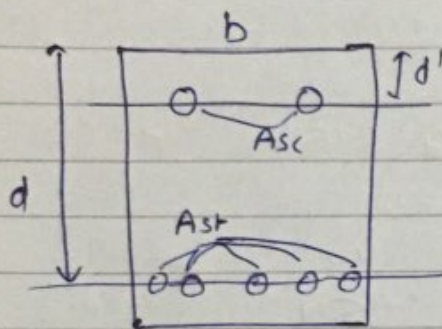


$$0.36 f_{ck} b \eta + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

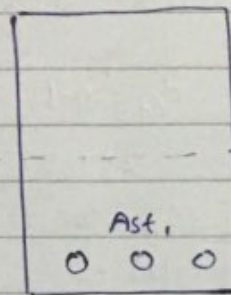
$$M = 0.36 f_{ck} b \eta (d - 0.42 \eta) + A_{sc} (f_{sc} - f_{cc}) (d - d')$$

in the above eqⁿ we've 3 variables and we can't calculate the R/F so to start with doubly R/F section are design as Balance section. i.e $X = X_{lim}$

Calculation of A_{sc} & A_{st} :-

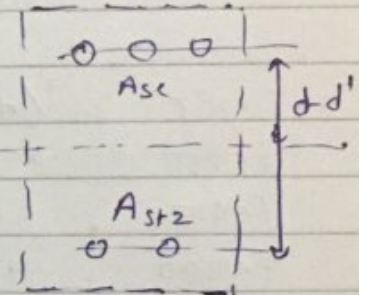


$$MOR = BM_u$$



$$MOR_1 = M_{lim}$$

$$A_{st1} = A_{st,lim}$$



$$MOR_2 = (B \cdot M_u - M_{lim})$$

$$A_{st} = A_{st1} + A_{st2}$$

$$A_{st1} = A_{st,lim} = \frac{M_{lim} (MOR)}{0.87 f_y (d - 0.42 x_{lim})} = \frac{P_{t,lim}}{100} \times b d$$

$$A_{st2} = \frac{M_u - MOR_1 (M_{lim})}{0.87 f_y (d - d')}$$

$$MOR_2 = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

or from $c_2 = T_2$

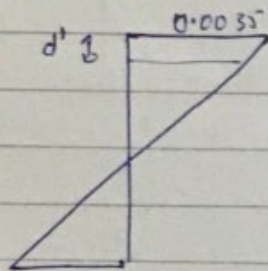
$$A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st2}$$

NOTE: A_{sc} value will be always greater than A_{st2} . Also we need to ensure that after providing R/F in the form of bars the section does not become OR.

The f_{sc} & f_{ec} value correspond to $X = X_{lin}$

f_{sc} value for different d'/d (valid only when $X = X_{lin}$)

Grade	$d'/d = 0.05$	$d'/d = 0.1$	$d'/d = 0.15$	$d'/d = 0.2$
Fe250	217.5	217.5	217.5	217.5
Fe415	355	352	342	329.2
Fe500	423.9	411.3	395.1	370.3

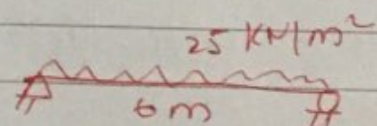
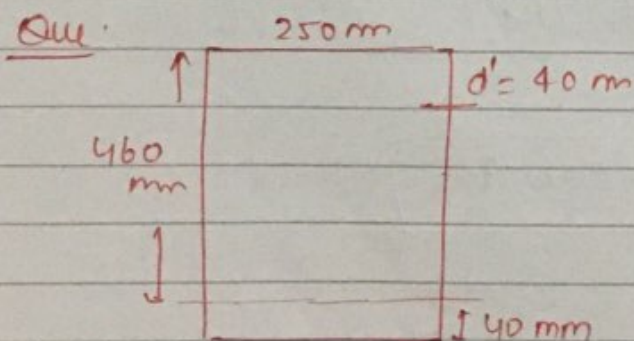


$$\epsilon_{sc} = \frac{0.0035}{X_{lin}} (X_{lin} - d')$$

$$= 0.0035 \left[1 - \frac{d'}{X_{lin}} \right]$$

$0.53 \rightarrow 250$
 $0.48 \rightarrow 415$
 $0.46 \rightarrow 500$

Linear interpolation shall be done for member d'/d



Fe415
M20

- Calculate amount of R/F
- Arrange bars in c/s
- Check the type of c/s designed in Option 2 and if OR then revise.

$$DL = 0.25 \times 0.5 \times 25 = 3.125 \text{ kN/m}$$

$$W_u = 1.5 (3.125 + 25) = 42.1875 \text{ kN/m}$$

$$M_u = \frac{W_u l^2}{8} = 109.04 \text{ kNm}$$

$$M_{lim} = 0.138 f_{ck} b d^2 = 146 \text{ kNm}$$

$$BM_u > M_{lim} \text{ Doubly R/F sec.}$$

$$A_{st} = A_{st1} + A_{st2}$$

$$A_{st1} = A_{st,lim} = \frac{P_{t,lim}}{100} b d = \frac{0.048 f_{ck} b d}{100} = 1104 \text{ mm}^2$$

$$BM_u - M_{lim} = 0.87 f_y A_{st2} (d - d')$$

$$A_{st2} = 289.10 \text{ mm}^2$$

$$\frac{d'}{d} = 0.087$$

$$\text{for Fe 415 } f_{sc} = 355 \text{ N/mm}^2 \quad (d'/d = 0.05)$$

$$f_{sc} = 352 \text{ N/mm}^2 \quad (d'/d = 0.1)$$

$$f_{sc} = 355 + \frac{(352 - 355)}{(0.1 - 0.05)} (0.087 - 0.05) = 352.8 \text{ N/mm}^2$$

$$\text{from } C_2 = T_2$$

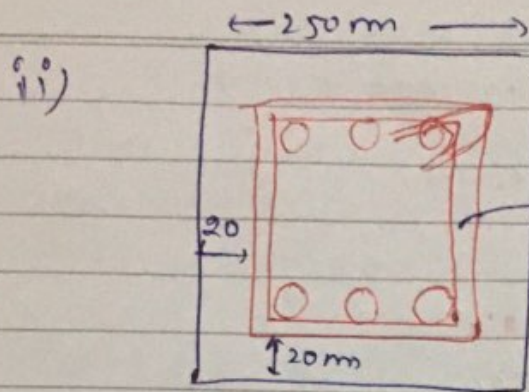
$$A_{sc} (f_{sc} - f_{cl}) = 0.87 f_y A_{st2}$$

$$A_{sc} = 303.6 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1393.1 \text{ mm}^2$$

$$A_{sc} = 303.6 \text{ mm}^2$$

$A_{st, min}$ check is not required because by default it will be higher.



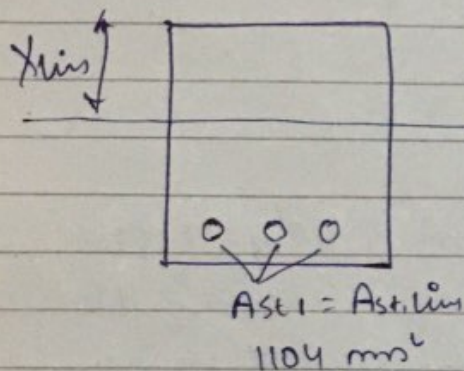
$$A_{sc, req} = 303.6 \text{ mm}^2$$

$$A_{sc, prov} = 3 - 12 \phi = 339 \text{ mm}^2$$

$$\phi_s = 8 \text{ mm}$$

$$A_{st, req} = 1393.1 \text{ mm}^2$$

$$A_{st, prov} = 3 - 25 \phi = 3 \times 490 = 1470 \text{ mm}^2$$



$$A_{sc} = \frac{0.87 f_y (A_{st, prov} - A_{st, min})}{(f_{sc} - f_{cc})}$$

$A_{st2} = A_{st, prov} - A_{st, min}$
= 366 mm²

NOTE.

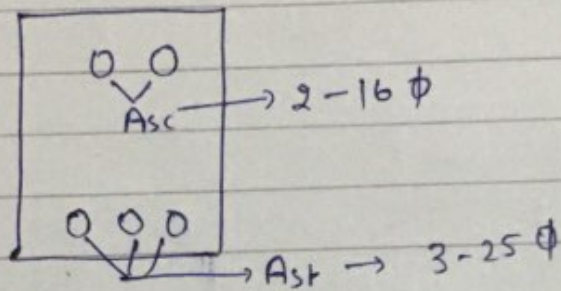
- To ensure the final CIS with RIF bars under RF or balanced, the compression RIF shall be calculated after providing Tensile RIF in the form of RIF bars and A_{sc}

$$A_{sc} = \frac{0.87 f_y (A_{st, prov} - A_{st, min})}{(f_{sc} - f_{cc})}$$

- NO $A_{st, min}$ check required in doubly RIF As A_{st} will be always greater than $A_{st, min}$

$$A_{sc} = \frac{0.87 f_y (1470 - 1104.00)}{(352.8 - 9)}$$

$$A_{sc} = 384.36 \text{ mm}^2$$

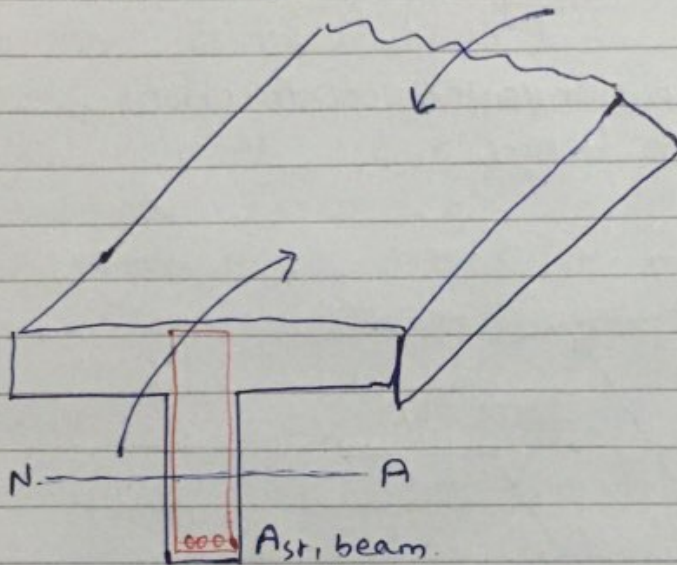
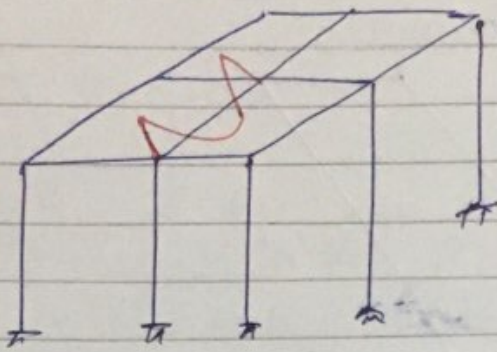


$$A_{sc \text{ prov}} = 2 \times 201$$

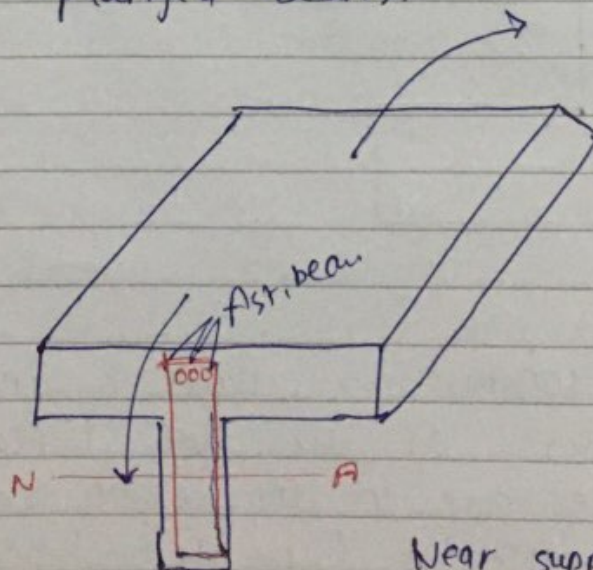
$$= 402 \text{ mm}^2$$

$$\approx$$

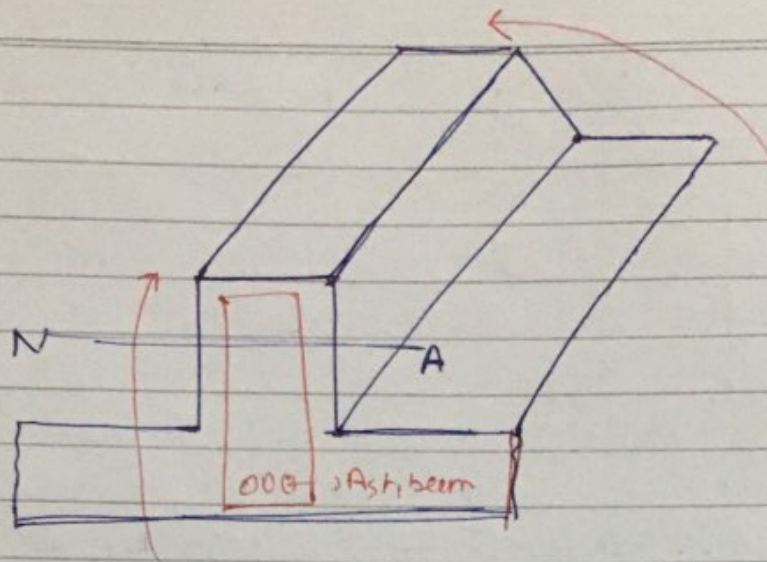
⑥ FLANGED SECTION



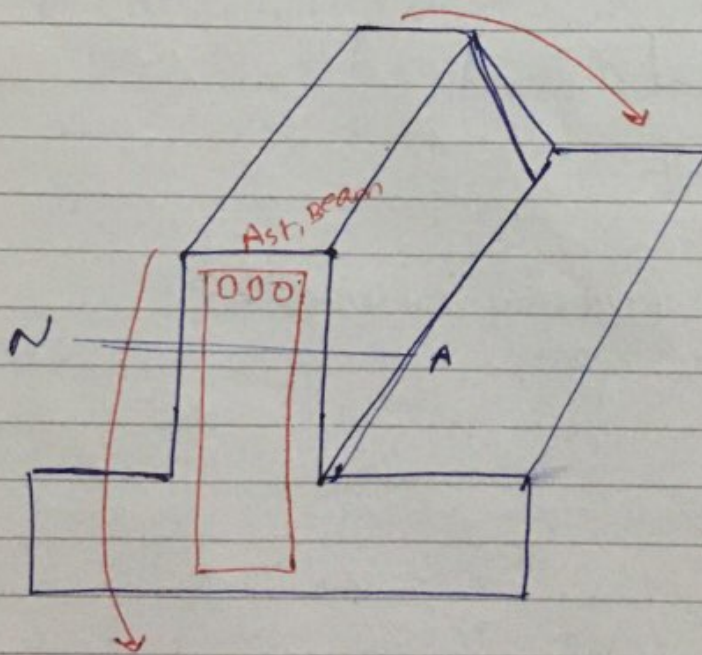
Near mid span (Normal construction)
flanged beam.



Near support, Normal construction
Rectangular beam.



Near mid span (inverted construction)
Rectangular beam.



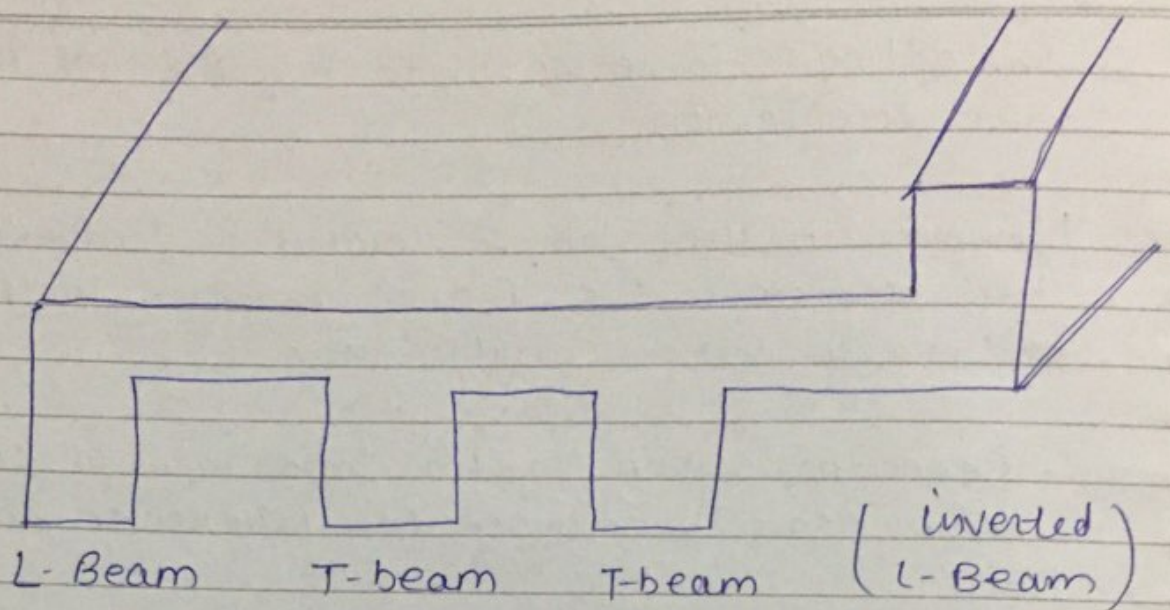
In monolithic construction slabs & Beams
are cast together. If the slab (flange)
in such cases are in compression zone,
they become effective (partly or wholly)

in adding significantly to the Area of concrete in compression.

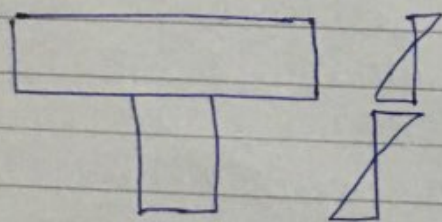
However if the slab is located in tension zone the concrete in the flange becomes ineffective in crack section analysis.

- Depending upon location and type of construction beams can be designed as Rectangular or Flange
- For Normal construction, near support in a continuous beam, BM is hogging. and it can be design as a rectangular secⁿ as the slab is in Tension. However away from support that is Near the mid span BM is sagging and slab is in compression, Hence it shall be design as a Flange beam.
- For inverted construction (recommended to increase Head room) and Generally provided in the Top Floor. The behaviour will be opposite to that of the beam in Normal construction.

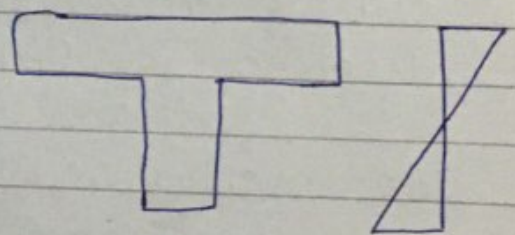
So we can say the same beam can be treated as Rectangular or Flange depending on Type of moment and construction.



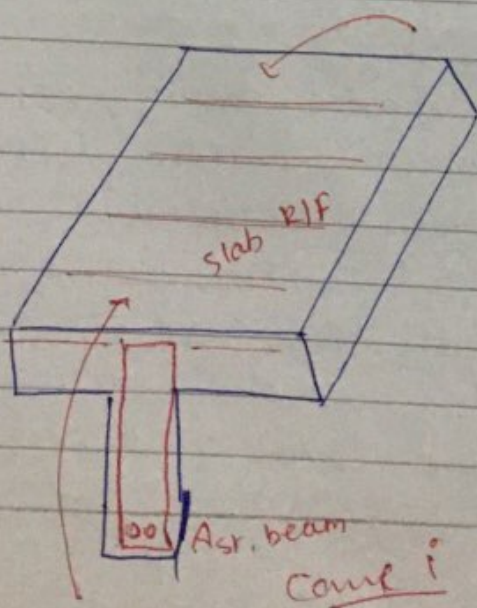
Only casting together may not ensure monolithic behaviour. Proper R/F shall be provided in region to ensure composite behaviour.

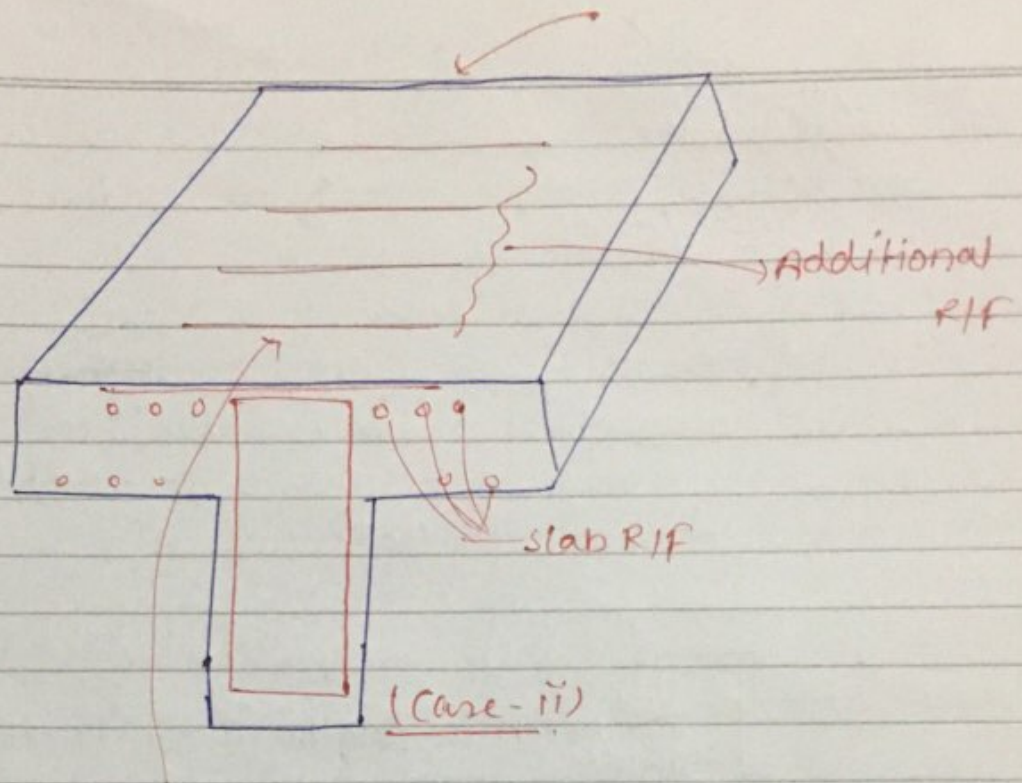


independent behaviour



Monolithic behaviour.



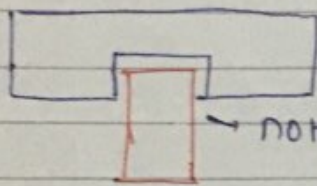
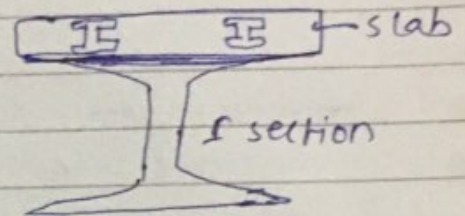
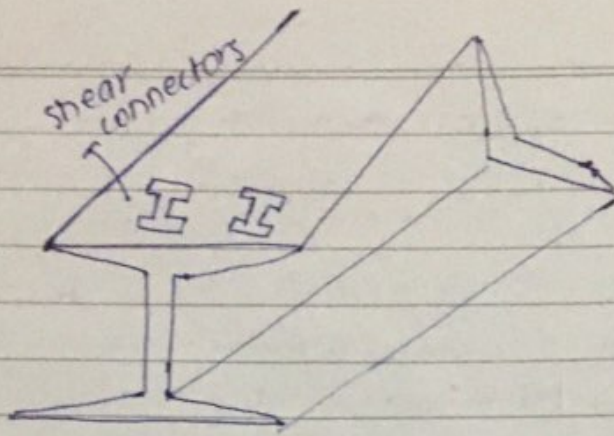


- When the bending dirⁿ of beam & slab are at 90° , NO need to provide additional R/F for monolithic behaviour (Case-i)

However when the Bending dirⁿ of beam and slab are same, Some additional R/F shall be provided to consider monolithic behaviour.

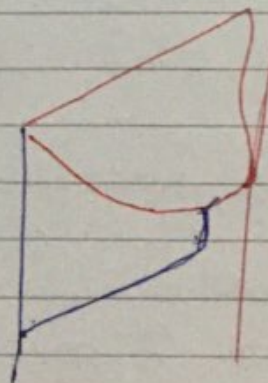
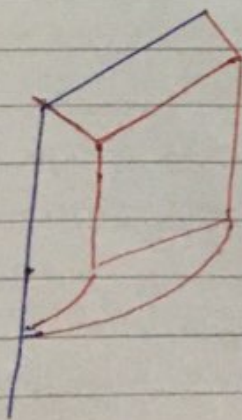
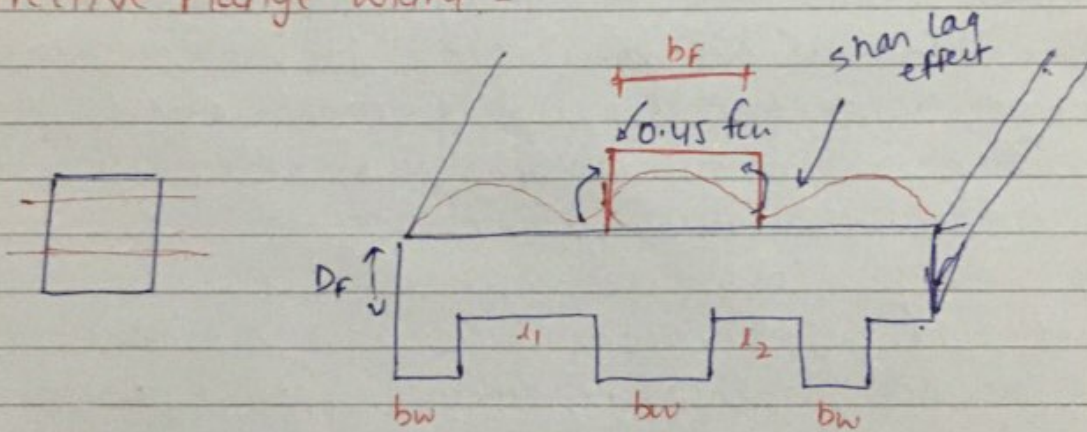
- sometime shear connectors are provided to develop monolithic action.
- Simply placing a slab over a Beam will not ensure monolithic behaviour

NOTES . Monolithic behaviour of Beam & slab also avoids buckling of compression Flange of beam when slab is in compression by providing lateral support to compression flange. Also lateral support can be achieved by other means.



notch in slab to avoid buckling of compression flange of beam.

Effective Flange width:-



- In plane compressive stress variation in the flange is not uniform the stress is max^m in the web region
- The Eff. Flange width is defined as a width of a hypothetical flange that resist compressive stress of uniform magnitude equal to the peak stress in the original wide flange such that the value of resultant compressive force is same.

b_f for beams :-

for L-Beam :- $b_f = b_w + 3 D_f + \frac{l_o}{12}$

$\nrightarrow b_w + l_{1/2}$

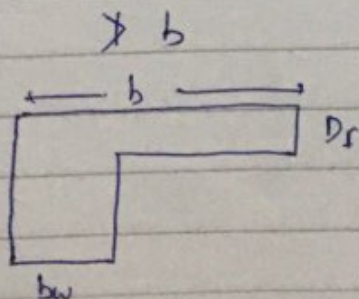
for T-beam :- $b_f = b_w + 6 D_f + l_o/6$

$\nrightarrow b_w + \left(\frac{l_1 + l_2}{2} \right)$

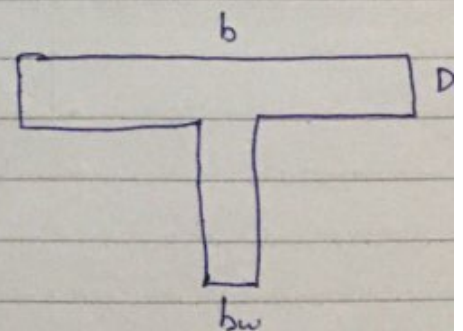
for isolated Beam :-

isolated L-Beam

$$b_f = b_w + \frac{0.5 l_o}{\frac{l_o}{b} + 4}$$



isolated T Beam



$$b_f = b_w + \frac{l_o}{\frac{l_o}{b} + 4}$$

$\nrightarrow b$

Isolated beams can be seen in staircases

b_w = width of web

b_f = eff. flange width

D_f = thickness of flange

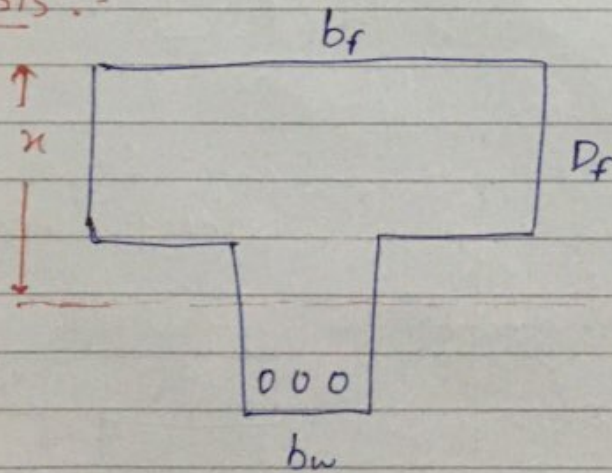
l_0 = distance b/w points of zero BM in Beams.

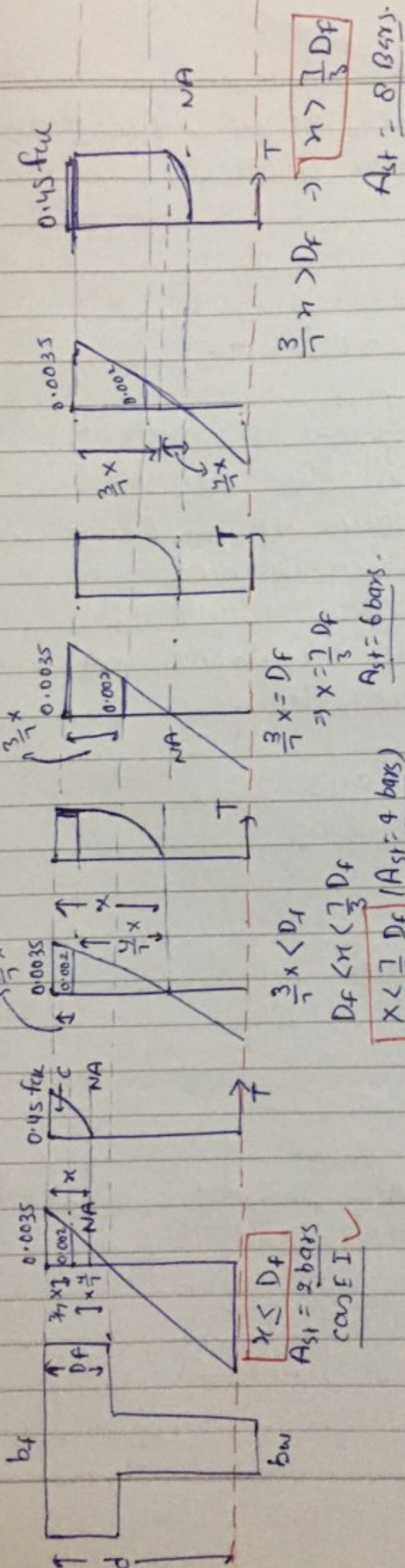
$l_0 = \begin{cases} \text{eff. span for simply supported beam and} \\ 0.7 \times \text{eff. span for continuous beams and} \\ \text{frames.} \end{cases}$

NOTE • b_f depends on span, web width, flange thickness, type of loading

• for example b_f is less for concentrated load and higher for UDL

Analysis :-





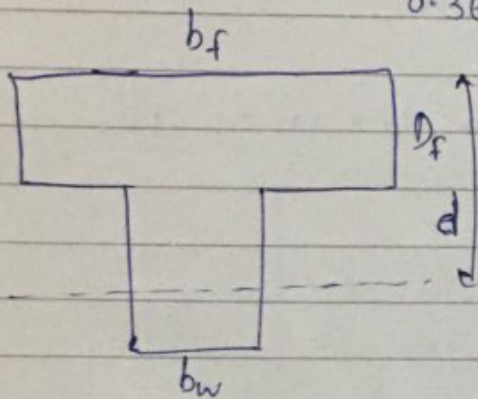
Zone-1 ($x < D_f$)

When NA lies in Flange, Analysis and design shall be done as a rectangular beam of width b_f

From $C = T$

$$0.36 f_{ck} b_f x = 0.87 f_y A_{st}$$

$$x = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$



$x \leq D_f$ implies NA in Flange

$x > D_f \rightarrow$ NA in Web

However the calculated NA depth is not the correct depth.

L.H.S

$$0.36 f_{ck} b_f D_f$$

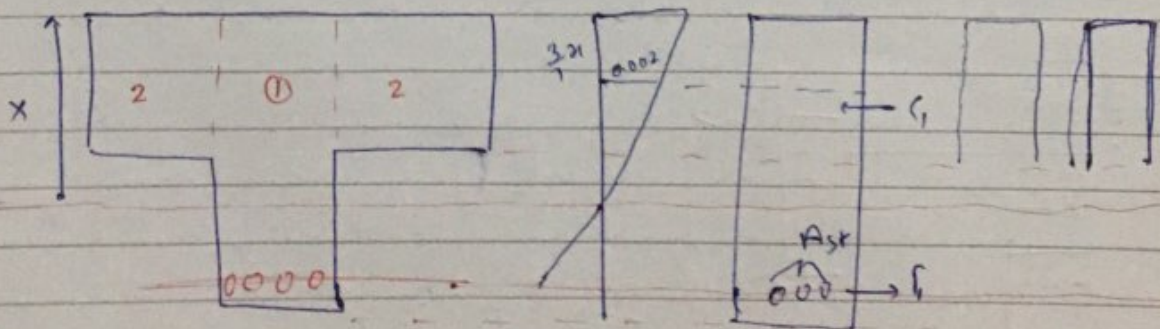
R.H.S

$$0.87 f_y A_{st}$$

$$L.H.S \geq R.H.S$$

NA is in Flange.

Zone-2 ($D_f < x < \frac{7}{3} D_f$)



$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

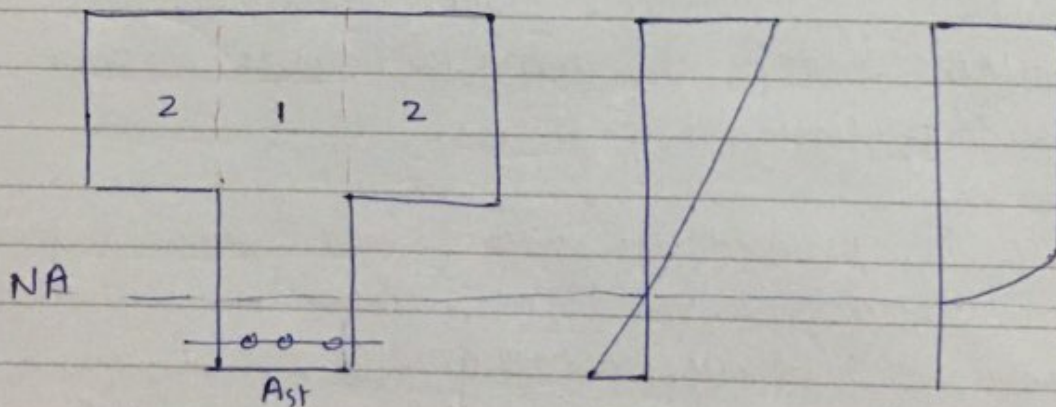
$$MOR = MOR_1 + MOR_2$$

$$= 0.36 f_{ck} b_w x (d - 0.42 x) + 0.45 f_{ck} (b_f - b_w) y_f (d - \frac{y_f}{2})$$

The flange portion will have rectangular stress upto $\frac{3}{7}x$ and truncated parabola upto D_f

in this case the flange can be consider having a constant stress of σ_c upto a depth of y_f
Where $y_f = 0.65 D_f + 0.15 x$

(iii) $(x > \frac{7}{3} D_f)$



$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

$$MOR = 0.36 f_{ck} b_w x (d - 0.42 x) + 0.45 f_{ck} (b_f - b_w) D_f (d - \frac{D_f}{2})$$

NOTE. in all the above cases f_{st} has been assumed $0.87 f_y$ by assuming UR and B

for OR secⁿ MOR shall be limited to MOR_{lim} of Flange secⁿ.

Steps in determining MOR :-

Step-1 calculate X_{lim}

Step-2 calculate X (can lie in any zone)

Step-3 if $x < X_{lim} \Rightarrow$ UR, B, calculate MOR for X by using the formulae of the zone in which X lies.

Step-4. if $x > X_{lim}$ OR.

$$MOR = MOR_{lim}$$

calculate MOR_{lim} by using the formulae of zone in which X_{lim} lies.

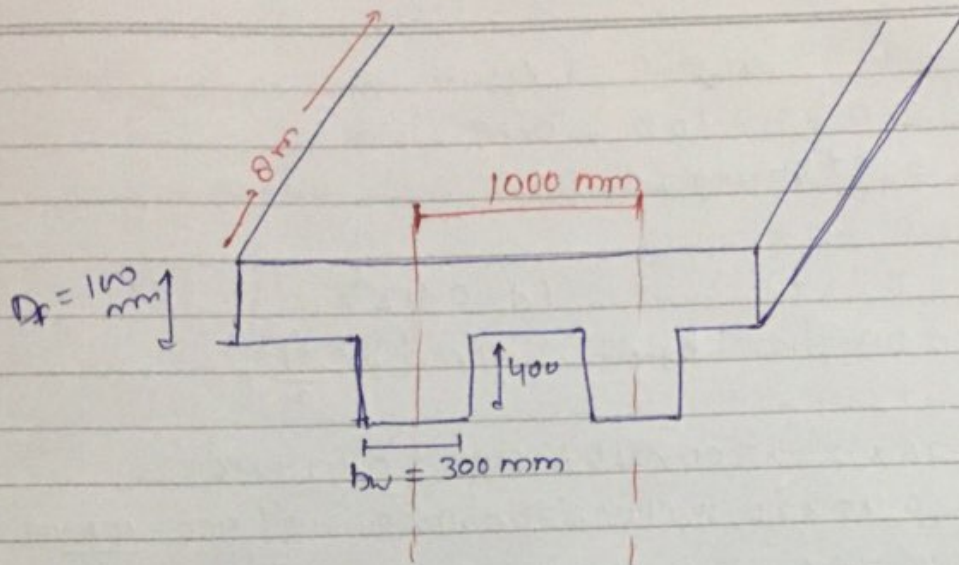
Que. A hall is covered by Beams and slab system and beams placed 1 m c-c, span of continuous beam = 8 m, thickness of slab = 100 mm, the size of beam below slab is 400 mm and width 300 assuming a T-beam near the mid span, calculate the following. M20, Fe 415, eff cover = 50 mm.

i) MOR_{lim}

ii) $A_{st, lim}$

iii) MOR when $A_{st} = 1963 \text{ mm}^2$

iv) MOR when $A_{st} = 2591 \text{ mm}^2$



$$b_f = b_w + 6 D_f + \frac{l_o}{6}$$

$$= 300 + 6 \times 100 + \frac{0.7 \times 8000}{6}$$

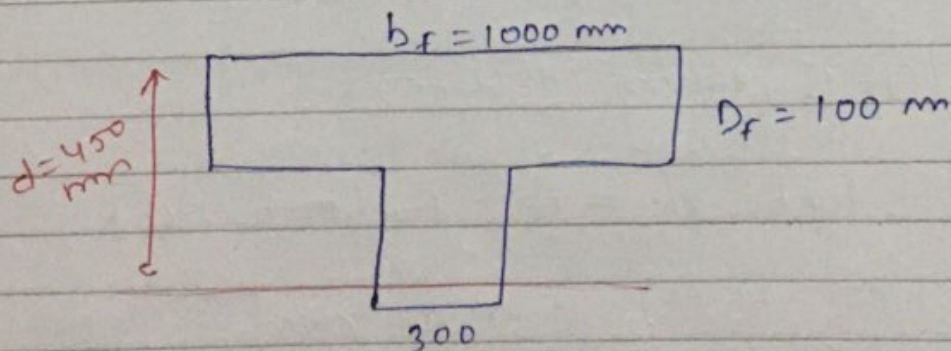
$$= 1833.33 > 1000$$

$$\Rightarrow \underline{b_f = 1000 \text{ mm}}$$

$$b_w = 300 \text{ mm}$$

$$D_f = 100 \text{ mm}$$

$$d = 400 + (100 - 50) = 450 \text{ mm}$$



$$X_{lim} = 0.48 d = 0.48 \times 450 = 216 \text{ mm}$$

$$\frac{7}{3} D_f = \frac{7}{3} \times 100 = 233.33 \text{ mm}$$

$$D_f < X_{lim} < \frac{7}{3} D_f$$

$$\begin{aligned} \gamma_f &= 0.65 D_f + 0.15 X_{lim} \\ &= 0.65 \times 100 + 0.15 \times 216 \\ &= 97.40 \text{ mm} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{MOR} &= 0.36 f_{ck} b_w X_{lim} (d - 0.42 X_{lim}) \\ &\quad + 0.45 f_{ck} (b_f - b_w) \gamma_f \left(d - \frac{\gamma_f}{2}\right) \\ &= 0.36 \times 20 \times 300 \times 216 \times (450 - 0.42 \times 216) + \\ &\quad 0.45 \times 20 \times (1000 - 300) \times 97.40 \left(450 - \frac{97.40}{2}\right) \\ &= 413.87 \text{ KNm} \end{aligned}$$

OR

$$\bar{X} = \frac{C_1 \times 0.42 X + C_2 \times \frac{\gamma_f}{2}}{C_1 + C_2}$$

$$\begin{aligned} &= \frac{0.36 \times f_{ck} b \times X_{lim} \times 0.42 X_{lim} + 0.45 f_{ck} (b_f - b_w) \gamma_f \times \frac{\gamma_f}{2}}{0.36 f_{ck} b_w X_{lim} + 0.45 f_{ck} (b_f - b_w) \gamma_f} \\ &= 66.85 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{MOR} &= (C_1 + C_2) \times \cancel{66.85} \times (d - 66.85) \\ &= 0.36 f_{ck} b_w X_{lim} + 0.45 f_{ck} (b_f - b_w) \gamma_f \left[450 - 66.85\right] \\ &= 413.87 \text{ KNm} \end{aligned}$$

ii) $A_{st, \text{lin}}$

$$C = T$$

$$0.36 \times f_{uc} \times b_w \times X_{\text{lin}} + 0.46 f_{uc} (b_f - b_w) Y_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times 216 + 0.45 \times 20 \times (1000 - 300) 97.4 = 0.87 \times 415 \times A_{st, \text{lin}}$$

$$A_{st, \text{lin}} = 2991.77 \text{ mm}^2$$

iii) $A_{st} = 1963 \text{ mm}^2$

Let us assume x lies in flange.

$$0.36 f_{uc} b_f x = 0.87 f_y A_{st}$$

$$x = 98.43 \text{ mm} < D_f < X_{\text{lin}}$$

correct assumption.

NA in flange, UR

$$\text{MOR} = 0.36 f_{uc} b_f x (d - 0.42 x) = 289.61 \text{ kNm}$$

iv) $A_{st} = 2591 \text{ mm}^2$

$$0.36 f_{uc} b_f x = 0.87 f_y A_{st}$$

$$x = 129.92 \text{ mm} > D_f, \text{ NOT correct.}$$

as NA is not in flange $\Rightarrow$

From formula of case 2.

Let us assume $D_f < x < \frac{7}{3} D_f$

$$0.36 f_{uc} b_w x + 0.45 f_{uc} (b_f - b_w) (0.65 D_f + 0.15 x) = 0.87 f_y A_{st}$$

$$x = 169.39 \text{ mm} < X_{\text{lin}} \quad \underline{\text{UR}}$$

$$y_f = 0.65 D_f + 0.15 x = 90.40 \text{ mm.}$$

$$MOR = 0.36 f_{uc} b_w x (d - 0.42 x) + 0.45 f_{uc} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

$$MOR = 369.18 \text{ kNm.}$$

Design :-

To find out the A_{st} value we need to obtain the NA depth and the zone in which NA lies

Steps

Step-1 calculate MOR_1 at $x = D_f$

$$MOR_1 = 0.36 f_{uc} b_f x (d - 0.42 x)$$

$$MOR_2 \text{ at } x = \frac{1}{3} D_f$$

$$MOR_2 = 0.36 f_{uc} b_w x (d - 0.42 x) + 0.45 f_{uc} (b_f - b_w) D_f \left(d - \frac{D_f}{2} \right)$$

Step-2 if $BM_u < MOR_1$, calculate A_{st} as follows

$$BM_u = 0.36 f_{uc} b_f x (d - 0.42 x)$$

$$0.36 f_{uc} b_f x = 0.87 f_y A_{st}$$

Step-3 if $MOR_1 < BM_u < MOR_2$

$$BM_u = 0.36 f_{uc} b_w x (d - 0.42 x) + 0.45 f_{uc} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

$$0.36 f_{uc} b_w x + 0.45 f_{uc} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

Step-4 if $BM_u > MOR_2$

$$BM_u = 0.36 f_{ck} b_w x (d - 0.42 x) + 0.45 f_{ck} (b_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

$$0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y A_{st}$$

calculate A_{st}

NOTE if in any case $x > x_{lim}$, the options available are

- 1) change the Grade of concrete or steel
- 2) change the dimension of C/S
- 3) Design Doubly R/F Flange section

$$A_{st, \text{provi}} \text{ shall be } \geq A_{st, \text{min}} \\ \leq A_{st, \text{max.}}$$

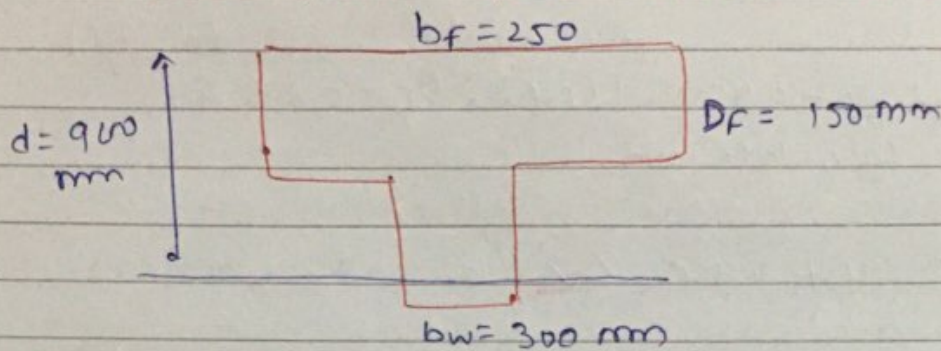
$$A_{st, \text{min}} = \frac{0.85 b_w d}{f_y}$$

$$A_{st, \text{max}} = 0.04 b_w D$$

• an approximate formula given in IS code

$$A_{st} = \frac{M_u}{0.87 f_y \left(d - \frac{D_f}{2}\right)}$$

Ques. for the following c/s, $A_{st} = ?$ for factored $B M_u$
 $= 2000 \text{ kNm}$. take M20, Fe 415



$$x_{lim} = 0.48 \times d = 432 \text{ mm}$$

$$\begin{aligned} MOR_1 &= 0.36 f_{ck} b_f D_f (d - 0.42 D_f) \\ &= 0.36 \times 20 \times 2500 \times 150 \times (900 - 0.42 \times 150) \\ &= 2259.90 \text{ kNm.} \end{aligned}$$

$$\left. \begin{array}{l} \frac{1}{3} D_f \\ = 350 \text{ mm} \end{array} \right\} \begin{aligned} MOR_2 &= 0.36 f_{ck} b_w x (d - 0.42 x) + 0.45 f_{ck} (b_f - b_w) \\ &\quad D_f \left(d - \frac{D_f}{2} \right) \\ &= 3019.5 \text{ kNm.} \end{aligned} \quad \{ x = 350 \text{ mm}$$

$$\begin{aligned} B M_u &= 0.36 f_{ck} b_w x (d - 0.42 x) + 0.45 f_{ck} (b_f - b_w) \\ &\quad (0.65 D_f + 0.15 x) \\ &\quad \left[d - \frac{(0.65 D_f + 0.15 x)}{2} \right] \end{aligned}$$

$$\cancel{3019.5} = x = 183.66 \text{ mm.} < x_{lim} \quad \underline{\text{UR}}$$

from $C = T$

$$y_f = 0.65 D_f + 0.15 x$$

$$y_f = 125.00 \text{ mm}$$

$$0.36 f_{ck} b_w x + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

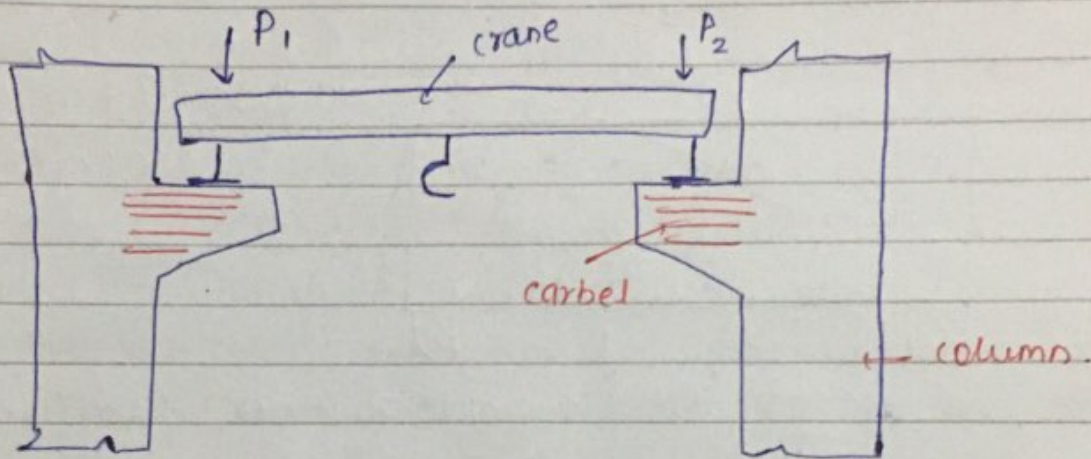
$$A_{st} = 7953.79 \text{ mm}^2$$

$$A_{st, \min} = \frac{0.85 b_w d}{f_y} = 553 \text{ mm}^2 < A_{st, \text{req.}}$$

⑦ DESIGN FOR SHEAR

Shear can develop due to change in BM (flexural shear) or torsion (torsional shear) or direct shear.

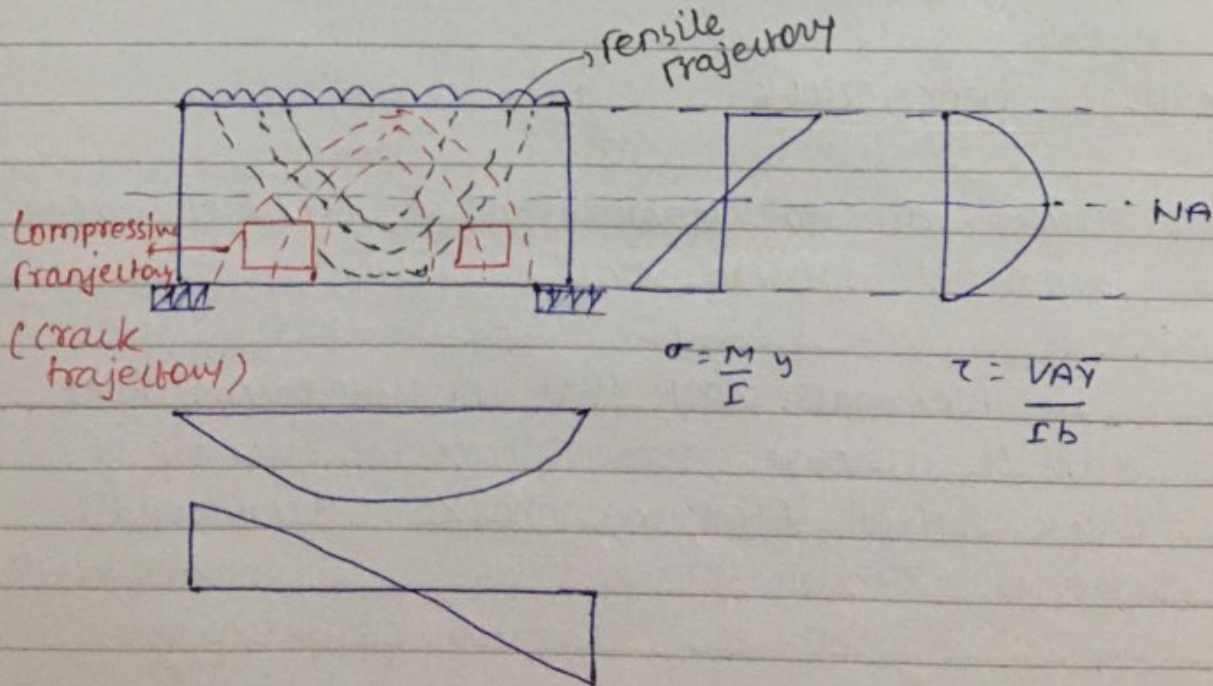
here we'll discuss flexural shear i.e. $V = \frac{dM}{dx}$

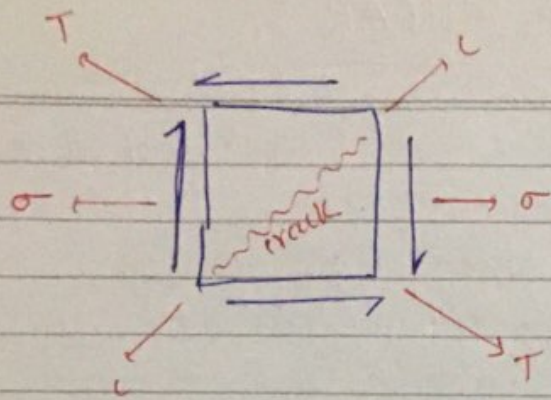


Direct shear in column corbel.

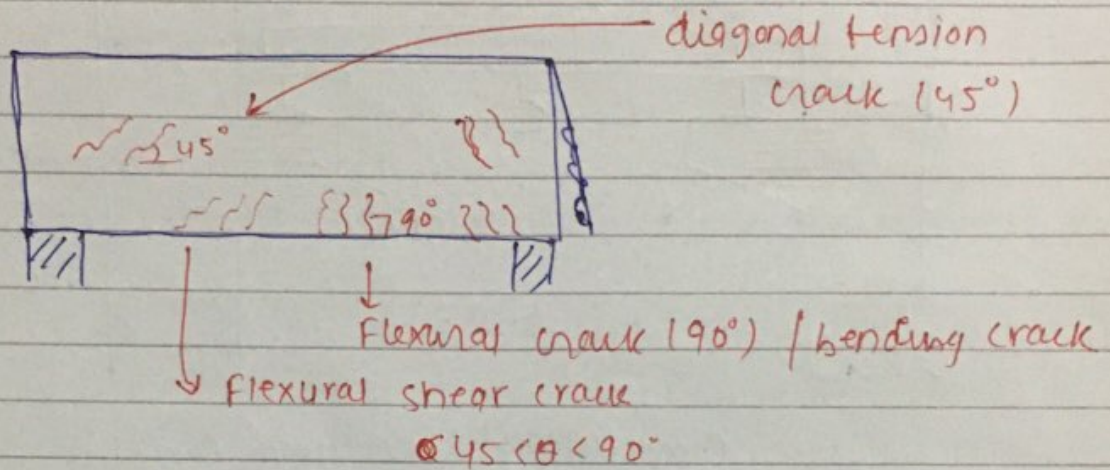
Modes of cracking (Failure):-

Diff. Modes of cracking can be there due to possible combination of BM & SF





$$\tan 2\alpha = \frac{2\tau}{\sigma}$$



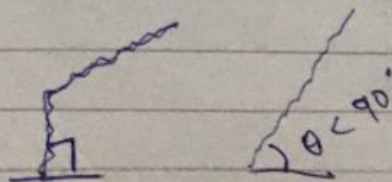
① Flexural crack:-

- it occurs due to higher flexural stresses and generally form near the mid span $\theta \approx 90^\circ$

• Flexural shear crack:-

it occurs due to combination of flexural cracks and diagonal tension crack

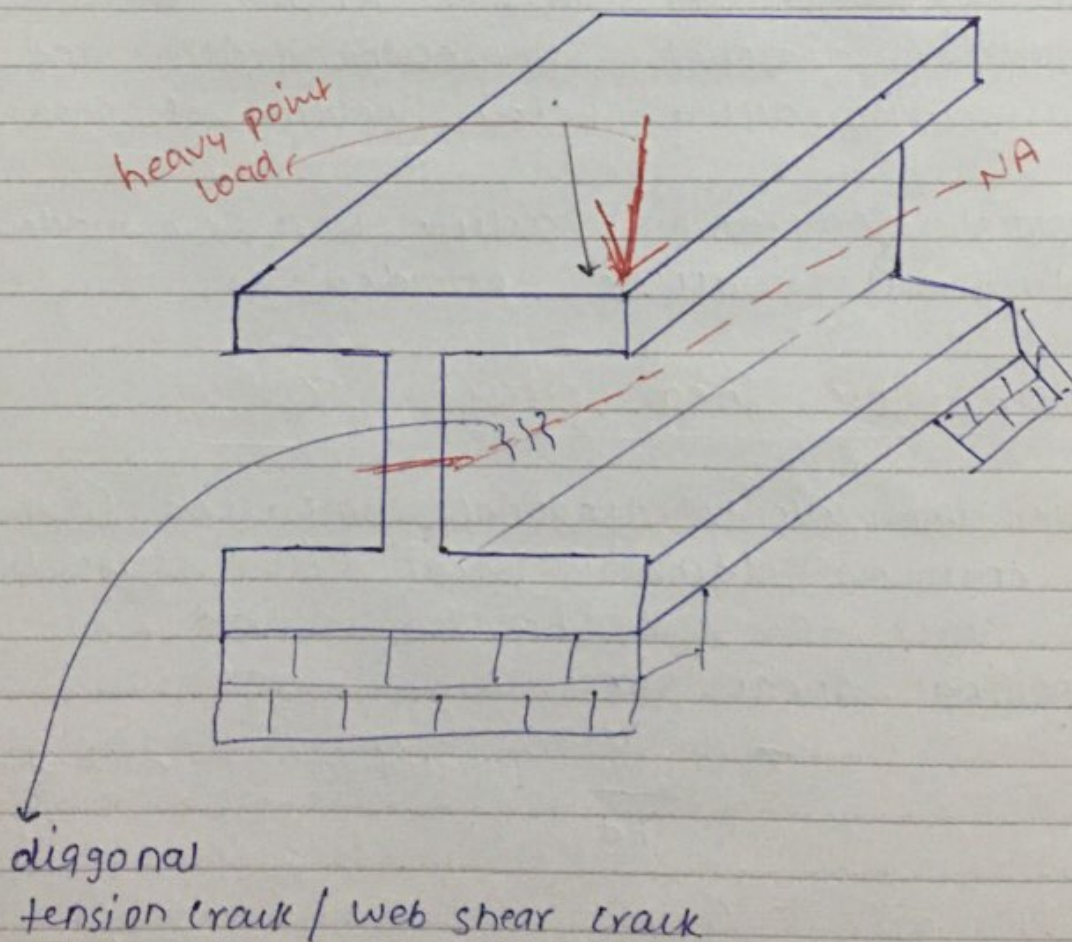
NOTE It is the flexural crack that usually forms first and due to increase shear stress at the tip of the crack, this flexural cracks extend into a diagonal tension crack



③ Diagonal Tension crack :-

it occurs due to large shear force + small BM and crack angle $\approx 45^\circ$ to the beam axis.

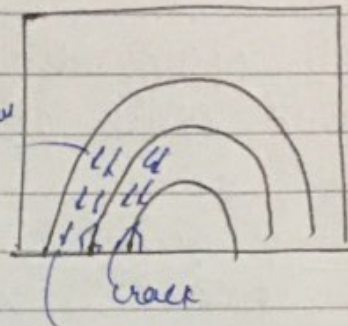
NOTE : In short span Beam (Like Deep Beam) and thin web or I-section subjected to high shear stresses due to heavy concentrated loads and relatively smaller BM, max. tensile stress is located near NA at 45° to the beam axis, and cracks occur near support. These are called Web shear crack / Diagonal Tension crack.



Diagonal compression failure / shear comp. failure :-

- It occurs due to large SF and characterized by crushing of concrete

Compression
strut



(diagonal comp.)

- Normally occurs in beam which are heavily reinforced against shear
- When the area of shear R/F is large, concrete becomes stronger in diagonal tension failure compare to diagonal compression failure and failure may occur before yielding of shear R/F
- Diagonal compression failure will be a sudden failure and shall be avoided.

Max. permissible shear stress :- $\tau_{c, max}$

In order to prevent diagonal compression failure and ensure yielding of shear R/F @ ultimate limit state, IS code has imposed a limit on nominal shear stress.

$$\tau_v = \frac{V_u}{bd} \quad (\text{valid for constant } d)$$

for beams $\tau_v \nless \tau_{c, max}$

for slab $\tau_v \nless 0.5 \tau_{c, max}$

If the above criteria is not satisfied then increase c/s or grade of concrete

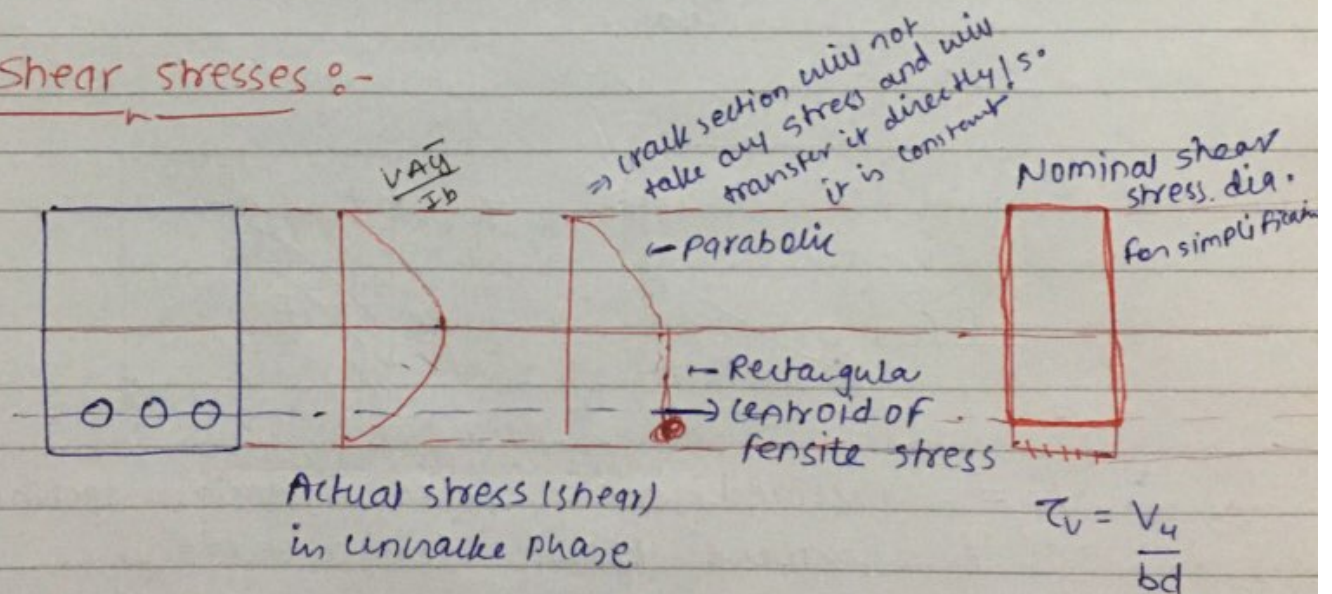
$\tau_{c, \max}$ Value as per IS code

Grade	$\tau_{c, \max}$ (MPa)
M20	2.0
M25	3.1
M30	3.5
M35	3.7
$\geq M40$	4.0

$$\tau_{c, \max} = 0.631 \sqrt{f_{cu}}$$

NOTE: The Max. permissible shear stress ($\tau_{c, \max}$) is based on diagonal compression failure.

Shear stresses :-



NOTE: • In RCC beam, Shear stress distribution is parabolic upto NA and ~~then~~ then constant upto centroid of Tension steel.

• Nominal shear stress τ_v is rectangular upto centroid of tension steel.

Nominal shear stress :-

$$\tau_v = \frac{V_u}{bd} \quad ; \text{ for constant } d$$

$b = b_w$ for Flanged section

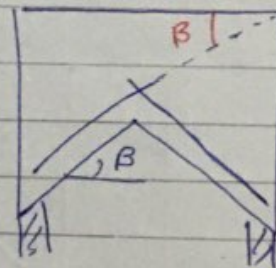
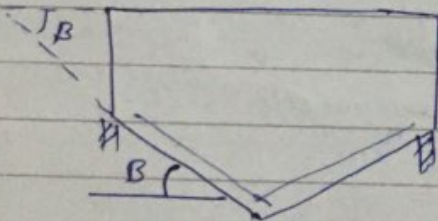
$V_u \rightarrow$ factored shear force

$b =$ width

$d \rightarrow$ eff. depth

Nominal shear stress τ_v (for Beams with varying depth d)

$$\tau_v = \frac{V_u \pm \frac{M_u}{d} \tan \beta}{bd}$$



$V_u =$ factored shear force @ design section

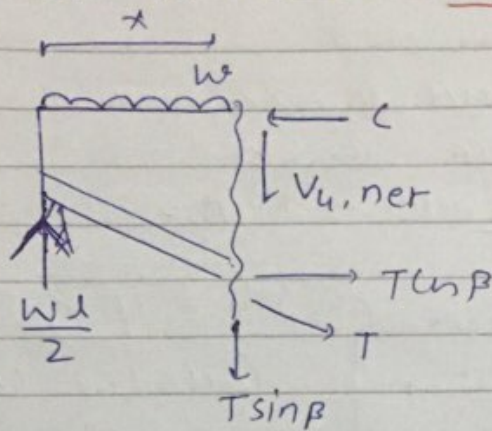
$M_u =$ factored BM @ design section

$d \rightarrow$ effective depth @ design section

$\beta =$ angle b/w the top and bottom edges of beam.

-ve sign is used when BM increases Numerically in the dirⁿ in which eff. depth increases,

+ve sign is used when BM increases numerically in the dir in which eff. depth decreases



$$\frac{Wl}{2} = wx + Vu,net + T \sin \beta$$

$$\Rightarrow Vu,net = \left(\frac{Wl}{2} - wx \right) - T \sin \beta$$

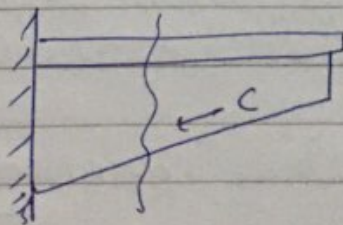
$$= Vu - T \sin \beta$$

$$= Vu - \frac{Mu}{d \cos \beta} \sin \beta$$

$$= Vu - \frac{Mu}{d} \tan \beta$$

$$Vu,net = Vu - \frac{Mu}{d} \tan \beta$$

$$\left\{ \begin{array}{l} Mu = T \cos \beta (d - 0.42x) \\ T = \frac{Mu}{d \cos \beta} \end{array} \right.$$



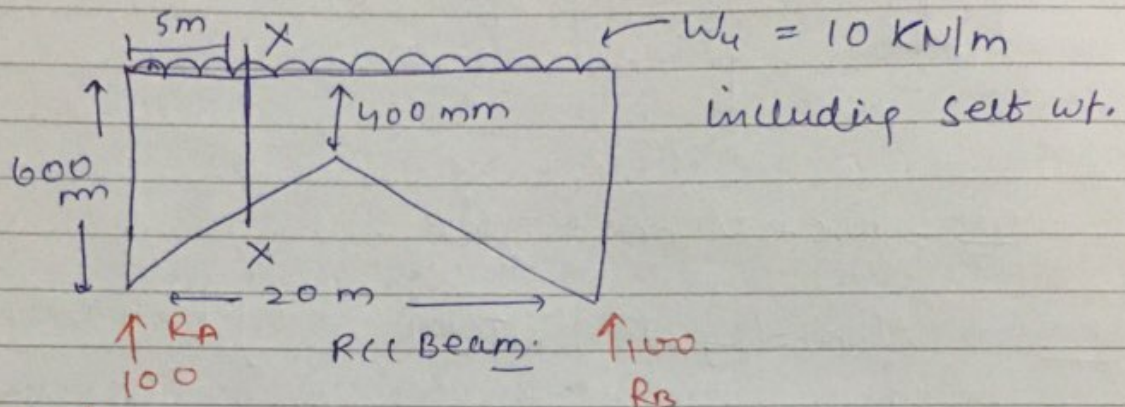
$$\tau_v = \frac{Vu - \frac{Mu}{d} \tan \beta}{bd}$$

In this case the help will be from compressive component.

Que. A haunched beam (varying depth) is simply supported at both ends as shown. obtain following

- 1) Design shear force @ XX
- 2) Design shear stress @ XX

Assume eff. cover 50 mm and b = 250 mm.



$$1) \quad V_{u, net} = V_u + \frac{M_u \tan \beta}{d}$$

$$V_u @ X-X = 100 - 10 \times 5 = 50 \text{ kN}$$

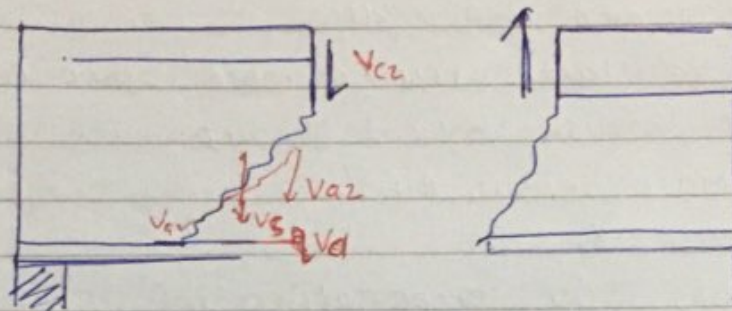
$$M_u @ X-X = 100 \times 5 - \frac{10 \times 5^2}{2} = 375 \text{ kNm}$$

$$d = 500 - 50 = 450$$

$$V_{u, net} = 50 + \frac{375}{0.45} \times \frac{200}{10000} = 66.67 \text{ kN}$$

$$\tau_v = \frac{V_{u, net}}{bd} = \frac{66.67 \times 10^3}{250 \times 450} = 0.593 \text{ MPa}$$

Shear Resistance Mechanism :-



V_{cz} = shear resistance offered by uncracked concrete

V_{az} = shear resistance offered by aggregate interlocking

V_d = shear resistance offered by dowel bars
(Main Tensile R/F)

V_s = shear resistance offered by shear R/F

$$V_u = V_{cz} + V_{az} + V_d + V_s$$

Relative proportion of various mechanisms depends on the loading stage, extent of cracking, material and geometrical proportions

- prior to flexural crack $\rightarrow$ only V_{cz}
- At start of flexural crack $\rightarrow V_{cz}, V_{az}, V_d$
- At diagonal tension crack $\rightarrow V_{cz}, V_{az}, V_d$ and V_s

NOTE:

- ① less crack width $\rightarrow$ High interlocking
- ② Angular aggregates have higher interlocking than rounded aggregates
- ③ Down resistance will be more when main Tensile R/F get support through vertical stirrup.
- ④ increase in Main Tensile R/F:-
 - a) increase V_d
 - b) controls the propagation of flexural cracks.
 - c) - contributes to increase the depth of NA due to higher Tensile R/F
 - d) - results in smaller crack and improves aggregates interlocking ~~so~~
so if $P_t \uparrow \Rightarrow V_{cz} \uparrow, V_{cz} \uparrow, V_d \uparrow$

Design shear strength of concrete :- (τ_c)

The combined effect of $V_{cz} + V_{cz} + V_d$ is called design shear strength of concrete.

τ_c is function of P_t and grade of concrete
 τ_c values (MPa)

$P_t = \frac{A_{st}}{b_d} \times 100$	M20	M25	M30
≤ 0.15	0.28	0.29	0.29
0.25	0.36	0.36	0.37
0.50	0.48	0.49	0.50
0.75	0.56	0.57	0.59
1.00	0.62	0.64	0.66
2.5	0.82	0.88	0.91
2.75	0.82	0.90	0.94
≥ 3.0	0.82	0.92	0.96

As τ_c is almost constant after certain % of tensile RIF, it can be concluded that beneficial effect of dowel action, control of crack propagation and increase in depth of uncracked concrete can not go on increasing and definitely with increase P_t .

Shear force that can be taken by concrete only is equal to $\tau_c \times b d$

The above τ_c value shall be multiplied by a factor

$$\phi = 1 + \frac{3 P_u}{A_g f_{uc}} \quad \rightarrow ?$$

$\phi > 1.5$

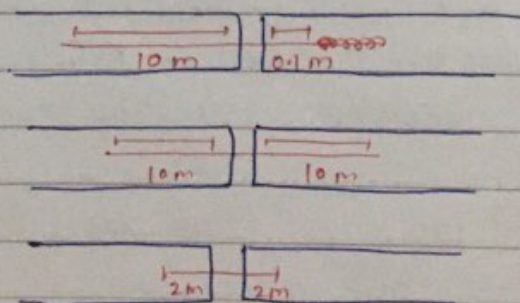
τ_c will increase due to compression confinement
So multiply τ_c with this factor.

When member is subjected to Axial compression P_u
(A_g = Gross Area of γ_s)

Axial compressive force increases the shear strength and hence decreases the chances of crack formation

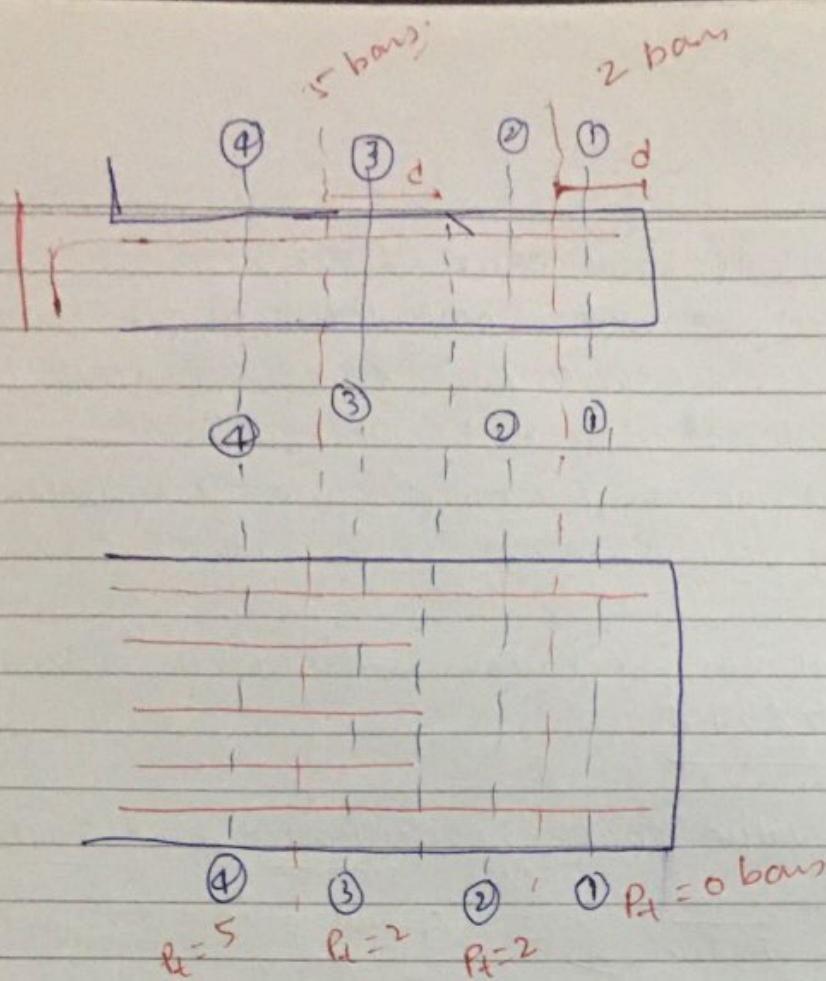
Axial Tension decreases the shear strength and increases the chances of crack formation.

NOTE:



$$2 = 3 > 1$$

$\approx$



In the above table of τ_c , A_{st} is the Area of Tension R/f which continues at least 1 eff. depth (d) beyond the section being considered except @ support where full area of Tension R/f may be used provided sufficient anchorage is available

Design shear strength of concrete in slab (solid slab)

Due to plate action, thinner the slab, greater the increase in shear strength

Design shear strength of slab = $K \tau_c$

K values

τ_c from the table given in beam shear strength.

Overall depth (mm) D	≤ 150	175	200	225	250	275	300
K	1.3	1.25	1.20	1.15	1.10	1.05	1.00

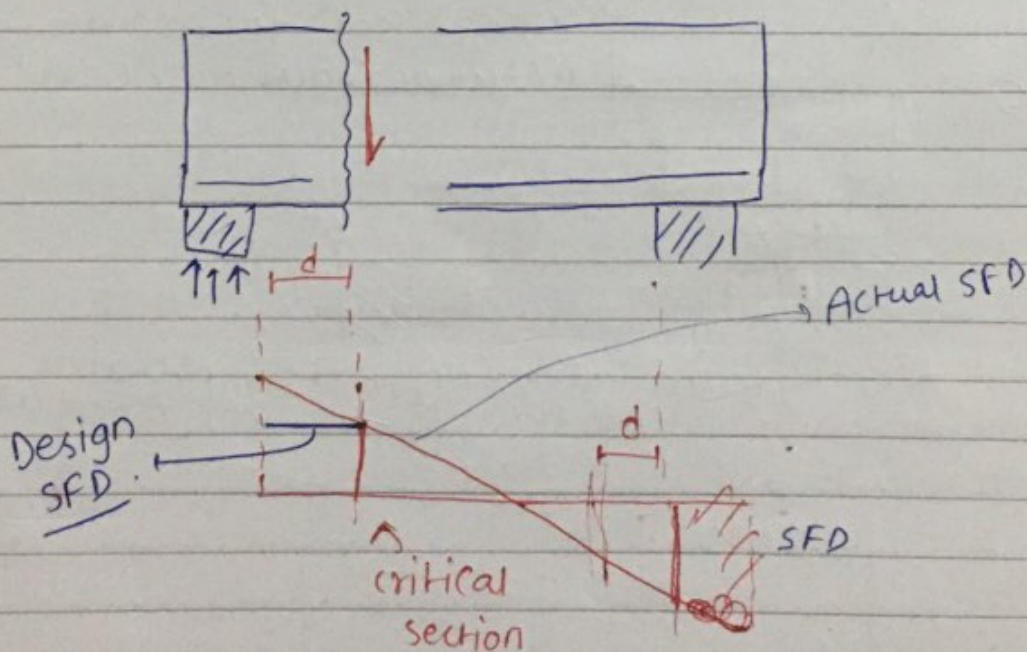
$$K = \begin{cases} 1.3 & D \leq 150 \text{ mm} \\ 1.6 - 0.002 D & 150 < D < 300 \\ 1 & D \geq 300 \end{cases}$$

The above concept is valid for one way shear (flexural shear) and not valid for punching shear or two way shear

Critical section for shear :-

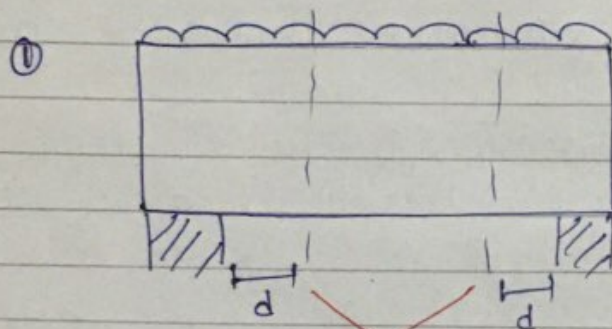
if support react introduces Transverse Compression

(compression confinement) in the end region of Member, shear strength of this region increases and chances of cracking decreases

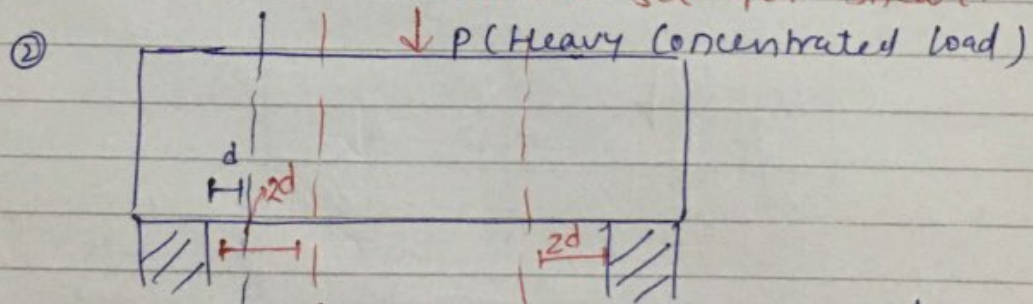


Beam Generally subjected to UDL or When principle load is located farther than $(2d)$ from the face of support, critical section (section with Max. shear stress) shall be taken as follows -

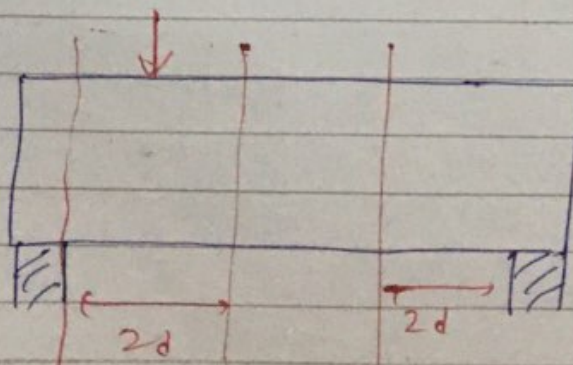
- 1) If compression introduced $\rightarrow$ at a distance d from face of support.
- 2) No compression introduced $\rightarrow$ Face of support



$\rightarrow$ critical secⁿ for shear.

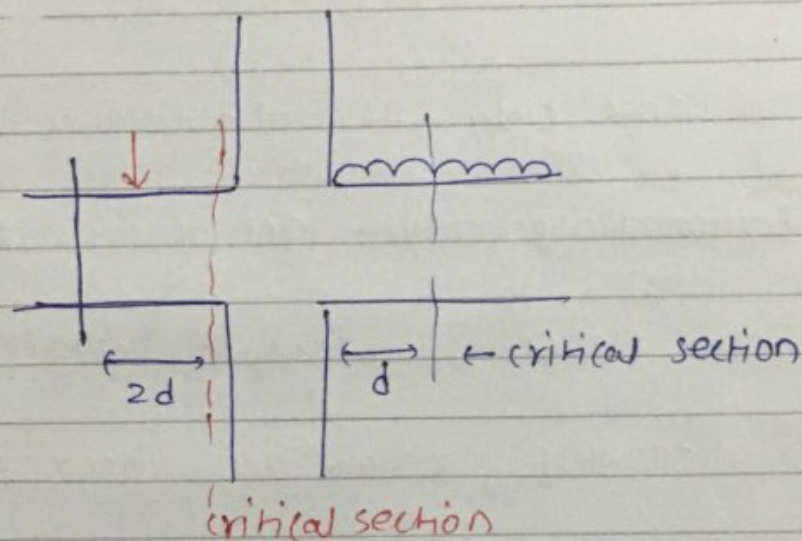
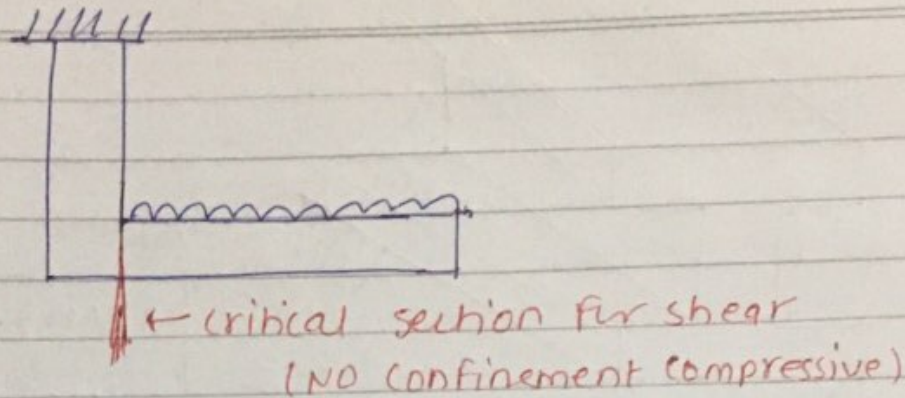


$\rightarrow$ critical secⁿ for shear (compression confinement)



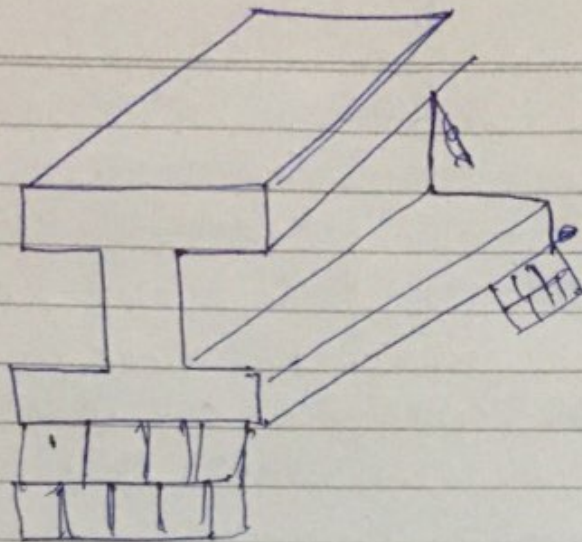
$\rightarrow$ critical secⁿ for shear

(No compression confinement)



NOTE. In case of compression confinement section located at a distance less than d from the face of support may be design for the same shear as that calculated at a distance d .

When principle load is within $2d$ from face of support, critical secⁿ is at face of support.



↓ COMP.
30 (90)
30 (60)
30 (30)

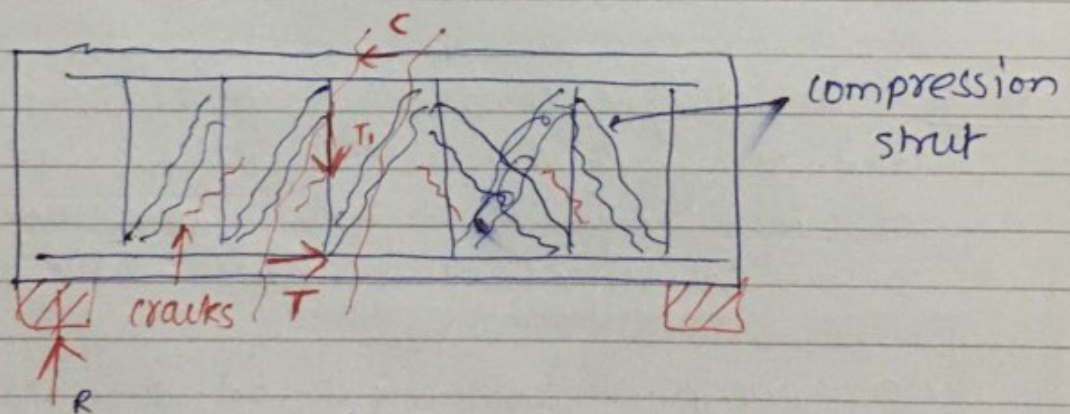
↙ COMP.
30 (30)
30 (60)
30 (90)

Load on Top flange only → at a distance 'd'

Load on bottom flange only → face of support

Load on both flanges → face of support

Action of shear RIF in resisting shear:-

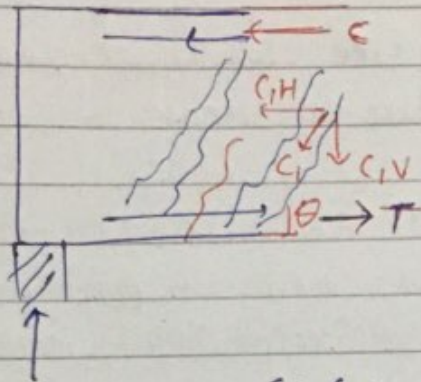


$$R = T_1$$

So, shear RIF will always feel Tensile stress.

NOTE • shear design is based on 2D truss analogy
• shear RIF / web RIF / transverse RIF will carry Tensile stress.

Influence of shear on longitudinal R/F :-



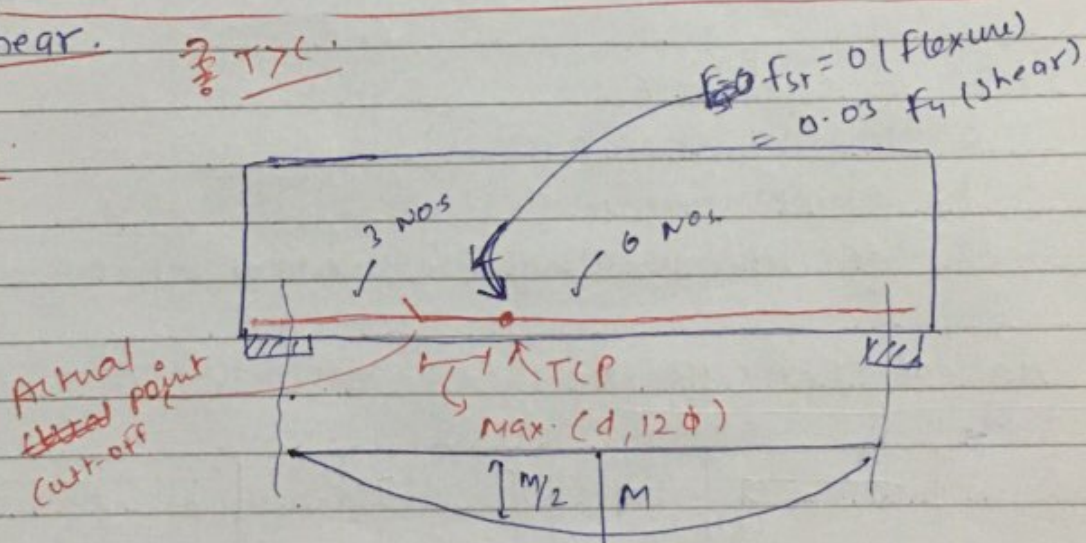
$$C + C_{1H} = T$$

$$T > C$$

as we move close to support, θ value decreases, due to inclined nature of crack and Hence stress in longitudinal R/F increases due to shear.

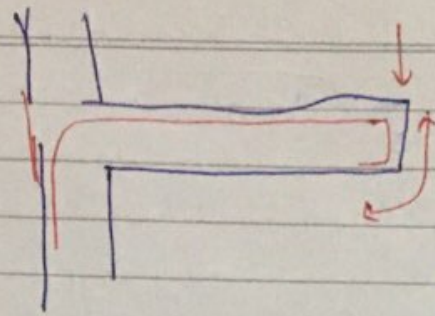
$$\frac{2}{3} T > C$$

NOTE



The effo of increase in Tension in longitudinal R/F due to shear is accounted as follows.

- For attachment the R/F shall extend beyond the point at which it is not Required from flexure criteria that is theoretical Ruff off point. for a distance = d or 12 phi whichever is Max. except support of simple span (simply supported and at the free end of cantilever with concentrated load.



to account for increase in stress in longitudinal R/F due to shear.

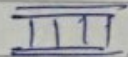
- b) at simple support and free end of cantilever encourage is provided to account for increase in Tension in longitudinal reinforcement due to shear

Design for shear (for Beams):-

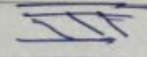
- ① Shear force resisted by concrete = $\tau_c \times b d$
that implies SF to be resisted by shear R/F = $(\tau_v - \tau_c) b d$

- ② 3 types of shear R/F

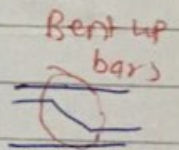
a) Vertical stirrup



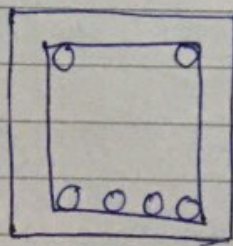
b) inclined stirrup



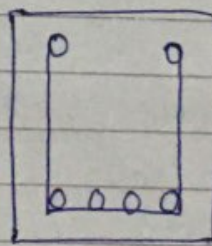
c) Bent up bars along with vertical stirrup



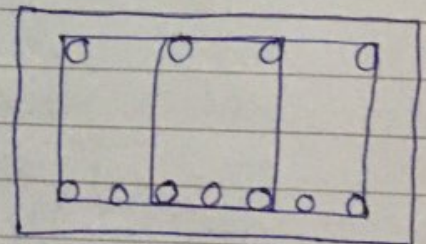
① Vertical Stirrups :-



2 legged closed stirrup



2 legged open stirrup

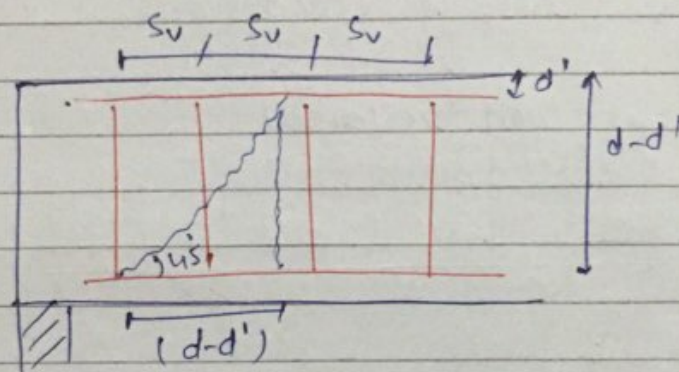


4 legged closed stirrups

→ Best for stress reversal that is dynamic loading

→ contributes significantly towards improving dowel action of longitudinal bars

→ closed stirrups are better as it resist torsion and helps in confining the compressive R/F in doubly R/F section and reduce chances of buckling



$$\text{No. of stirrups in one crack length} = \left(\frac{d-d'}{S_v} \right) \approx \frac{d}{S_v}$$

$$\begin{aligned} \text{Total force that can be resisted by shear R/F} \\ = 0.87 f_y A_{sv} \frac{d}{S_v} \end{aligned}$$

$$\Rightarrow 0.87 f_y A_{sv} \cdot \frac{d}{S_v} \geq (\tau_v - \tau_c) b d$$

A_{sv} = Area of shear R/F for 2-legged.

$$A_{sv} = 2 \times \frac{\pi}{4} \phi^2$$

S_v → spacing of shear R/F

spacing (vertical)

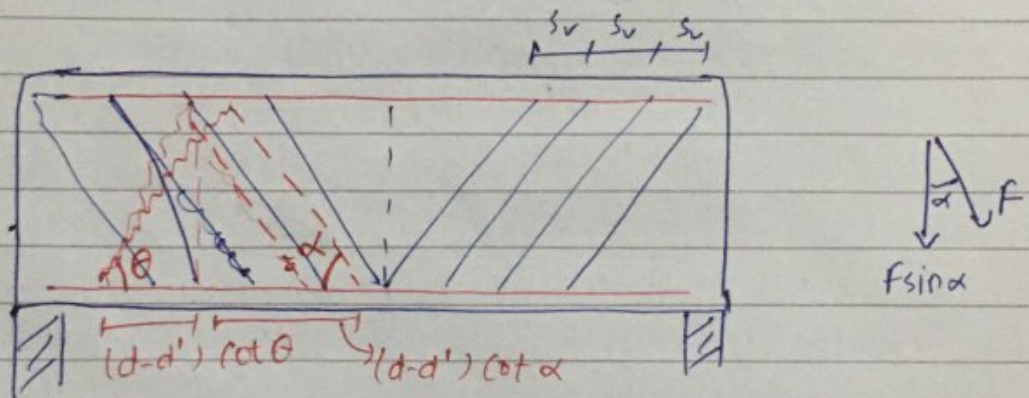
$$S_v \nlessgtr 0.75d \rightarrow \left(\begin{array}{l} \text{To ensure at least} \\ \text{one shear R/F in a} \\ \text{crack length} \end{array} \right)$$
$$\nlessgtr 300 \text{ mm}$$

NOTE: Whatever the grade of steel is
 $f_y \geq 415 \text{ MPa}$.

applicable for all R/F. that is vertical,
 inclined or Bent up bars.

(2) Inclined stirrup:-

- Most eff. in reducing width of inclined cracks for Normal loading condition.
- Ineffective in load reversal



$$\text{No. of shear stirrup} = \frac{(d-d')}{S_v} (\cot \theta + \cot \alpha)$$

$$\text{Total force} = 0.87 f_y A_{sv} \frac{d}{S_v} (\cot \theta + \cot \alpha) \quad \theta = 45^\circ$$

$$\text{force on the vertical dir}^n = 0.87 f_y A_{sv} \frac{d}{S_v} (\cot \theta + \cot \alpha) \sin \alpha$$

$$= 0.87 f_y A_{sv} \frac{d}{S_v} (\sin \alpha + \cos \alpha)$$

$$0.87 f_y A_{sv} \frac{d}{S_v} (\sin \alpha + \cos \alpha) \geq (\tau_v - \tau_c) b d$$

α that is angle b/w beam axis and stirrup shall not be less than 45° o/w there is chances of slippage of inclined stirrups along the longitudinal bars

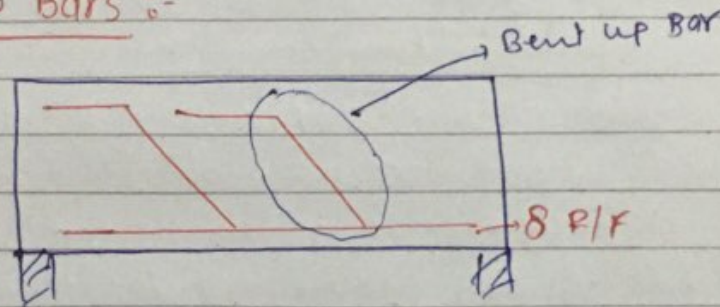
Spacing (inclined)

$$S_v \leq d$$

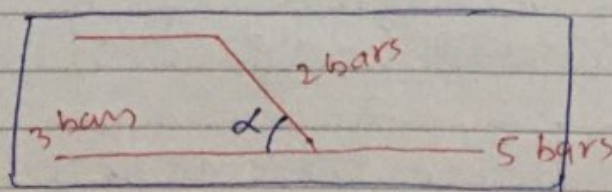
$$\leq 300 \text{ mm}$$

$$f_y \leq \underline{415 \text{ MPa}}$$

③ Bent up Bars :-



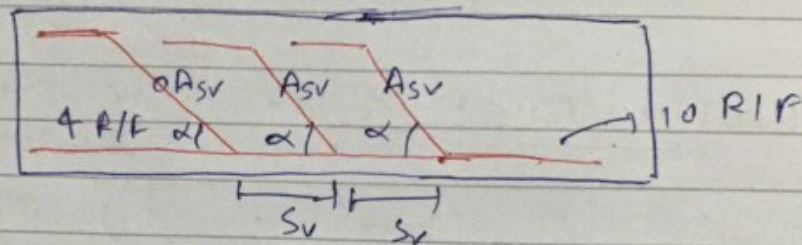
- It is the main Tensile R/F Bent @ suitable locations
- Doesn't enhance the dowel action and Not effective in load reversal. Hence vertical stirrups are always provided along with bent up bars.
- For a single bar or single group of parallel bars all bent at same location



Max. shear force that can be resisted by bent up bars = $0.87 f_y A_{sv} \sin \alpha$

$A_{sv} \rightarrow$ Area of bent up bars at the same location
~~Area~~

for a series of bent up bars at different locations



Max. shear force that can be resisted by bent up bars = $0.87 f_y A_{sv} \frac{d}{S_v} (\sin \alpha + \cos \alpha)$

$A_{sv} =$ Area of bent up bars @ one location

NOTE: When bent up bars are provided their contribution towards shear resistance shall not be more than half the total shear to be resisted through shear R/F

Acceptable contribution of bent up bars =

$$\text{Min} \left[\frac{(\tau_v - \tau_c) b d}{2}, V_{us} \right]$$

$V_{us} =$ Max. shear to be resisted by bent up bars.

$$(\tau_v - \tau_c) bd = 100 \text{ kN}$$

$$V_{us} = 130 \text{ kN}$$

Acceptable contribution
of bent up bars

$$= \min\left(\frac{100}{2}, 130\right)$$

Vertical stirrup for = ~~50~~

$$100 - 50 = 50 \text{ kN}$$

$$(\tau_v - \tau_c) bd = 100 \text{ kN}$$

$$V_{us} = 30 \text{ kN}$$

Acceptable contribution of
bent up bars = $\min\left(\frac{100}{2}, 30\right)$
= 30 kN

Vertical stirrup for = $100 - 30$
= 70 kN.

The vertical stirrups provided along with bent up bars shall be Greater than equal to Min. shear R/F

Minimum shear R/F :-

Minimum shear R/F shall be provided in the form of stirrups (Generally vertical stirrups) in all beams when $\tau_v > 0.5 \tau_c$

if $\tau_v < 0.5 \tau_c$ then no minimum shear R/F required

Also for structure for minor R/F such as lintels, No shear R/F if $\tau_v \leq \tau_c$

Minimum shear R/F as per IS code is

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

$$0.87 f_y A_{sv} \frac{d}{s_v} \geq 0.4 bd$$

$$\frac{A_{sv} (0.87 f_y)}{b s_v} \geq \frac{(\tau_v - \tau_c) bd}{0.4 bd}$$

NOTE { $0.5 \tau_c \leq \tau_v \leq \tau_c + 0.4$ → provide R/F for 0.4 MPa
 $\tau_v > \tau_c + 0.4$ MPa → provide R/F for $(\tau_v - \tau_c)$

PURPOSE OF MINIMUM SHEAR R/F (PRIORITY WISE)

- ① control the propagation of diagonal cracks i.e resist the principle tension and alters the pattern of cracking.
- ② prevent sudden formation of inclined cracks and reduces chances of abrupt failure i.e ensure ductile behaviour in shear
- ③ Improves dowel action of longitudinal bars
- ④ Helpful in controlling crack due to creep, shrinkage, Temp, differential settlement.
- ⑤ Take care of secondary Torsion
- ⑥ Hold main R/F in place while casting and act as Ties for compression steel and make them more effective.

Design steps:-

- i) calculate $\tau_v = \frac{V_u}{bd}$ or $\frac{V_u \pm \frac{M_u \tan \alpha}{d}}{bd}$
- ii) $\tau_v \nless \tau_{c, \max}$ (beams)
 $\tau_v \nless 0.5 \tau_{c, \max}$ (slabs) | if not satisfied then increase f_{ck} or ρ_s

iii) calculate τ_c based on P_t and f_{ck}

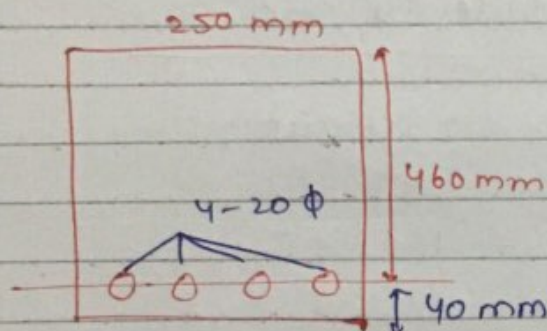
iv) provide shear R/F and check minimum shear criteria also

v) Check with Max. spacing criteria

Que. for the following section design

1) Vertical stirrups

2) Inclined stirrups



M20

Fe 500

$V_u = 200 \text{ kN}$

P_t	$\tau_c (\text{MPa})$
0.75	0.56
1.00	0.62
1.25	0.67

Step-1

Nominal shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{200 \times 10^3}{250 \times 460} = 1.74 \text{ MPa}$$

$$\tau_{c, \text{max}} = 0.63 \sqrt{f_{ck}} = 2.8 \text{ MPa}$$

$$\tau_{c, \text{max}} > \tau_v$$

$\Rightarrow$ safe in diagonal compression

$$P_t = \frac{A_{st}}{bd} \times 100$$

$$P_t = \frac{4 \times 314}{250 \times 460} \times 100 = 1.09\%$$

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)}{(1.25 - 1)} (1.09 - 1)$$

$$\tau_c = 0.638 \text{ MPa}$$

as $\tau_v - \tau_c > 0.4$, provide R/F for $\tau_v - \tau_c$

i) assume ~~two~~ 2 leg 8 mm dia stirrup

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2$$

$$0.87 f_y \times A_{sv} \times \frac{d}{S_v} \geq (\tau_v - \tau_c) bd$$

$$\frac{0.87 \times 415 \times 100 \times 460}{(1.74 - 0.638) \times 250 \times 460} \leq 131.05 \text{ mm}$$

$$S_v \leq 131.05 \text{ mm}$$

$$S_v = \text{Min} \begin{cases} 0.75d \rightarrow 460 = 345 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

$\Rightarrow$ provide 2 leg, 8 mm dia stirrup

@ 130 mm c-c.

$$ii) \quad 0.87 f_y A_{sv} \times \frac{d}{s_v} (\sin \alpha + \cos \alpha) \geq (\tau_v = \tau_c) b d$$

$$\alpha = 45^\circ$$

$$0.87 \times 415 \times 100 \times \frac{460}{s_v} (\sin 45 + \cos 45) \geq$$

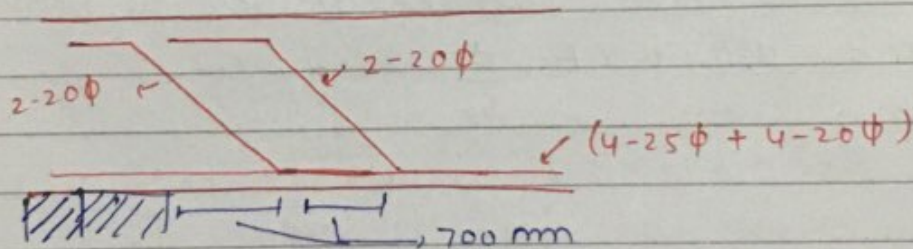
$$(1.74 - 0.638) 250 \times 460$$

$$s_v \leq 185.84 \text{ mm}$$

$$s_v \nlessgtr \begin{cases} d \rightarrow 460 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

provide 8 mm dia 2 legged inclined stirrup
@ 185 mm c/c

Que.



$$b = 300 \text{ mm}, d = 550 \text{ mm}, M20, Fe415$$

$$V_u = 250 \text{ kN}$$

P_t	τ_c
1	0.62
1.25	0.67

- ① In the above beam calculate the acceptable contribution of Bent up bars
- ② Design shear force for vertical stirrup.

$$i) \quad \tau_v = \frac{V_u}{bd} = \frac{250 \times 10^3}{300 \times 550} = 1.515 \text{ MPa}$$

$$< \tau_{cr, \max.} \quad \underline{\text{OK.}}$$

$$P_t = \frac{A_{st}}{bd} \times 100 = \frac{4 \times 490}{300 \times 550} \times 100 = 1.18\%$$

$$\tau_c = 0.62 + \frac{(0.67 - 0.62)}{(1.25 - 1)} \times (1.18 - 1) = 0.656 \text{ MPa}$$

$(\tau_v - \tau_c)$ has to be resisted by bent up bars + vertical stirrups.

$$= (\tau_v - \tau_c) bd = (1.515 - 0.656) \times 300 \times 550$$

$$= 141.7 \text{ kN}$$

Max^m force that can be resisted by bent up

$$bqr = 0.87 f_s \times A_{sv} \frac{d}{S_v} (\sin \alpha + \cos \alpha)$$

$$= 0.87 \times 415 \times \frac{2 \times 314 \times 550}{700} (\sin 45^\circ + \cos 45^\circ)$$

assume
 $\alpha = 45^\circ$

$$= 251.9 \text{ kN}$$

Acceptable contribution of bent up bars =

$$= \min \left\{ \frac{141.74}{2}, 251.9 \right\}$$

$$= 70.87 \text{ kN}$$

ii) Design S.F for vertical stirrups

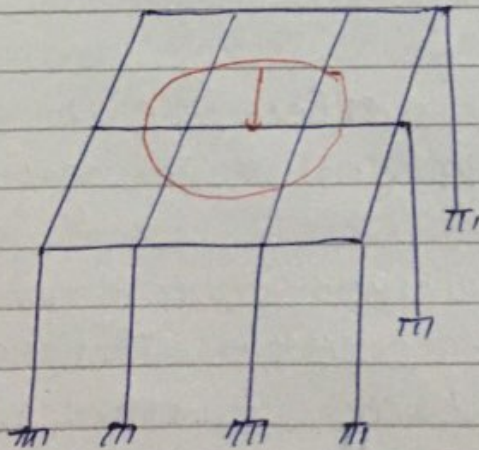
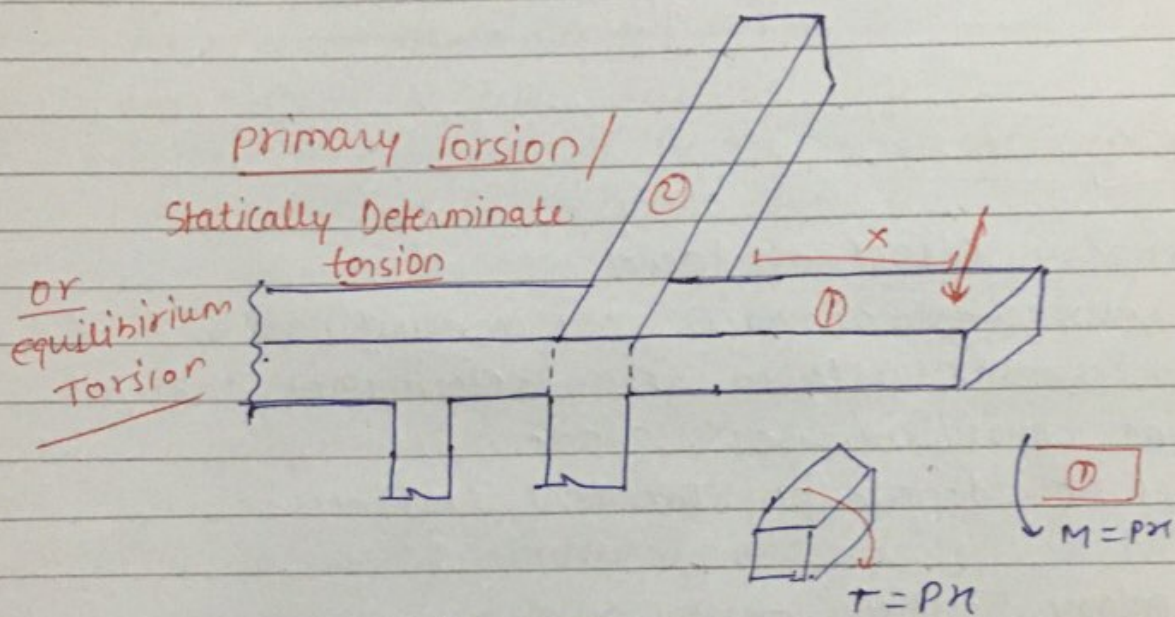
$$= 141.74 - 70.87$$

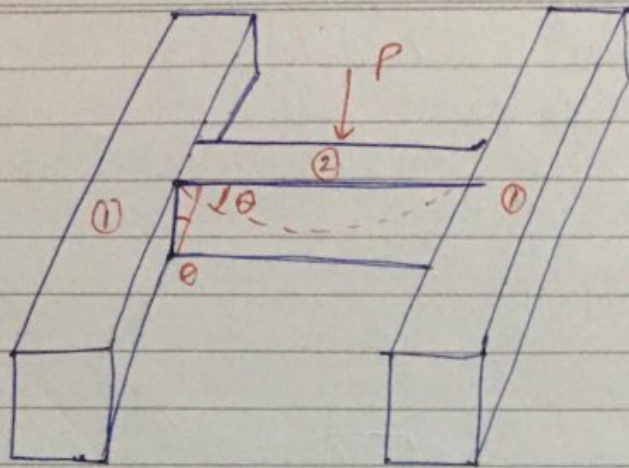
$$= 70.87 \text{ kN}$$

⑧ Design for Torsion

Torsional moment due to unsymmetrical loading, Geometry etc. exist along with BM, SF and axial force. Generally shear stresses are induced in Beam due to Torsion

Types of Torsion:-





$$EI \rightarrow M_1, V_1$$

$$EI, I_g \rightarrow M_2, V_2, \tau$$

$$M_1 > M_2, V_1 > V_2$$

Secondary Torsion /

Statically indeterminate
torsion /
compatible torsion

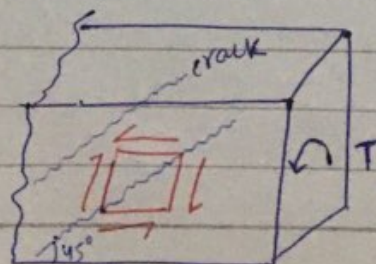
primary Torsion :-

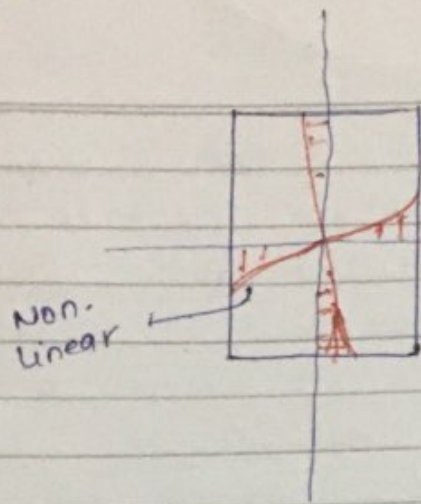
- primary effect of load
- develops to maintain static equilibrium
- can be calculated from determinant analysis using equilibrium eqⁿ alone
- does not depend on Torsional stiffness.

Secondary Torsion :-

- induced as a secondary effect i.e. by rotations
- Twisting moment is dependent on Torsional stiffness etc.
- develops to maintain compatibility condition
- statically indeterminate and require Torsional stiffness to obtain it.

Torsional stress :-





$$\tau_{c, \max} = \frac{T}{\alpha b^2 D}$$

b = smaller dimension
 α - depends on D/b

Max. shear stress due to Torsion in case of rectangular Beam occurs at center line of wider face.
 Diagonal Tension reduced by Torsion is similar to Tension produced by Flexural shear

NOTE: Torsion is a 3D problem including shear and warping of C/S. That is plane section no longer remains plane.

Design: - ① During structural analysis if Torsional stiffness is neglected then beam designed for obtained BM & SF is assumed to cater for compatibility Torsion.

However for equilibrium torsion we need to design the beam.

IS code design method (based on skew bending theory) recommends to design a beam with BM, SF and Torsion by equivalent BM & SF (hypothetical BM & SF)

This concept is applied for equilibrium Torsion only
 we generally don't design for compatibility torsion.

once Torsion crack is formed, the crack will penetrate inwards from the outer surface and can lead to Sudden failure unless torsional R/F is provided.

equivalent moment / Equivalent shear (V_e)

$$\Rightarrow V_e = V_u + 1.6 \frac{T_u}{b}$$

$V_u \rightarrow$ factored Transverse shear

$T_u \rightarrow$ factored Torsional moment

$b \rightarrow$ width of beam. ($b = b_w$ for flange section)

$$\Rightarrow M_{e1}$$

$$M_{e1} = M_u + M_t$$

$$M_t = \frac{T_u}{1.7} \left[1 + \frac{D}{b} \right]$$

$M_u \rightarrow$ factored BM

$D \rightarrow$ overall depth of Beam

$b \rightarrow$ width of Beam ($b = b_w$ for flanged secⁿ)

Design Steps:-

1) calculate $\tau_{ve} = \frac{V_e}{bd}$ and τ_{max}

2) calculate A_{st} for M_u if $M_u > M_{u,lin}$ of singly R/F section $\Rightarrow$ Design a Doubly R/F section

Step-3 calculate τ_c based on P_t of step-2

Step-4 Case-1 $0.5 \tau_c \leq \tau_v \leq \tau_c$

then provide min shear RIF as in the case of flexural shear ($0.87 f_y A_{sv} \frac{d}{s_v} \geq 0.4 b d$)

The spacing of stirrups in this case shall be as per flexural shear spacing criteria.

(i.e. for vertical stirrup spacing $\nless 0.75 d$ m
 $\min \{ 300 \text{ m}$

$$\checkmark \tau_c < \tau_{ve} < \tau_{c, \max}$$

$$\tau_{ve} \gg \tau_{c, \max}$$

Case-2 if $\tau_c < \tau_{ve} \leq \tau_{c, \max}$

$\Rightarrow$ design both longitudinal RIF

(for New BM) and shear RIF

Design longitudinal RIF for M_{e1} ($M_u + M_t$)

if $M_{e1} > M_{um}$ for a singly RIF $\Rightarrow$ Design a Doubly RIF section.

$$\text{for singly RIF } A_{st} = \frac{f_{ck}}{2 f_y} \left[1 - \sqrt{1 - \frac{4.6 M_{e1}}{f_{ck} b d^2}} \right] b d$$

if $|M_t| > |M_u|$ i.e. Numerical Value of

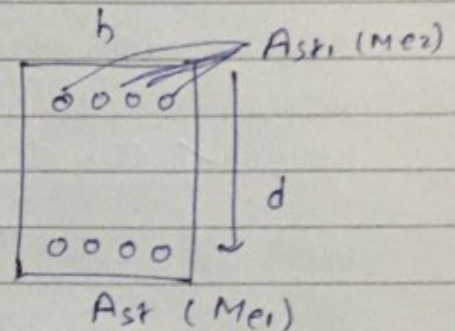
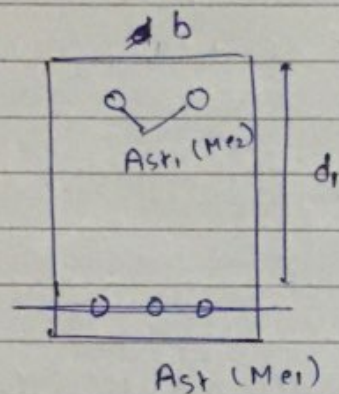
~~exp~~ M_t exceeds Numerical value of M_u , then longitudinal RIF shall be provided on the flexural compression face for M_{e2}

$$M_{e2} = |M_t| - |M_u|$$

$$A_{st1} = \frac{f_{cu}}{2f_y} \left[1 - \sqrt{1 - \frac{4.6 M_{e2}}{f_{cu} b d^2}} \right] b d$$

A_{st1} = tensile RIF provided on the flexural compression face

if $|M_{e1}| < |M_{e2}|$ then no need to provide Tensile RIF on the flexural compression face.



NOTE - The A_{st1} is over and above A_{st} in case of Doubly RIF section.

Design Shear RIF by using the following two formulas -

formula-1. $0.87 f_y A_{sv} \frac{d}{S_v} \geq \tau_{ve} b d$

corresponds to $\tau_c \rightarrow$ New tensile RIF provided for M_{U1}

formula-2 $A_{sv} = \frac{T_u S_v}{b d_1 (0.87 f_y)} + \frac{V_u S_u}{2.5 d_1 (0.87 f_y)}$

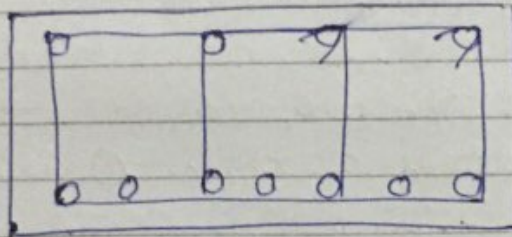
or $0.87 f_y A_{sv} \frac{d_1}{S_v} \geq \frac{T_u}{b_1} + \frac{V_u}{2.5}$

A_{sv} = Area of the outer two Legged.

b_1 = c/c distance b/w corner bars along the width

d_1 = c/c distance b/w corner bars along the depth

NOTE: ① Since the cracks are spiral, four sided closed stirrups $\perp$ to axis of member are required ② Only the outer two legged shall be considered because inner legs are not efficient in shear due to Torsion.



Only Flexural shear $\rightarrow$ 4 Legged

Flexural shear + Torsion shear $\rightarrow$ 2 Legged

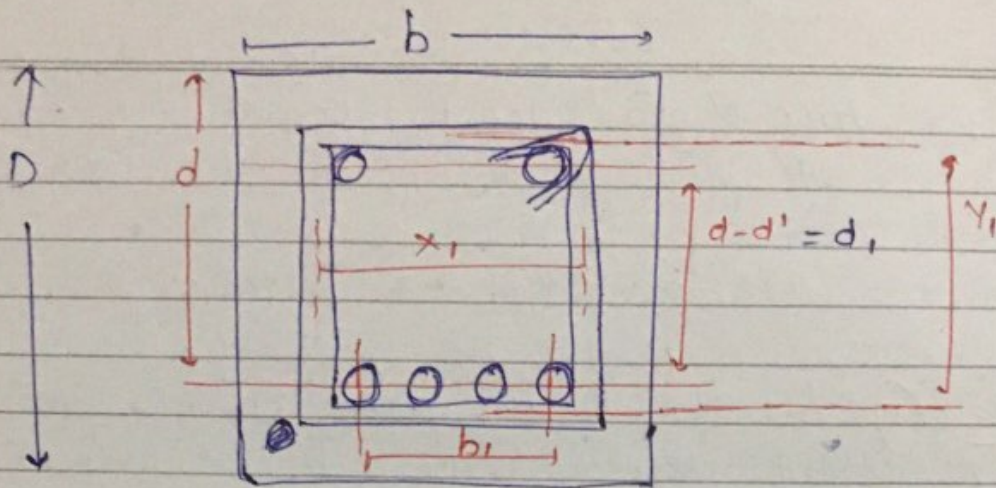
$\rightarrow$ No restriction on characteristic strength of stirrups unlike in flexural shear case i.e. for f_{esd} we can take 500 MPa.

$$\text{spacing of stirrups} \leq \min \left\{ \begin{array}{l} x_1 \\ \frac{x_1 + y_1}{4} \\ 300 \text{ mm} \end{array} \right.$$

x_1 = shorter dimension of stirrup (c/c distance)

y_1 = longer dimension of stirrup (c/c distance)

Arkan



Case-3

if $\tau_{ve} > \tau_{c, max}$

then either increase the c/s or increase the grade of concrete

$$\left\{ \begin{array}{l} 0.5 \tau_c < \tau_{ve} \leq \tau_c \rightarrow \text{① } \times \rightarrow M_u \\ \tau_c < \tau_{ve} \leq \tau_{c, max} \rightarrow \text{② } \checkmark \rightarrow M_u + M_t \rightarrow M_{eq} \rightarrow A_{st} \\ \tau_{ve} > \tau_{c, max} \times \end{array} \right.$$

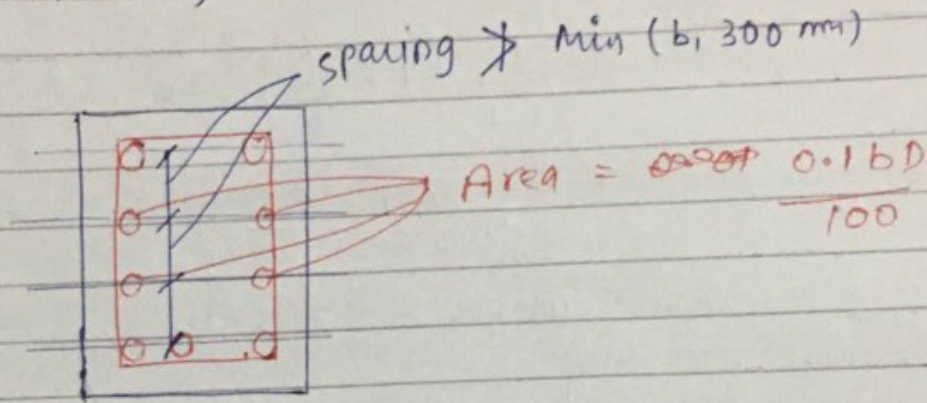
$M_t > M_u \rightarrow A_{st1}$

Recommendation for longitudinal bars :-

longitudinal Bars shall be placed as close to the corner as possible and there shall be at least one longitudinal Bar at each corner.

When the c/s dimension (b, D) of the member exceeds 450 mm, additional longitudinal bars shall be provided along the two faces for crack control due to torsion.

Total Area of side face R/F shall not be less than 0.1% of Web Area (that is $0.001 b D$) and distributed equally on the two faces at a spacing not exceeding b or 300 mm whichever is minimum.



Que. Design R/F for the concrete Beam of Rectangular secⁿ for the following data.

$D = 850 \text{ mm}$, $d = 800 \text{ mm}$, $b = 300 \text{ mm}$
 $M_u = 200 \text{ kNm}$, $V_u = 100 \text{ kN}$, $T_u = 50 \text{ kNm}$
 $f_{cu} = 20 \text{ MPa}$, $f_y = 415 \text{ MPa}$

P_k	0.25	0.5	0.75	1.00	1.25
τ_c (MPa)	0.36	0.48	0.56	0.62	0.67

$$V_e = V_u + 1.6 \frac{T_u}{b}$$

$$= 100 + 1.6 \times \frac{50}{0.3}$$

$$= 366.67 \text{ kN}$$

$$\tau_{ve} = \frac{V_e}{bd} = \frac{366.67 \times 10^3}{300 \times 800} = 1.527 \text{ MPa}$$

$$\tau_{c, \max} = 2.8 \text{ MPa}$$

here $\tau_c < \tau_{ve} \leq \tau_{c, \max}$. case - ii

$$M_{e1} = M_u + M_t$$

$$M_t = \frac{T_u}{1.7} \left(1 + \frac{D}{b} \right) = 0$$

$$= \frac{50}{1.7} \left(1 + \frac{850}{300} \right) = 112.74 \text{ kNm.}$$

$$M_{e1} = 200 + 112.74 = 312.74 \text{ kNm.}$$

$$M_t < |M_u|$$

No need to provide tensile R/f on the flexural compression side

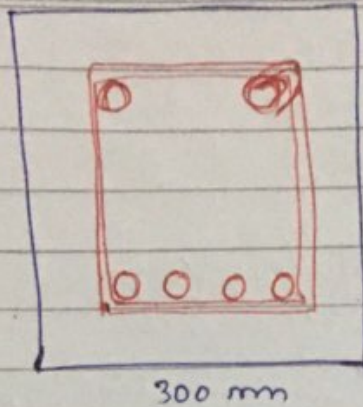
$$\begin{aligned} M_{\min} &= 0.138 f_{uc} b d^2 \\ &= 0.138 \times 20 \times 300 \times 800^2 \\ &= 529.29 \text{ kNm.} \end{aligned}$$

$M_{\min} > M_{e1}$, can design a singly R/f section.

$$A_{st} = \frac{f_{uc}}{2f_y} \left[1 - \sqrt{1 - \frac{4.6 M_{e1}}{f_{uc} b d^2}} \right] b d$$

$$A_{st} = \frac{20}{2 \times 415} \left[1 - \sqrt{1 - \frac{4.6 \times 312.74 \times 10^6}{20 \times 300 \times 800^2}} \right] 300 \times 800$$

$$A_{st} = 1209.83 \text{ mm}^2$$



HANGER BAR DESIGN:- $= 0.2\%$ of Gross Area

$$= 0.2 \times bD / 100$$

$$= 0.2 \times 300 \times 850 / 100 = 510 \text{ mm}^2$$

provide 2 hanger bars of 12 mm dia
 Area = $2 \times 113 = 226 < 510 \text{ mm}^2$ O.K.

Shear R/F design:-

$$\tau_{ve} = 1.527 \text{ N/mm}^2$$

$$A_{st} = 4 \times 25 \phi$$

$$= 1256 \text{ mm}^2$$

$$\left\{ P_t = \frac{A_{st}}{bd} \times 100 \right.$$

$$P_t = 0.52\%$$

$$\tau_c = 0.48 + \frac{(0.56 - 0.48)}{(0.75 - 0.50)} (0.52 - 0.50)$$

$$\tau_c = 0.486 \text{ MPa}$$

$$\tau_c < \tau_{ve} < \tau_{c, \max.} \quad \checkmark \quad \left\{ \text{case (ii)} \right.$$

Formula-1

$$0.87 f_y A_{sv} \frac{d}{S_v} \geq (\tau_{ve} - \tau_c) b d$$

$$\Rightarrow S_v \leq 115.6 \text{ mm}$$

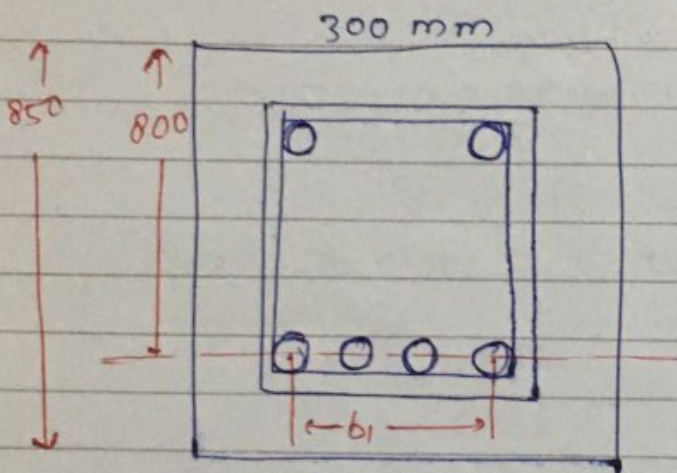
Formula - 2

$$A_{sv} \geq \frac{T_u}{f_y}$$

$$0.87 \times f_y \times A_{sv} \times \frac{d_1}{S_v} \geq \frac{T_u}{b_1} + \frac{V_u}{2.5}$$

$$0.87 \times 415 \times 100 \times \frac{750}{S_v} \geq \frac{50 \times 10^6}{224} + \frac{100 \times 10^3}{2.5}$$

$$S_v \leq 102.87 \text{ mm}$$



$$b_1 = 300 - 2 \times (20 + 8 + \frac{20}{2}) = 224 \text{ mm}$$

$$d_1 = 800 - 2 \times (20 + 8 + \frac{20}{2}) = 764 \text{ mm} \quad 800 - 50 = 750 \text{ mm}$$

$$S_v \leq \min \left\{ \begin{array}{l} x_1 \\ \frac{x_1 + y_1}{4} \\ 300 \text{ mm} \end{array} \right.$$

$$x_1 = 224 + 2 \left(\frac{\phi_s}{2} + \frac{\phi_m}{2} \right)$$

$$= 224 + 20 + 8$$

$$= 252 \text{ mm}$$

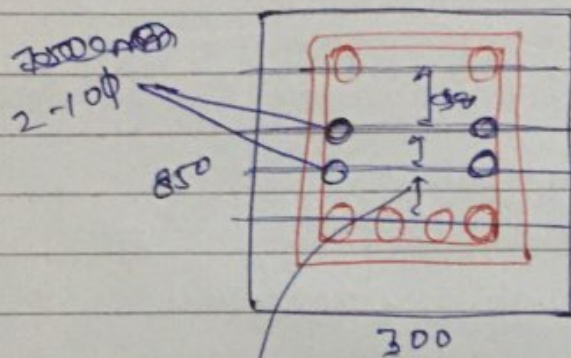
$$y_1 = \frac{\phi_s}{2} + \frac{12}{2} + d_1 + \frac{\phi_m}{2} + \frac{\phi_s}{2}$$

$$= 774 \text{ mm}$$

$$S_v \leq \begin{cases} 252 \\ 256 \\ 300 \text{ mm} \end{cases}$$

S_v provide 8mm dia 2-legg closed stirrups @ 100 mm c/c.

side face R/F:-



$$\frac{750}{3} = 250 < (b, 300) \text{ mm}$$

OK

side face R/F on each face.

$$= \frac{0.05 b D}{100}$$

$$= \frac{0.05}{100} \times 300 \times 850$$

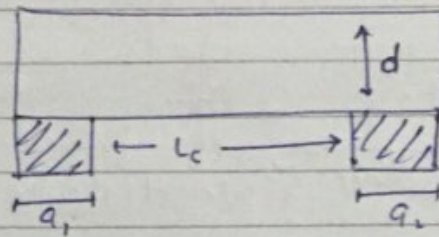
$$= 127.5 \text{ mm}^2$$

9) DESIGN OF BEAMS

EFFECTIVE SPAN :- (BEAMS & SLABS)

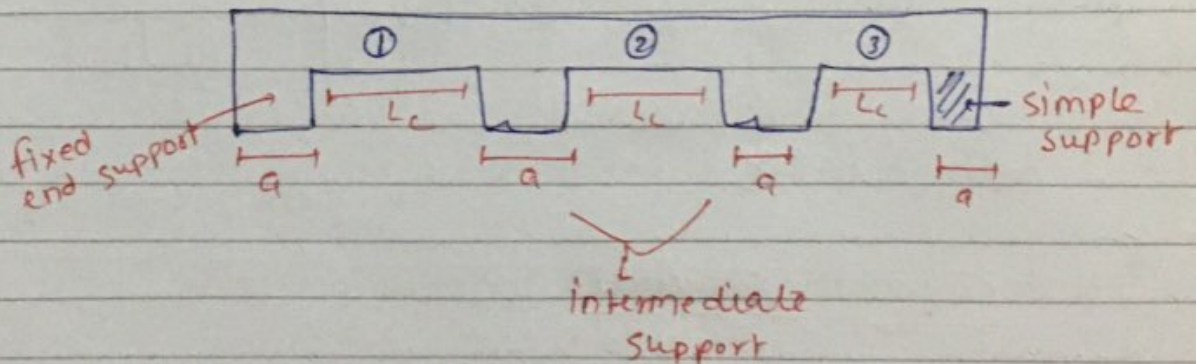
It is the length eff. in Bending

i) simply supported :-



$$L_{eff} = \min \left\{ \begin{array}{l} L_c + d \\ L_c + \frac{(a_1 + a_2)}{2} \end{array} \right. \quad \left(\text{c/c distance b/w supports} \right)$$

ii) continuous :-



a) when $a \leq \frac{L_c}{12}$

$$\Rightarrow L_{eff} = \min \left\{ \begin{array}{l} L_c + a \\ L_c + d \end{array} \right. \quad \text{for beams 1, 2, 3.}$$

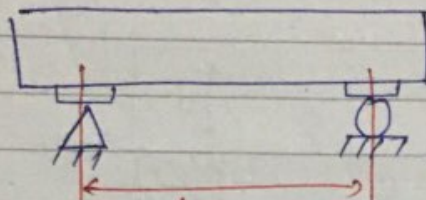
* b) when $a > \frac{L_c}{12}$ or 600, whichever is Min =

→ $L_{eff} = L_c$ for 1 end fix & other continuous or for intermediate span. } for beams 1 & 2

→ $L_{eff} = \min \left\{ \begin{array}{l} L_c + d/2 \\ L_c + a/2 \end{array} \right.$ for end span with one end freely supported (simple support) and other end continuous (beam 3)

a = width of discontinuous support that is simple support.

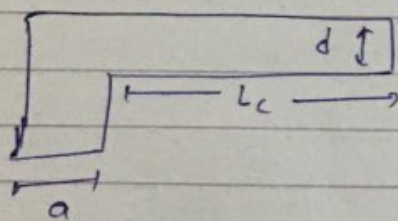
iii) Span with roller or rocker bearing :-



$l_{eff} = c/c$ distance b/w bearings

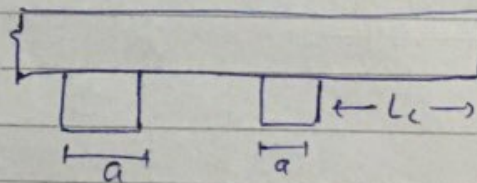
iv) cantilever :-

Independent cantilever



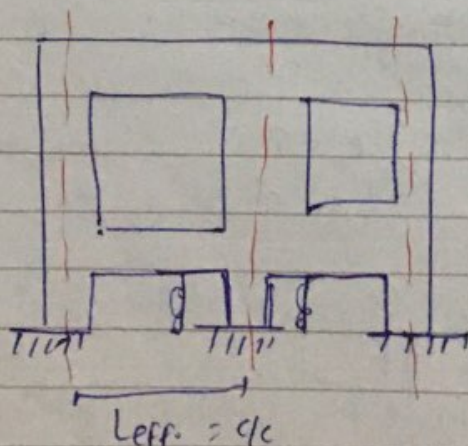
$$l_{eff} = L_c + \frac{d}{2}$$

When cantilever is a part of continuous beam.



$$l_{eff} = L_c + \frac{a}{2}$$

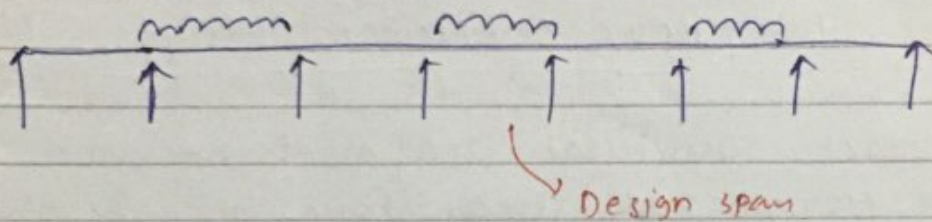
v) for frame structures :-



Live load arrangement for max. BM $\Rightarrow$

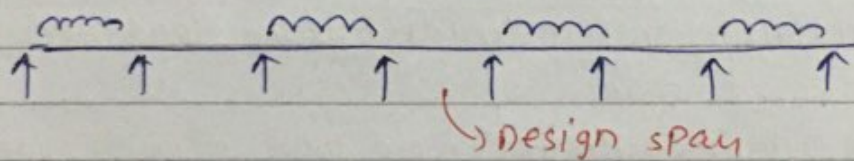
as dead load is permanent, LL shall be arranged to obtain the Max. BM for design

i) Max. sagging (+ve) BM in a given span



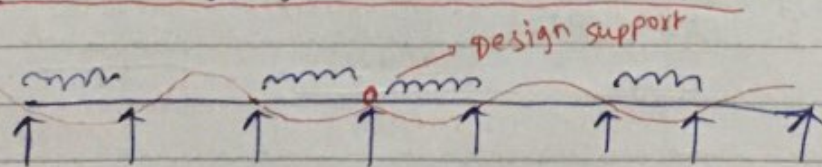
load the same span + alternate span.

ii) Maximum Hogging (-ve) BM in a span



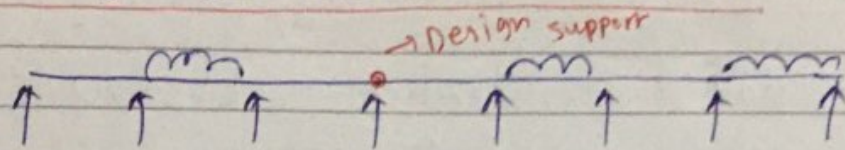
load the adjacent span + alternate span

iii) Max^m hogging Moment @ a support :-



load the adjacent span of support + alternate span

iv) Max. sagging moment @ a support



load the next span to adjacent of support
+ alternate spans.

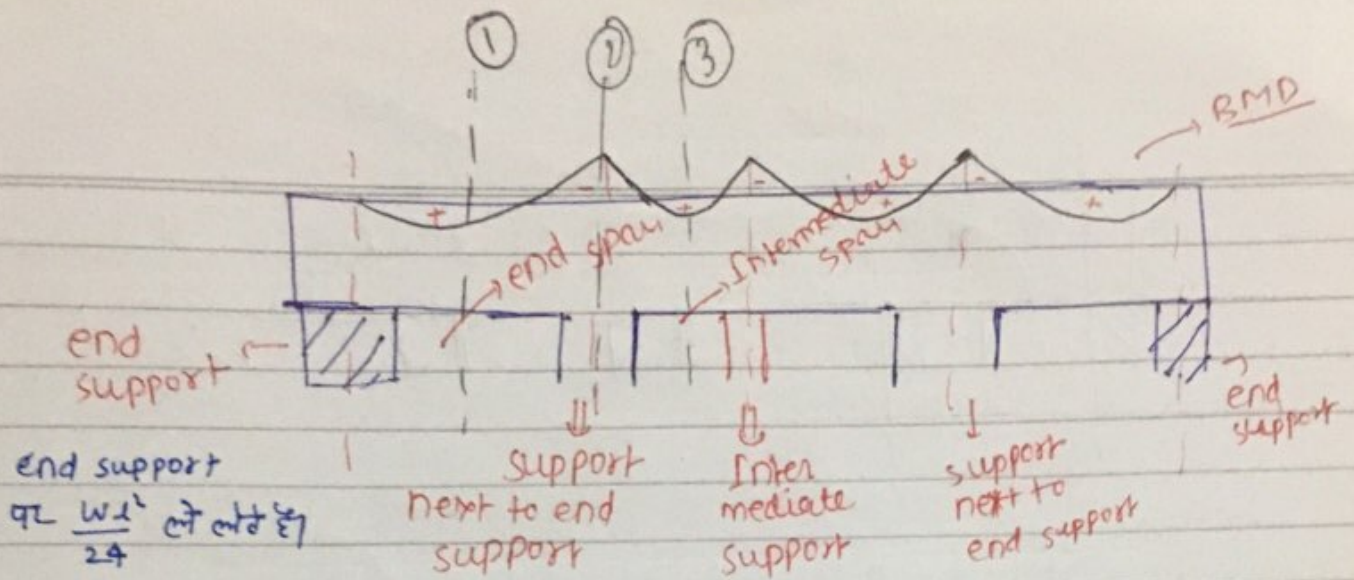
Moment and shear coefficients (Beams & one way slab) :-

When exact structural analysis is not done, Bending moment and shear force used in the design may be obtained by using coefficients given in Is code provided the following conditions are satisfied.

- 1) uniform c/s
- 2) substantially uniformly distributed load over 3 or more spans
- 3) spans don't differ by more than 15% of longest

B.M coefficients: -

Types of load	span moments		Support moments	
	Near Middle of end span	At middle of intermediate span	Support Next to end support	At other intermediate support
D.L	$+\frac{1}{12}$	$+\frac{1}{16}$	$-\frac{1}{10}$	$-\frac{1}{12}$
L.L	$+\frac{1}{10}$	$+\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{9}$



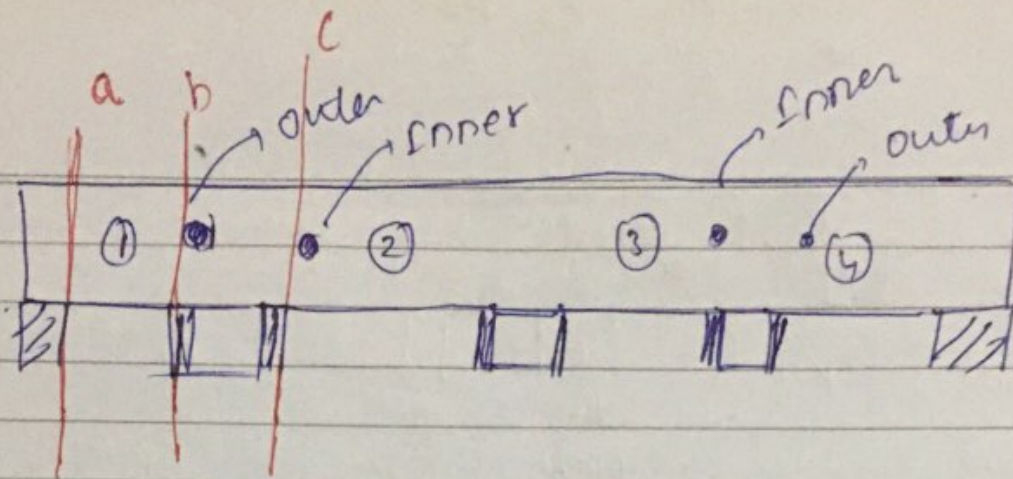
$$B.M_1 = \frac{1}{12} w u_{DL} l_{eff}^2 + \frac{1}{10} w u_{LL} l_{eff}^2$$

$$B.M_2 = -\frac{1}{10} w u_{DL} l_{eff}^2 - \frac{1}{9} w u_{LL} l_{eff}^2$$

NOTE If the span on the two sides of support are different or the loading are different, support moment will be calculated from the two sides and average will be taken.

Shear coefficients :-

Types of load	At end support	Support Next to the end support		Other intermediate supports
		outer side	inner side	
D.L	0.40	0.60	0.55	0.50
L.L	0.45	0.60	0.60	0.60



$$SF_A = 0.40 W_{u,DL} L_c + 0.45 W_{u,LL} L_c$$

$$SF_B = 0.60 W_{u,DL} L_c + 0.60 W_{u,LL} L_c$$

$$SF_C = 0.55 W_{u,DL} L_c + 0.60 W_{u,LL} L_c$$

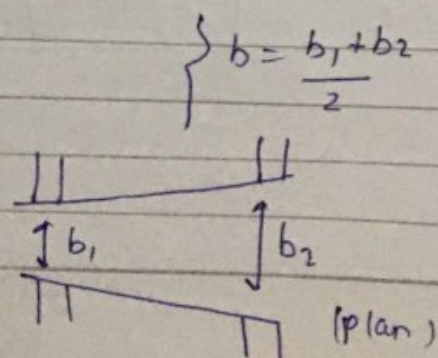
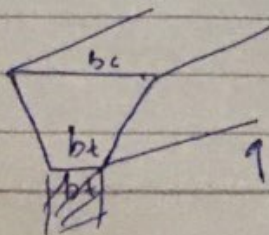
NOTE - The respective span length and load shall be considered

Slanderness Limits for Beams:-

(To ensure lateral stability):-

- If length of a beam is very high compare to c/s dimension, compression flange can buckle
- for simply supported and continuous beams, clear distance b/w lateral restrain shall not be greater than $\text{Min} \left(60b, \frac{250b^2}{d} \right)$

b = width of compression flange, Mid way b/w the lateral restrain
 d = eff. depth.



for cantilever, clear distance b/w free end
and lateral restraint $\geq \min \begin{cases} 25b \\ 100 \frac{b^2}{d} \end{cases}$

BOND :-

Refers to the adhesion b/w concrete & steel and is responsible for transfer of force from ~~bar~~ steel to concrete.

3 Mechanisms for bond

i) chemical adhesion :-

Gum like property in the product of hydration

ii) Frictional Resistance :-

Surface Roughness of the R/F and Grip exerted by shrinkage of concrete

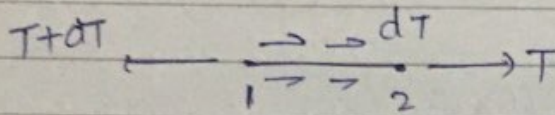
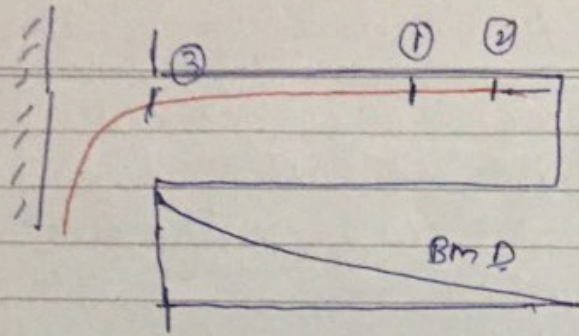
iii) Mechanical Interlock :-

Due to Ribs on Deform bar
(NOT available in plain bar) $\rightarrow$ HYSD Bar.

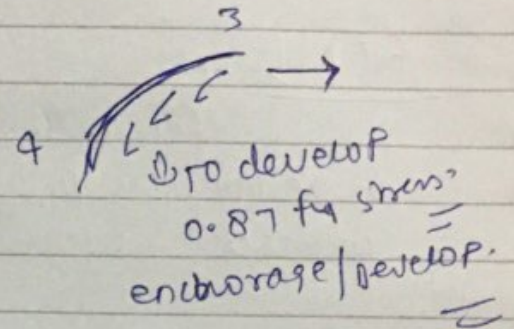
• Depending on the location, Bond can be called as

i) flexural Bond / Local Bond (Due to change B.M)

* ii) anchorage Bond / development bond (This is due to the cut off point or end point where we require to develop a desired level of stress.)



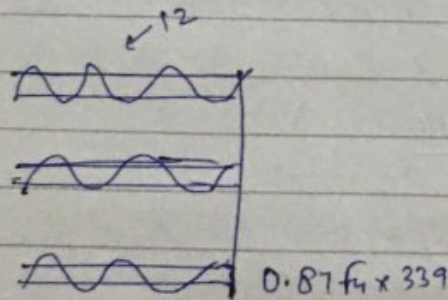
flexural/local
 (To equate the tensile stress)



NOTE: Bond stress :- stress on surface of R/F due to load !

Bond strength :- MAX. Bond stress that a bar can take ! (permissible bond stress)

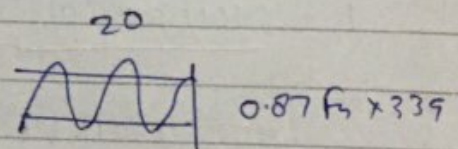
We'll mainly Deal with Anchorage/Development Bond !



bond stress ↓

Bond strength ↑

Area ↑



bond stress ↑

Bond strength ↓

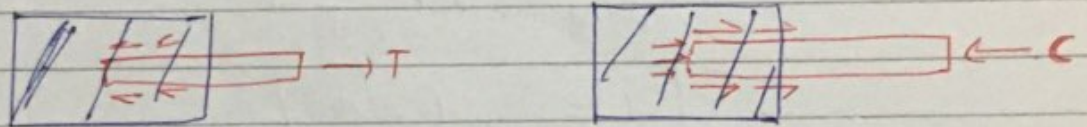
Area ↓

=

Development length & Anchorage length:-

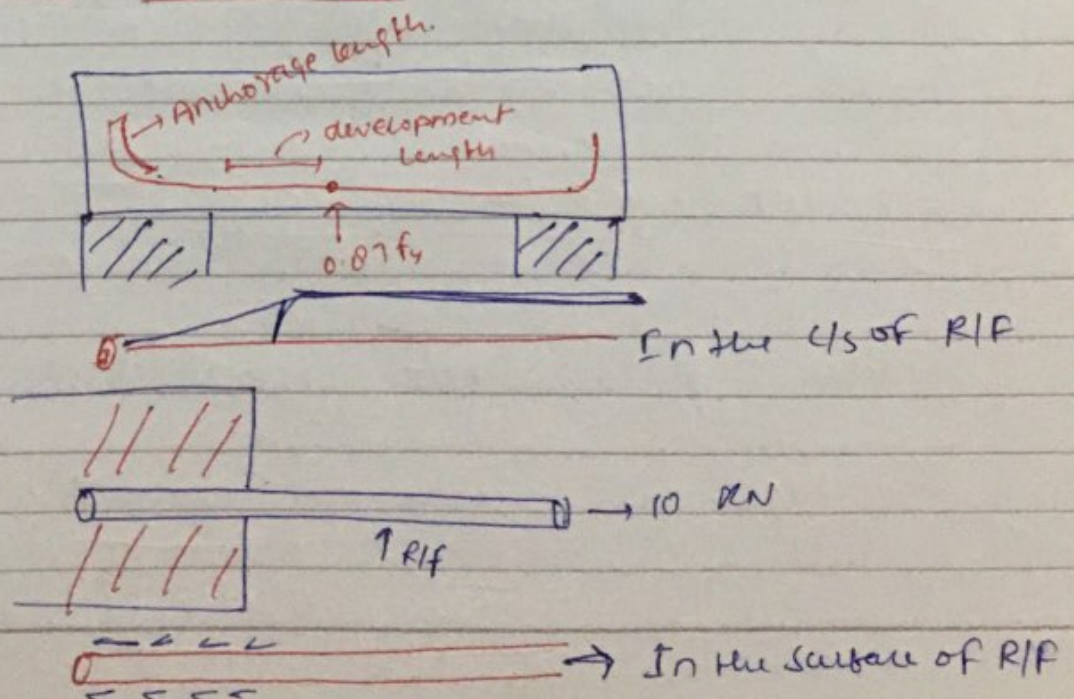
Certain minimum length of Bar is required on either side of stress point to transfer the force to the surrounding concrete through bond without slip to avoid pulling out of bar under Tension or pushing in under compression.

This minimum length is called development length.

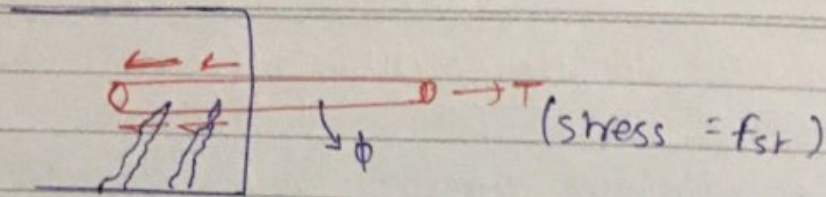


- However when development length can not be provided straight, bends, Hooks or Mechanical encourage can be provided to supplement with an equivalent embedment length.

- Development length can also be called an anchorage length when the embedded bar is not subjected to flexural Bond.



* τ_{bd} is more in compression than Tension!



along 1/3 (stress)

$\lll \rightarrow$ not same, due to cracks. (τ_{bd}) on surface

τ_{bd} (constant)

$$\pi \phi l \tau_{bd} = f_{st} \cdot \frac{\pi}{4} \phi^2$$

*
$$l = \frac{f_{st} \phi}{4 \tau_{bd}}$$

$l \rightarrow$ development length to develop a stress of f_{st} .

The bond stress variation is non uniform
However for design purpose it is considered as constant

τ_{bd} = Design bond stress / permissible bond stress / bond strength.

When $f_{st} = 0.87 f_y$, $l = l_d$.

$$l_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

τ_{bd} for plain Bar in Tension :-

Grade	M20	M25	M30	M35	$\geq M40$
τ_{bd} (MPa)	1.2	1.4	1.5	1.7	1.9

$$\tau_{bd} = 0.16 (f_{ck})^{2/3}$$

NOTE • For Bars in compression τ_{bd} shall be increased by 25%

• for deform bars τ_{bd} shall be increased by 60%

* for same stress level L_d in compression = $0.8 L_d$ in Tension.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} \rightarrow \text{Tension}$$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd} \times 1.25} \rightarrow \text{comp.}$$

L_d for a stress of $0.87 f_y$

Grade of steel	Tension		Compression	
	M20	M25	M20	M25
Fe 250	46 ϕ	39 ϕ	37 ϕ	32 ϕ
Fe 415	47 ϕ	41 ϕ	38 ϕ	33 ϕ
Fe 500	57 ϕ	49 ϕ	46 ϕ	40 ϕ

$$L_d \text{ for MS} = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{54.375 \phi}{\tau_{bd}}$$

$$L_d \text{ for HYSD} = \frac{0.87 \times f_y \times \phi}{4 \tau_{bd} \times 1.6} = \frac{56.41 \phi}{\tau_{bd}}$$

* L_d for MS bar will be always less than that of HYSD Bar

l_d for Bundled Bars :-

No of bars
bundled

% increase in l_d for
individual Bar

∞ (2 bars)

10%

∞ (3 bars)

20%

∞ (4 bars)

33%

Anchorage Value for bends & hooks :-

Bends and hooks can be provided to obtain required development length.

- Deformed bars may be used without end anchorage if development length criteria is satisfied. However Hooks shall be provided for plain bars as extra safety.

When bars are in Tension :- (Main R/F)

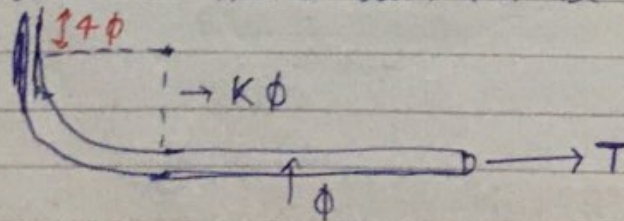
anchorage value (equivalent horizontal length to resist the same force) shall be taken as 4ϕ for each 45° bend subject to a Maximum of 16ϕ .

projection beyond all bends is taken as 4ϕ

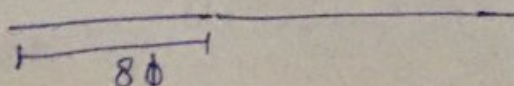
(o/w the bends or hooks will not be considered as effective)

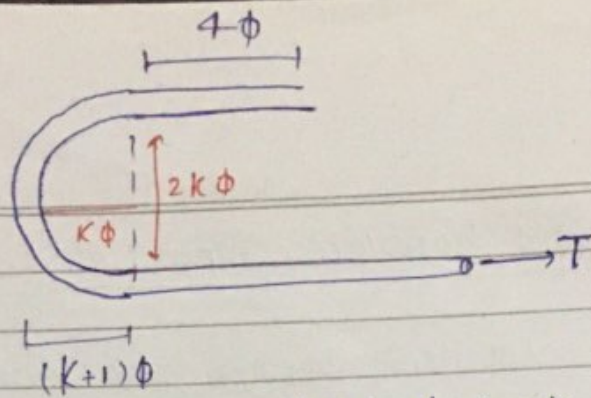
$$\frac{\lambda (K+0.5)\phi}{2} + 4\phi$$

K for MS $\rightarrow 2$
K for HYSD $\rightarrow 4$



Standard bend 90° (8ϕ)



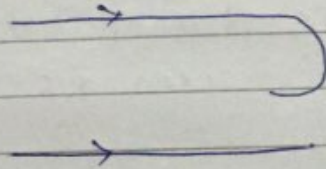


Standard hook 180° (16ϕ).

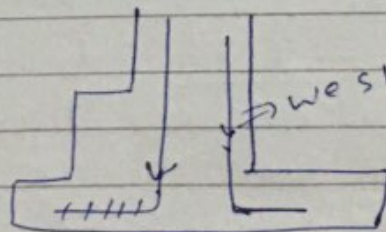
NOTE: ① As MS bar is more ductile, less radius is required for bend and hence $K=2$ for MS
 $K=4$ for HYSD.

K shall not be used for anchorage value calculation

in Compression: -

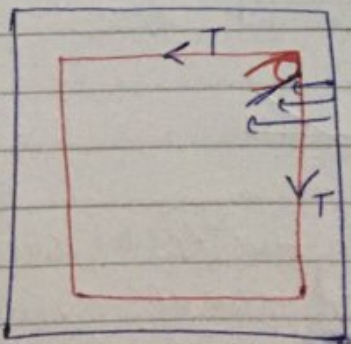


Bends and hooks are not effective in anchoring bars in compression and hence consider the projected length only.



We still bend the bar because it can be beneficial while Solution Bends (BM)

Anchorage Requirement for shear R/F :-



for stirrups and transverse Ties

Complete development length and anchorage is considered to have been provided if the following conditions are satisfied.

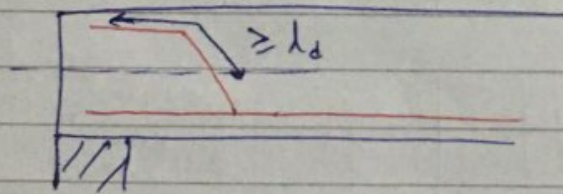
Angle

Extension beyond end of curve.

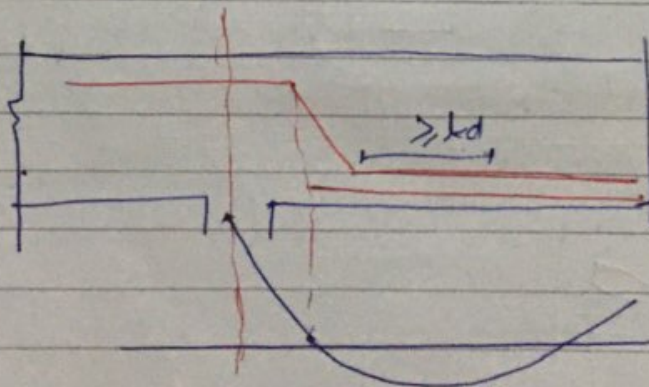
90°	→	8 ϕ
135°	→	6 ϕ
180°	→	4 ϕ

dia of shear R/F $\leq$ dia of bars about which shear R/F is bent

for Bent up bars



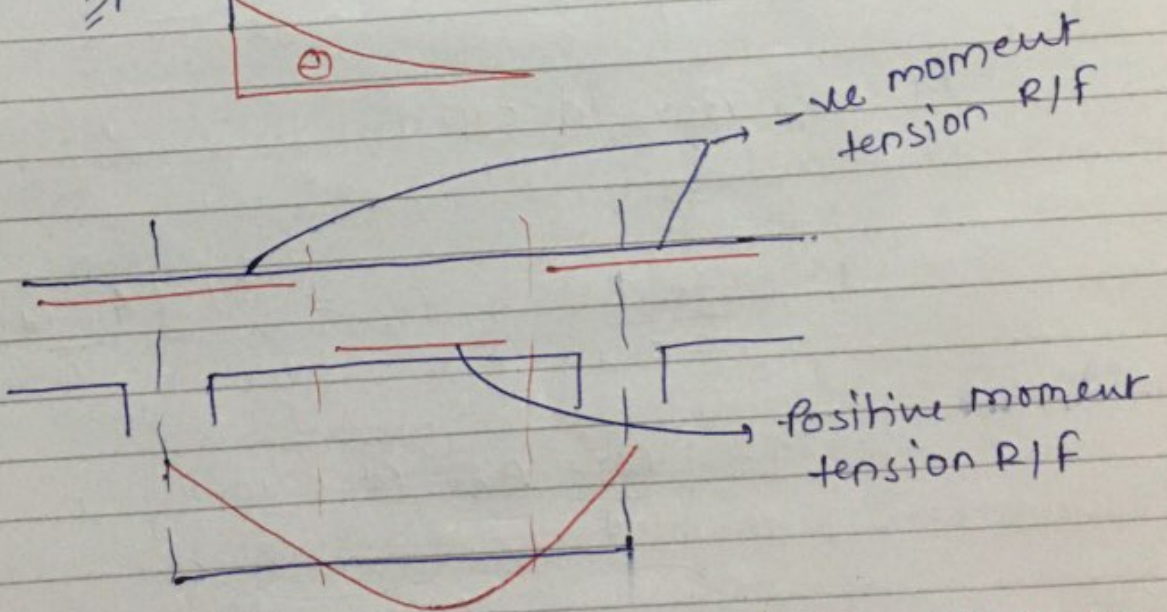
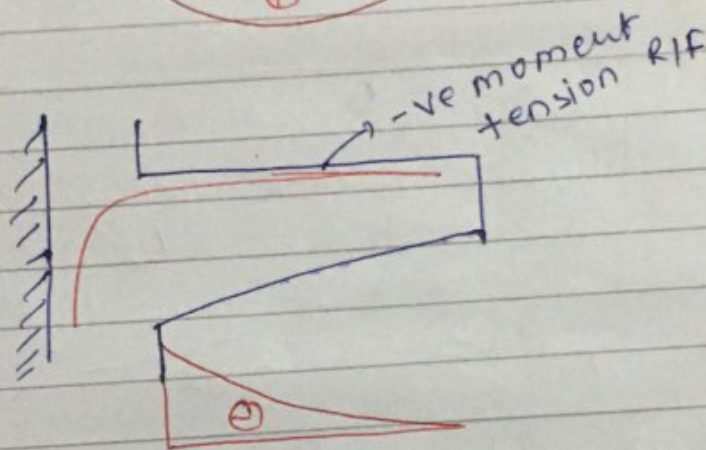
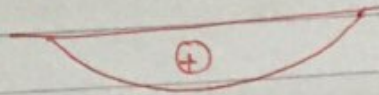
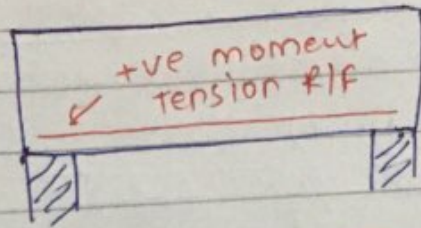
When bar is terminated into compression zone, the l_d shall be provided from mid depth of beam.



When bar is terminated in the tension zone, the l_d shall be provided from end of incline.

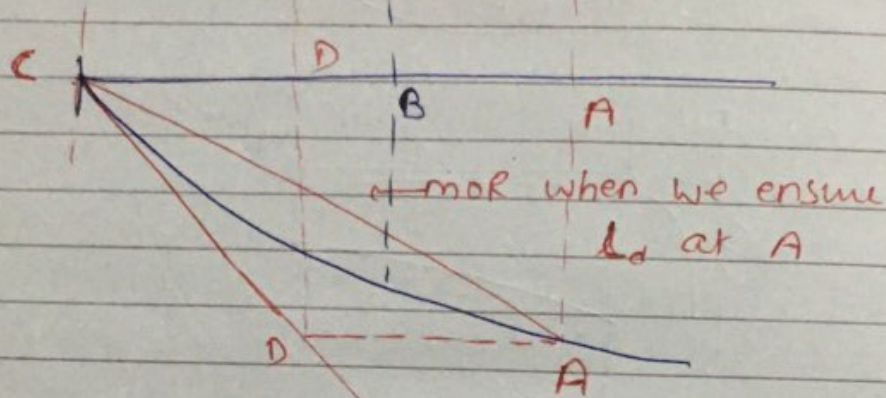
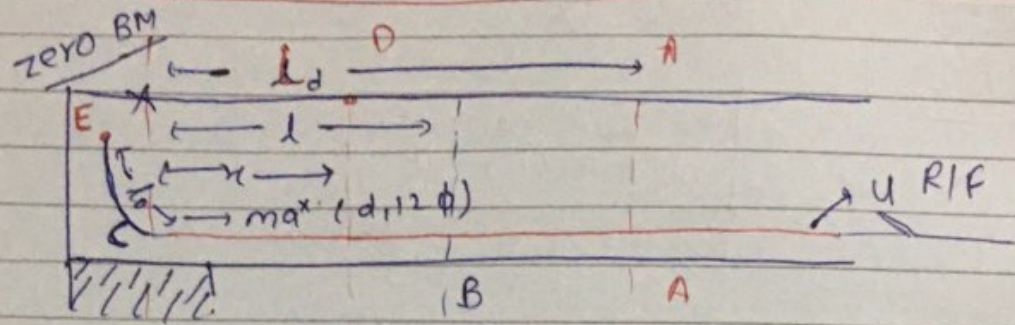
Development length requirement:-

NOTE.



Development Length Required (+ve moment ten.

Development length Requirement (+ve moment tension R/F)



$$M = f_{st} A_{st} (d - 0.42x)$$

$\downarrow$
4-R/F

$$l = \frac{f_{st} \phi}{4 \tau_{bd}} \Rightarrow f_{st} = l \left(\frac{4 \tau_{bd}}{\phi} \right)$$

$$M = \left[\frac{4 \tau_{bd}}{\phi} A_{st} (d - 0.42x) \right] l$$

Constant

$$M = \text{constant} \times l$$

$$\text{When } l = l_d, M = \text{MOR}$$

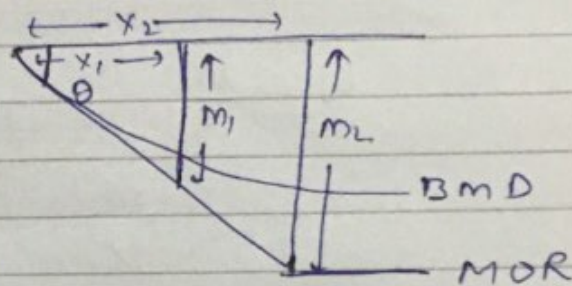
$$\text{When } l = 0, M = 0$$

$$y = m x$$

In order to envelop the BMD diagram, the MOR diagram shall be ADE

It means we need to ensure stress of $0.87 f_y$ not at point A but at point D

$$\tan \theta = \frac{M}{x} \Rightarrow x = \frac{M}{\tan \theta} = \frac{M}{V}$$



$$\tan \theta = \frac{M_2}{x_2} = \frac{M_1}{x_1}$$

$$\Rightarrow x_2 = \frac{M_2}{V}, \quad x_1 = \frac{M_1}{V}$$

length required to develop stress of $0.87 f_y \leq$ length available to develop a stress of $0.87 f_y$.

$$l_d \leq x$$

$$l_d \leq \frac{M}{V}$$

$$\star \quad l_d \leq \frac{M}{V} + l_0$$

Note:

Adequate Flexural strength can invariably be ensured by providing a full development length l_d to a point E beyond a (critical section D) located at a distance $\frac{M}{V}$ from the secⁿ of O, BM

$M = MOR$ at point of O, BM assuming a stress of $0.87 f_y$ that is consider all R/F available @ point of O, BM and assume a stress of $0.87 f_y$

$V = \text{factored SF @ point of } \ominus, \text{ BM}$

$l_o = \text{total anchorage value beyond center of support}$
However l_o shall not be greater than $\max. (d, 12\phi)$
 $l_o \nless \max. (d, 12\phi)$

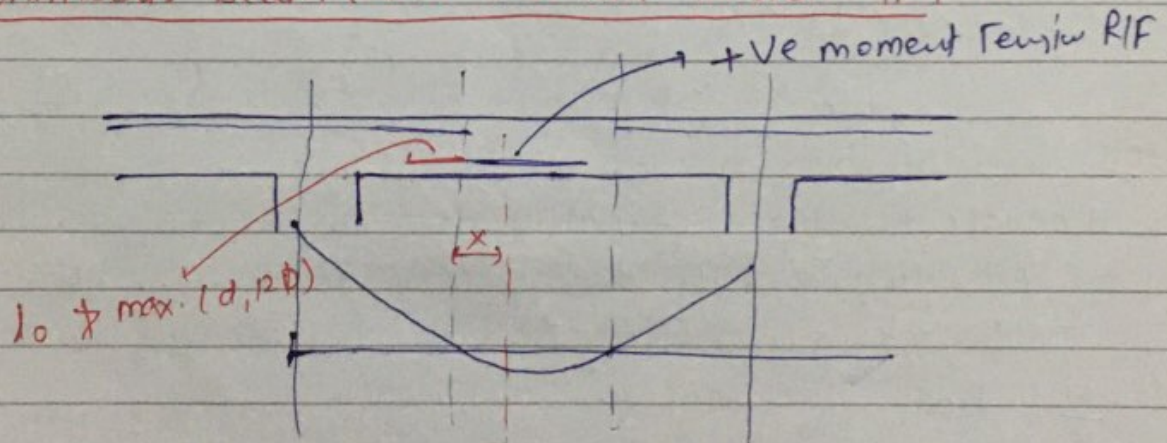
At simple support (+ve moment tension R/F):-

$$\begin{aligned} l_d &\leq \frac{m}{V} \\ 0.8 l_d &\leq \frac{m}{V} \\ l_d &\leq 1.25 \frac{m}{V} \end{aligned} \quad \left\{ \begin{array}{l} \text{due to compression confinement} \\ l_d \text{ value will } \downarrow \text{ because} \\ \text{shear strength } \uparrow. \end{array} \right.$$

$$l_d \leq 1.3 \frac{m}{V} + l_o$$

 for positive moment tension R/F at simple support.
=

In Continuous Beam (+ve moment tension R/F):-



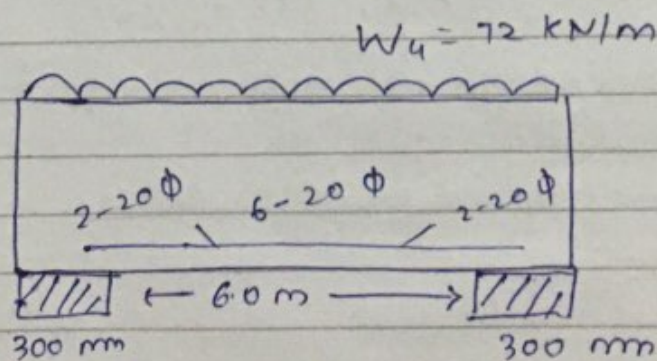
$$l_d \leq \frac{m}{V} + l_o$$

 $\rightarrow$ NO comp. confinement

+ve moment tension R/F at
point of Inflection (POI)

NOTE: Here V = factored shear force at point of O.B.M
 However in RCC design Question We can
 consider shear force at face of support
 (higher SF) than SF @ P.O.I, and check the
 criteria
 If it is satisfied with higher SF then it will be
 satisfied with the lower value

Ques: A simply supported beam having a
 clear span of 6m is supported on brick
 wall of 300 mm the beam carries a factored
 load of 72 kN/m including self wt. size of
 beam is 300 x 700 mm. The eff. depth is 660 mm.
 The beam is R/F with 6-20 ϕ @ center.
 If only 2 bars are continued into the support,
 check the development length criteria @ support
 assuming M20 & Fe 415 also detail the R/F
 @ support



$$D = 700 \text{ mm}, \quad d = 660 \text{ mm}$$

$$b = 300 \text{ mm}, \quad \text{M20, Fe 415}$$

At simple support

$$L_d \leq 1.3 \frac{m}{v} + l_0$$

$$L_d = 47 \phi = 940 \text{ mm.}$$

$$M = 0.87 f_y A_{st} \left(d - \frac{f_y A_{st}}{f_{ck} b} \right)$$

$$= 0.87 \times 415 \times 2 \times 314 \left(660 - \frac{415 \times 2 \times 314}{20 \times 300} \right)$$

$$= 139.8 \text{ kNm.}$$

$$x_{\text{eff}} l_{\text{eff}} = \min \left\{ \begin{array}{l} l_c + d \rightarrow 0.66 \\ l_c + a \rightarrow 0.3 \end{array} \right\} = 6.3 \text{ m}$$

$$V = \frac{w l_{\text{eff}}}{2} = 226.80 \text{ kN}$$

$$1.3 \frac{m}{v} = 801.32 \text{ mm.}$$

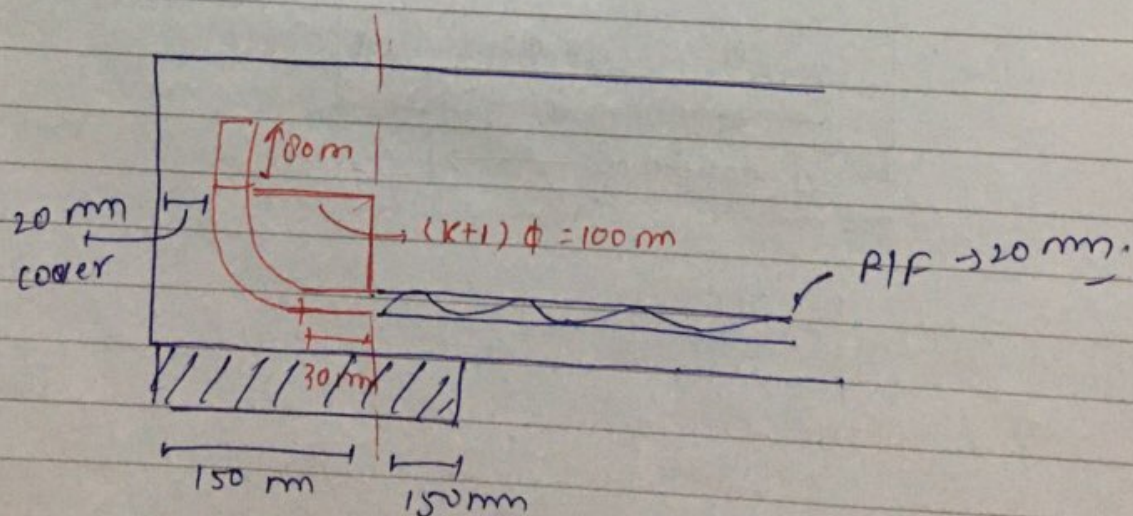
We require a $l_0 = 138.68 \text{ mm.}$

$$\left\{ \begin{array}{l} l_0 = l_d - 1.3 \frac{m}{v} \end{array} \right.$$

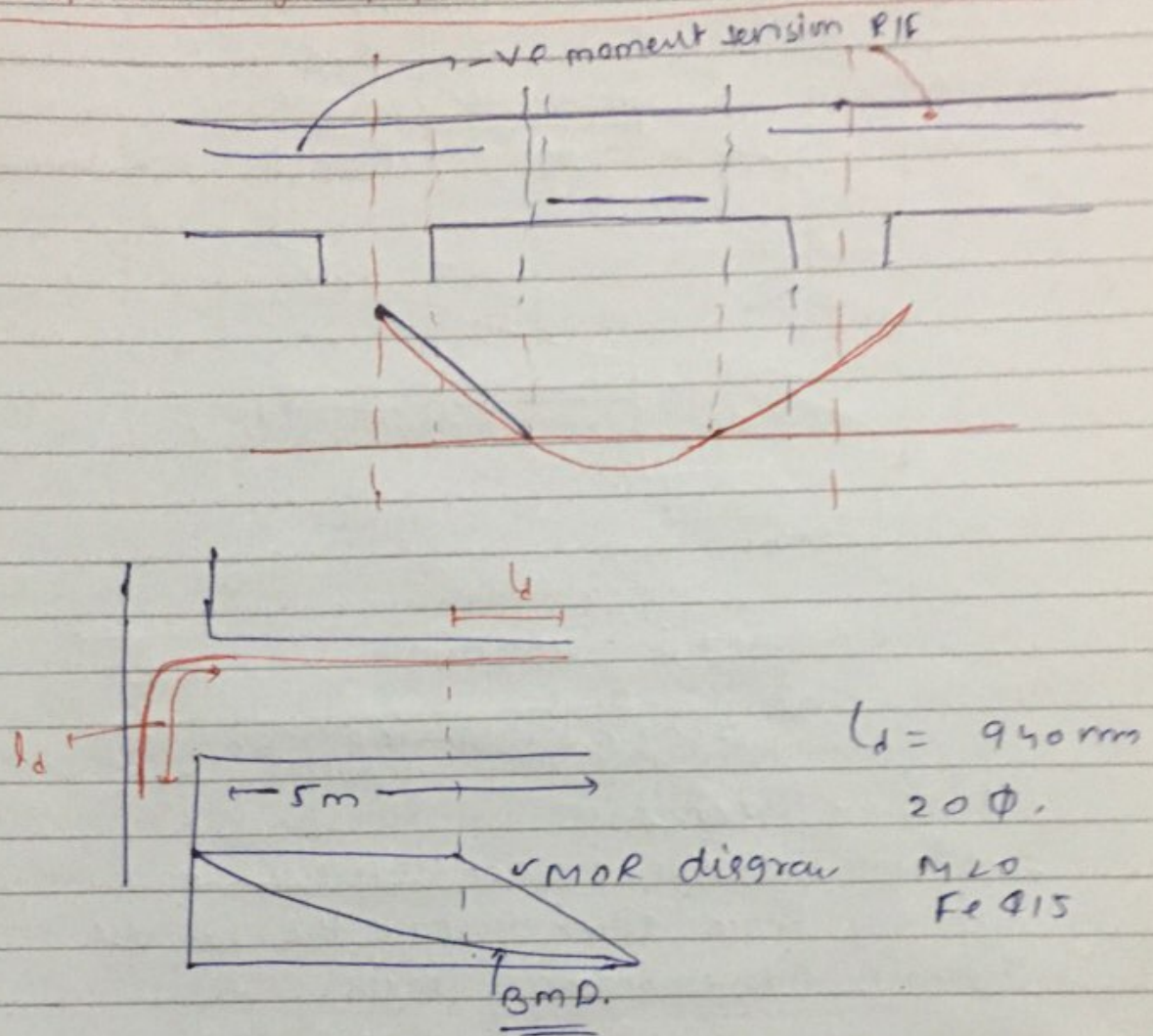
$$l_0 \geq \max. (d, 12\phi)$$

$$= \max. (660, 240)$$

$$l_0 \leq 660 \text{ mm.}$$

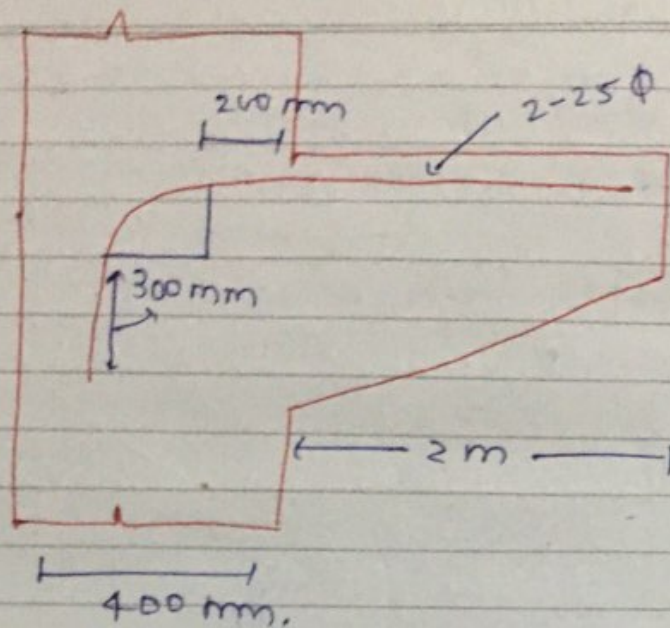


Development length requirement (-ve moment tension R/F)



In case of -ve moment tension R/F, we need to ensure l_d on either side of the Max. stress point (Max. Bm location)

Que check the adequacy of the anchorage provided for the longitudinal tension R/F in the beam shown below and suggest modification if required to ensure a stress of $0.87 f_y$ in the bar at Max stress point



$$L_d = 47 \phi = 1175 \text{ mm}$$

$$200 + 8\phi + 300 - 4\phi = 600 \text{ mm}$$

~~embed~~

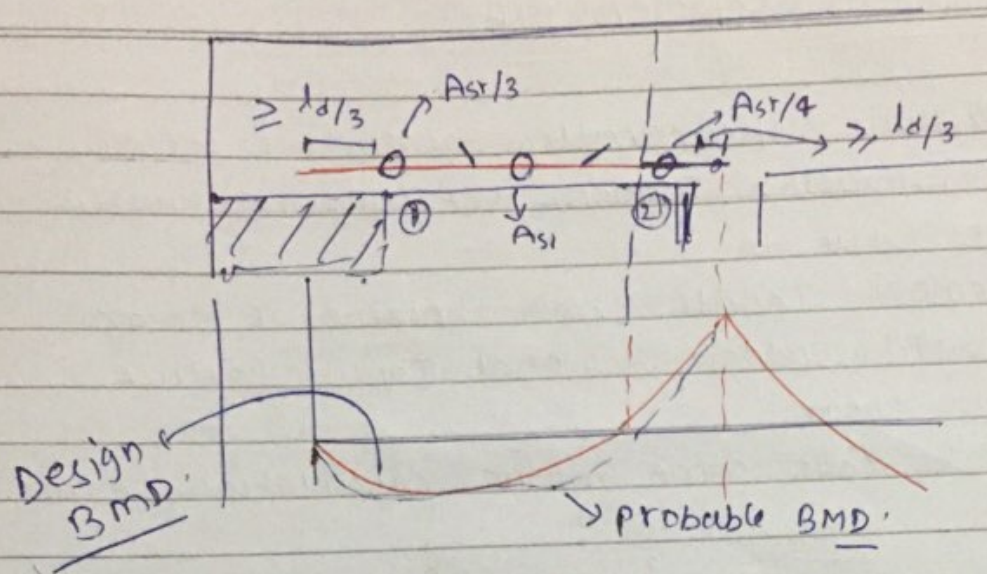
Embedment length is not sufficient on the support side and hence either decrease the bar dia or increase the embedment length

Additional requirement for main R/F :-

This is required to provide some additional safety due to change in BMD because of minor settlement of support, minor calculation error etc.

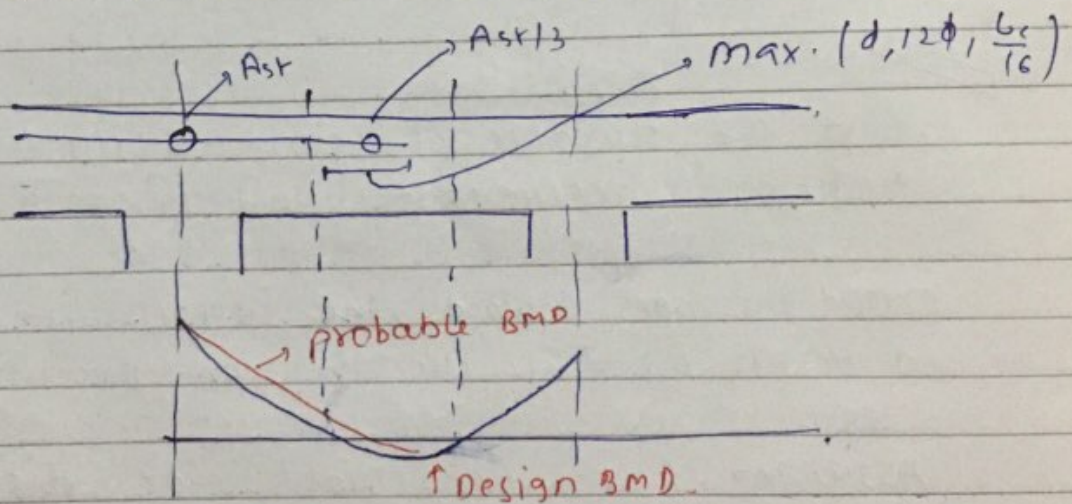
For +ve moment Tension R/F :-

at least $\frac{1}{3}$ rd of the Tensile R/F in simple member (simply supported) and $\frac{1}{4}$ th of Tensile R/F in continuous member shall extend along the same face of the member into the support to a length $L_d / 3$



For -ve moment Tension R/F:-

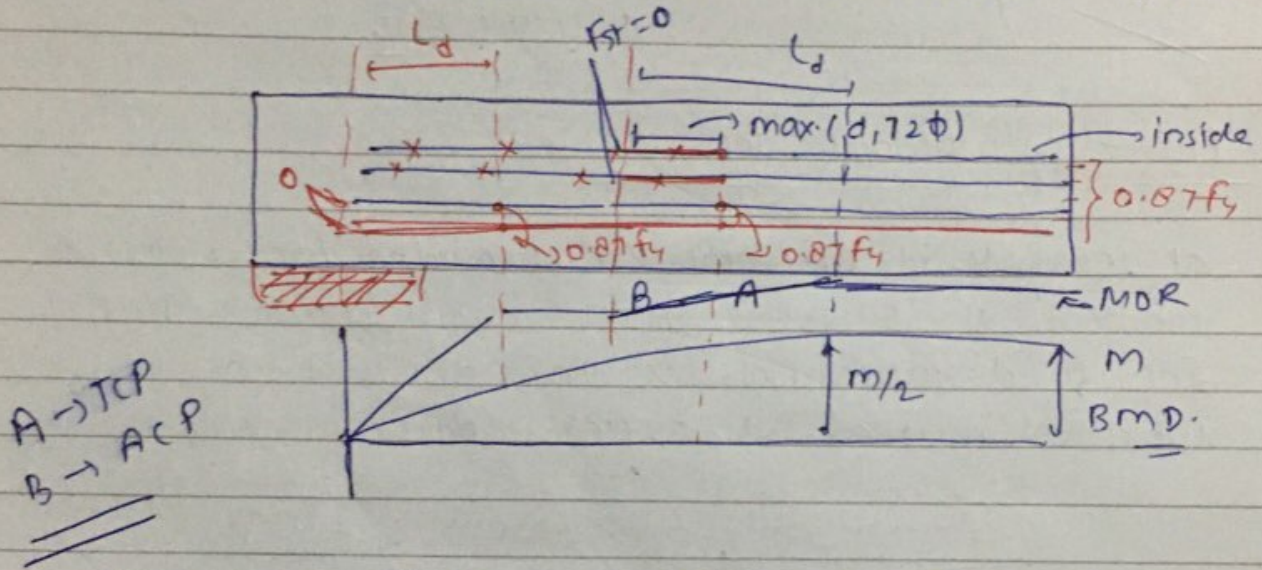
at least $\frac{1}{3}$ rd of Total R/F provided for negative moment at support shall extend beyond POI for a distance not less than d , 12ϕ or $L_c/16$, whichever is Max. L_c = clear span



Curtaiment of tension RIF

if $MOR > BMD$, theoretically some RIF can be curled, however Tension RIF are not curled at TCP due to

- increase in Tensile stress because of shear
- To avoid sudden diagonal tensile failure due to shear
- To take care some minor calculation error.



find the distance of point A (TIP) as per
unhement requirement, that is 50', 30' etc.

extend the bars towards the support upto a distance d or 12ϕ whichever is max. (point B)

Assuming B as ACP Draw MOR diagram

ensure that $MOR > BMD$ at every cis
or extend actual cut off point towards
the support until $MOR > BMD$.

Additional Restriction at Actual cut off point:-

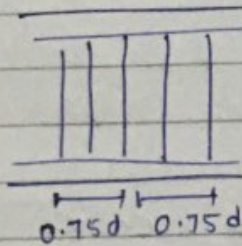
Flexural R/F shall not be terminated in a Tension zone unless any one of the following cond. is satisfied.

- ① The shear @ actual cut off point does not exceed $\frac{2}{3}$ rd that permitted, including the shear strength of web R/F

$$\tau_v \leq \frac{2}{3} (\tau_c + \tau_s) \quad , \quad \tau_c + \tau_s \geq 1.5 \tau_v$$

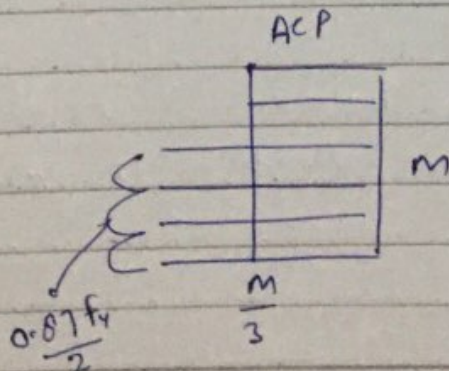
$\uparrow$ Nominal $\downarrow$ strength $\downarrow$ shear R/F

- ② Stirrups Area in excess of that required for shear and Torsion is provided over a distance = $0.75d$ from each cut off point



spacing ~~57.5~~ $\leq 0.75d$
 it any stress comes from Tensile R/F then shear R/F will take care of that!

- ③ for 36 mm and smaller bars the continuing bars provide double the Area required for flexure @ the cut off point and shear does not exceed $\frac{3}{4}$ th that permitted.



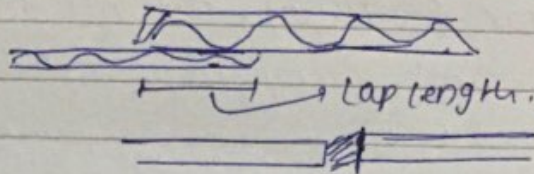
$$\tau_v \leq \frac{3}{4} (\tau_c + \tau_s)$$

Splicing :-

It is required when bars are placed short of their length or bar diameter has to be change

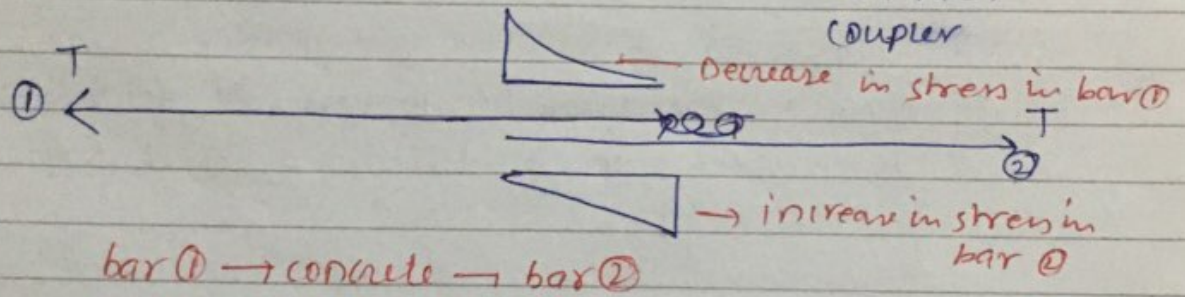
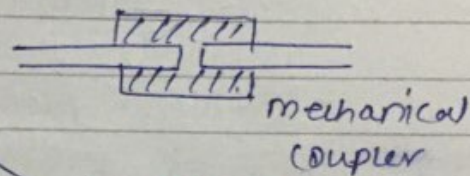
Types of splices :-

(i) Lap splices



(ii) Welded splices

(iii) Mechanical splices



Codal Recommendations :-

splices shall be away from the section of Max. stress and shall be stagger.

splices in flexural member should not be at sections where the BM is ~~80%~~ more than 50% of MOR

NOT more than 50% bars shall be spliced at a location q/w special measures need to be taken.

NO lap splice for $\phi > 32 \text{ mm}$

for pre.

Bar in	lap length
flexural tension	$\text{Max. } (L_d, 30 \phi)$
Direct tension	$\text{Max. } (2L_d, 30 \phi)$
compression	$\text{Max. } (L_d, 24 \phi)$

NOTE: for 2 bars of diff. dia lap length is calculated based on dia of smaller bar

Side face R/F :-

provided when depth of web in a beam exceeds 750 mm
To control shrinkage and Temp crack in case of
Normal beam having no Torsion

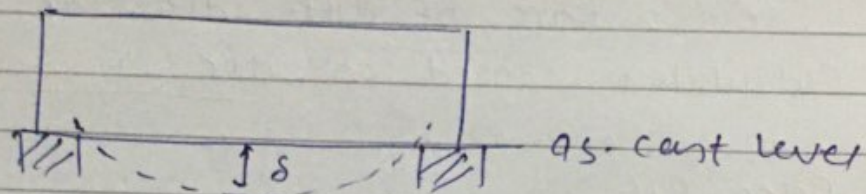
Area of side faces R/F ~~greater~~ ^{0.1% of} $\geq$ Web Area on both the faces.
and spacing $\leq \min. \{ b, 300 \}$ mm.

Check for Deflection (serviceability criteria)
(Both Beams & Slabs) :-

Deflection including the effect of Temp., creep and shrinkage after erection of partition and application of finishes that is after all DL shall not exceed $\frac{\text{span}}{350}$ or 20 mm whichever is less

This is to prevent Damage to partitions and finishes.

- (long term)
- Final deflection due to all loads including the effect of creep, shrinkage and Temp. measured from as-cast-level of support, floor or all other horizontal member shall not exceed $\text{span}/250$



However it is very difficult to calculate the exact deflection due to various factors and Generally deflection is controlled by limiting $\frac{\text{Span}}{\text{eff. depth}}$

As per IS code the deflection limit given above may be Generally assume to be satisfied if $(\frac{l}{d})$ ratio for span of ~~8.10 m~~ upto 10 m is $\geq$ the following specified values.

Beam type	l/d
Cantilever	7
Simply supported	20
Continuous	26

for span above 10 m, the above values shall be multiplied by $\frac{10}{\text{span (m)}}$, except for

cantilever in which exact deflection shall be calculated and check.

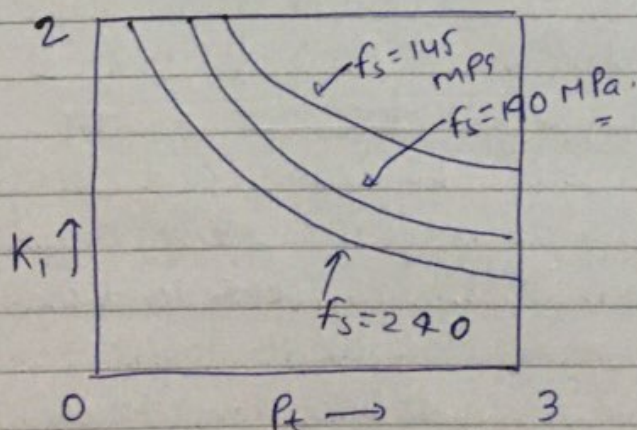
The above obtained value shall be modified for

- 1) Area and stress of Tensile R/F (K_1)
- 2) Area of compression steel (K_2)
- 3) Type of Beam that is rectangular or Flange (K_3)

$$\left(\frac{l}{d}\right)_{\text{Actual}} \leq \left(\frac{l}{d}\right)_{\text{basic}} K_1 K_2 K_3$$

$$\Rightarrow d_{\text{actual}} \geq \frac{l}{\left(\frac{l}{d}\right)_{\text{basic}} K_1 K_2 K_3} \quad l \rightarrow l_{\text{eff.}}$$

K_1 (P_t % and stress in it)



f_s = stress in steel at service load

$$f_s = 0.58 f_y \frac{A_{st, \text{req}}}{A_{st, \text{prov.}}}$$

$$\begin{cases} 1.5 W \rightarrow 0.87 f_y \\ W \rightarrow \frac{0.87}{1.5} f_y = 0.58 f_y \end{cases}$$

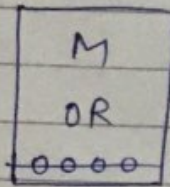
In the above graph, value of K_1 increases as P_t ↑ and F_s ↓. It means for low value of P_t and F_s depth required is less from deflection criteria.

Under given service load lower value of P_t & F_s implies two conditions

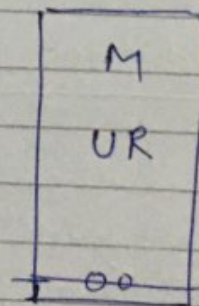
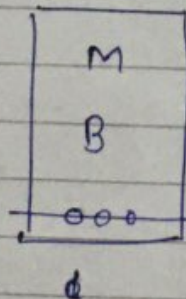
- 1) Either the c/s assume from Bending criteria is large
- 2) Or for a given c/s the design load is less.

In case 1) EI is high and in case 2 load is less for a given c/s both these cases will result in lesser stress across the c/s and hence lesser creep and lesser long term deflection

So we have a scope to decrease the depth (from deflection consideration) that was assumed from Bending criteria



EI is less
and stress is more
hence very less
scope to decrease
depth from deflection
criteria



EI is high and stress
is less and hence
high scope to decrease
the depth from
deflection criteria

Assume c/s $\rightarrow$ obtain f_s and P_t $\rightarrow$ Calculate K_1
from Bending

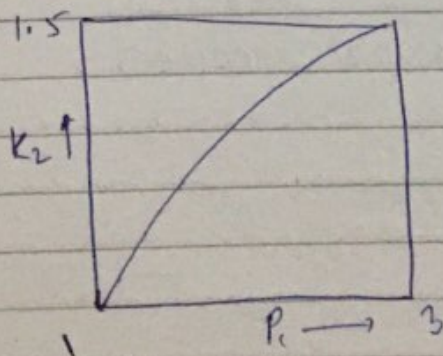
↓
check depth requirement
for deflection criteria
Check scope $\leftarrow$ to increase or decrease depth
Again calculate New P_t and repeat procedure.

• higher value of P_t and f_s means means lesser c/s or higher load so higher creep and higher long term deflection and hence we may require to increase the depth to account for deflection criteria

• If $A_{st, provided} > A_{st, req.}$ then $f_s \downarrow$ and hence less creep and lesser long term deflection but we shall not provide $A_{st} > A_{st, lim}$ o/w during excessive overloading the failure will be brittle.

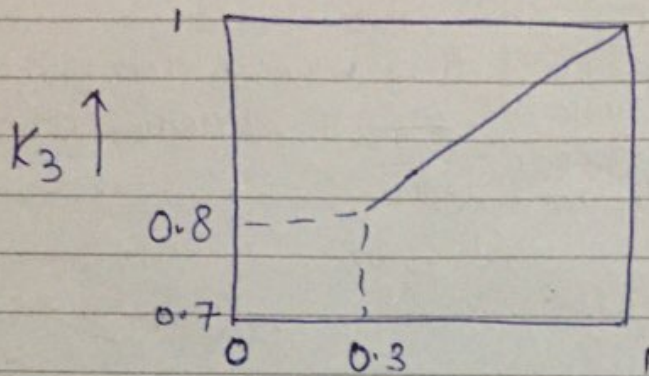
for K_1 in case of continuous beams and slabs the value of P_t and f_s should be calculated at mid span region. However in case of cantilever it should be calculated at support.

K_2 Value :-



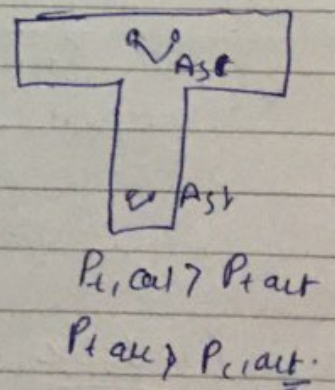
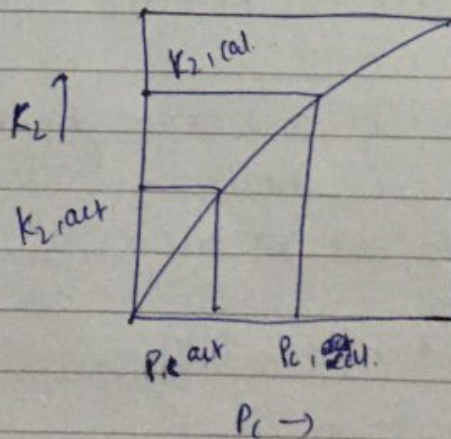
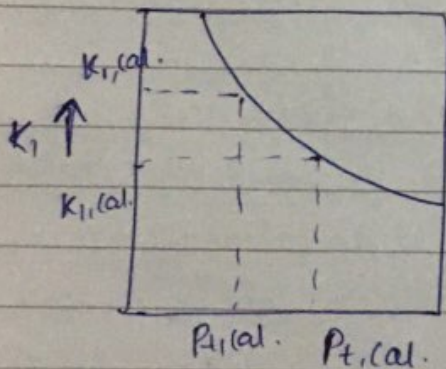
As compression steel reduces, long term effect of creep & shrinkage, $K_2 \uparrow$ as $P_c \uparrow$. higher P_c will dec↓ the depth requirement from deflection consideration.

K_3 (type of beams) :-



$\left(\frac{\text{web width}}{\text{flange width}} \right) \longrightarrow$

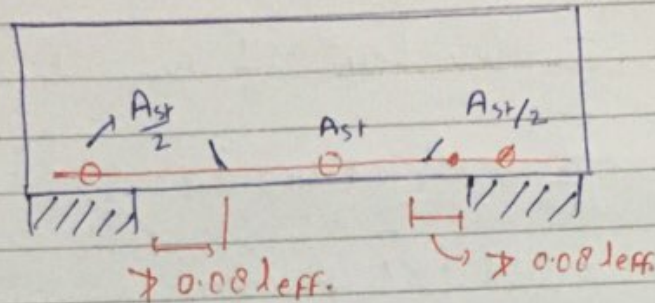
In Flanged beam P_t and P_c obtained by ~~b~~ $b = b_w$ and hence K_1 and K_2 will be higher than actual. To account for this K_3 was introduced. However K_3 gives inconsistent result and it is no more in use.



NOTE for a singly RIF Rectangular flexural member $K_2 = K_3 = 1$

Typical R/F details :-

provided in sp 34 and it is simplified development rule that will satisfy all the detailing criteria



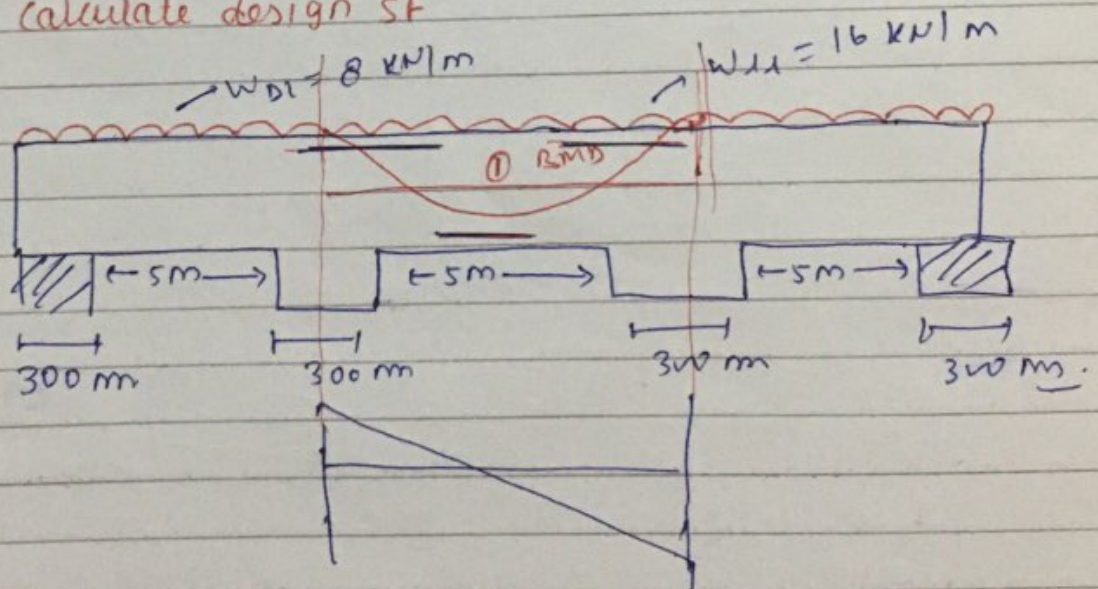
In case of partially restrain Member 35% of the R/F ($0.35 A_{st}$) shall also be provided for negative moment at the support and fully anchored.

Que. for the beam 1

- calculate design BM
- calculate design SF

M20
Fe 415

$b = 300$
 $D = 450$
 $d = 400$



$$i) \text{ B.M}_{\text{sagging}} = \frac{1}{16} W_{UDL} l_{\text{eff}}^2 + \frac{1}{12} W_{TL} l_{\text{eff}}^2$$

$$\frac{l_c}{12} = \frac{5000}{12} = 416.67 \text{ mm}$$

$$a < 300 \text{ mm} \Rightarrow a < \frac{l_c}{12} \text{ , so } l_{\text{eff}} = \min \left\{ \begin{array}{l} l_c + a \\ l_c + d \end{array} \right\} = 5.3 \text{ m}$$

$$= \frac{1}{16} \times 1.5 \times 8 \times (5.3)^2 + \frac{1}{12} \times 1.5 \times 16 \times (5.3)^2$$

$$= 77.24 \text{ KNm}$$

$$BM_{\text{hogging}} = -\frac{1}{10} W_{u,ol} l_{eff}^2 - \frac{1}{9} W_{u,cl} l_{eff}^2$$

$$= -\frac{1}{10} \times 1.5 \times 8 \times (5.3)^2 - \frac{1}{9} \times 1.5 \times 16 \times (5.3)^2$$

$$= -108.61 \text{ KNm}$$

$$\text{ii) } SF = 0.55 W_{u,ol} L_c + 0.60 W_{u,cl} L_c$$

$\downarrow \quad \downarrow \quad \downarrow \quad \searrow$
 $1.5 \times 8 \quad 5 \quad 1.5 \times 16 \quad 5$

$$SF = 105 \text{ KN}$$

⑩ ONE-WAY SLAB

slabs are plate elements and generally analysed as flexural member

Type of slab :-

① Based on shape

- Rectangular
- square
- circular etc.

② Based on load Transfer

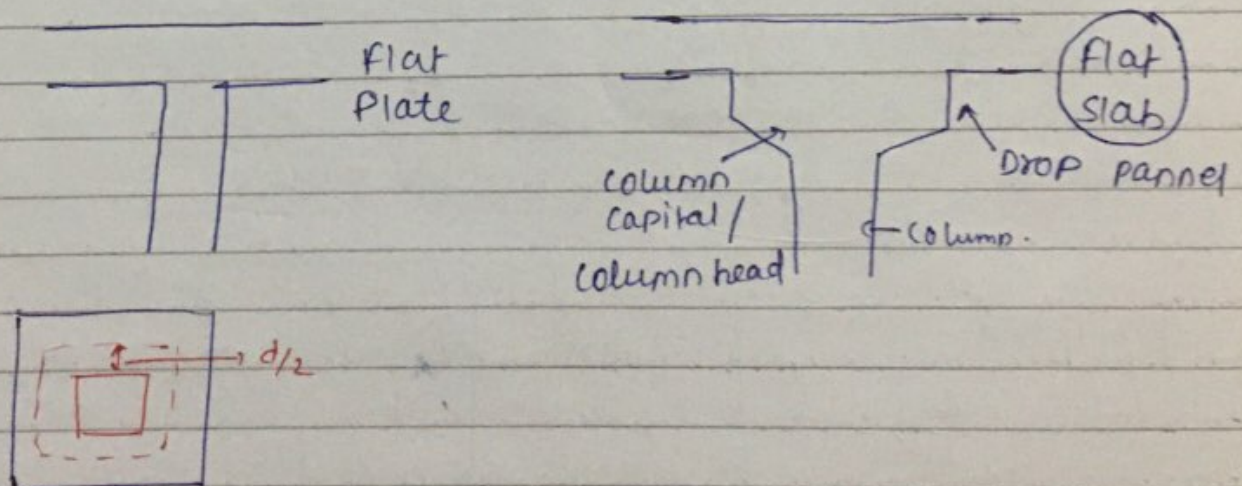
- solid slab (load to beam)
 - flat slab
 - flat plate
- } load to column

③ Based on load Type

- slab with UDL
- slab with point load (concentrated load)

④ Based on Bending dirⁿ :- / span dirⁿ :-

- one way slab / one way bending / one way spanning
- Two way slab / two way bending / two way spanning.



IN case of Flat slab we consider two way shear / punching shear & the critical secⁿ for shear shall be at a distance $d/2$ from the periphery of the column / capital / Drop panel.

$d \rightarrow$ eff. depth of slab.

One way slab and Two way slab :-

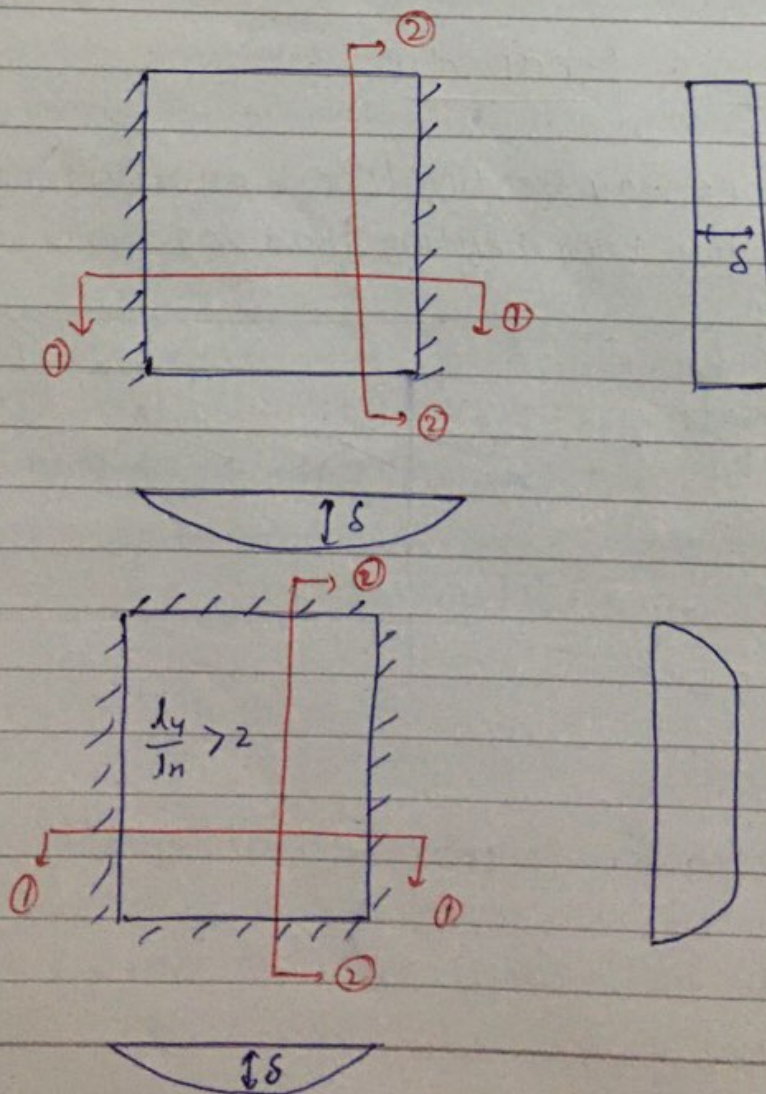
- a slab will be called one way if ① it is supported on two opposite edges only, ② it is supported on all 4 edges and $\frac{l_y}{l_x} > 2$ $\frac{l_y}{l_x}$ is called as aspect ratio

$l_y \rightarrow$ eff. span in the longer dirⁿ

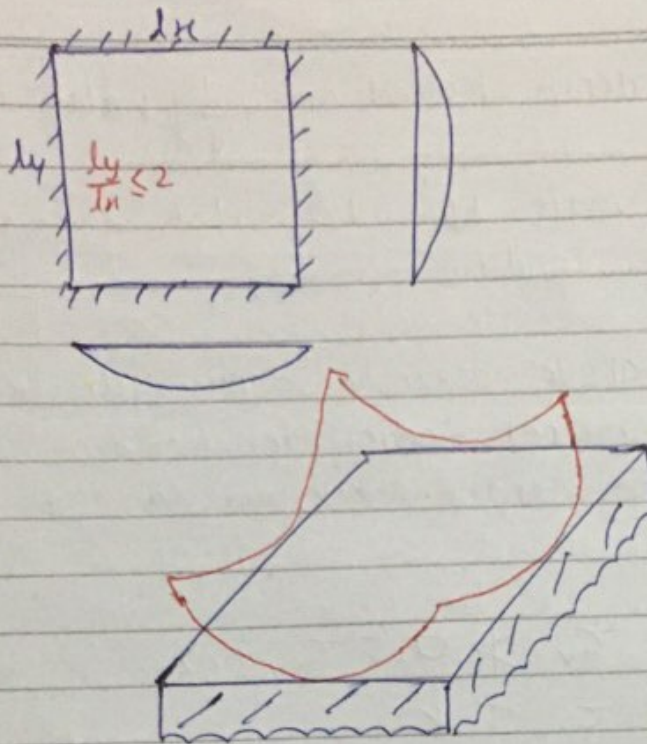
$l_x \rightarrow$ eff. span in the shorter dirⁿ.

A slab supported on all 4 edges with $l_y \neq l_x \leq 2$ is called as Two-way slab.

Deflected shape :-

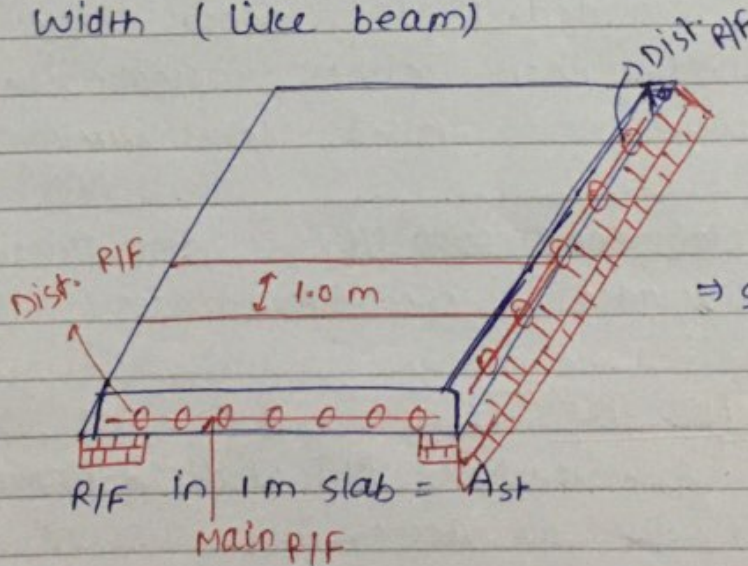


Two way



One way slab design: -

One way slab are generally designed considering 1 m width (like beam)



$$\Rightarrow \text{spacing} = \frac{1000}{\left(\frac{A_{st}}{\pi \phi^2} \right)}$$

$$= \frac{1000 \times \pi \phi^2}{A_{st}}$$

Deflection in slab Generally Governs the depth requirement

- calculate the depth based on $\left(\frac{l}{d}\right)$ ratio and K_1 value.

Generally K_1 varies b/w 1.2 - 1.6, however take 1.2 in exam condition.

- Based on the above assumed depth calculate A_{st} eff. span, critical section for bending and shear and development length check are same as in case of beam.

Shear design :-

$$\tau_v \nlessgtr 0.5 \tau_{c, \max}$$

$$\tau_v \leq K \tau_c$$

$$K = \begin{cases} 1.3 & D < 150 \\ 1.6 - 0.002D & 150 < D < 300 \\ 1 & D \geq 300 \text{ mm} \end{cases}$$

Shear stresses in slab are generally low and shear RIF are not provided. If the shear criterion does not satisfy then either increase the depth or Grade.

We go for depth because

In case of slab compression RIF are not provided it can be provided in exceptional cases.

Main & distribution RIF :-

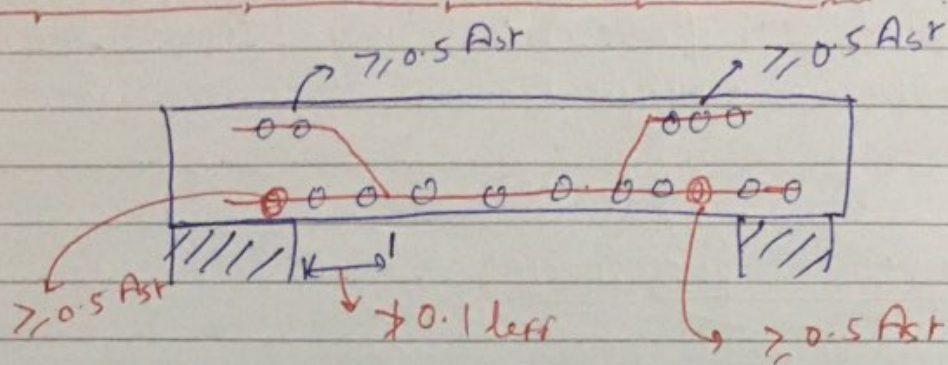
- Main RIF shall be provided along the bending dirⁿ that is across the support for two sides support slab and along the shorter span for 4 sides support slab.

- Distribution RIF / Secondary RIF / Temp. & Shrinkage RIF shall be provided in the transverse dirⁿ of Main RIF

RIF requirement and spacing :-

- Mild steel RIF in each dirⁿ of slab $\geq 0.15\% bD$
- for HYSD it shall be $\geq 0.12\% bD$
- Max. Dia of bar shall not exceed $\frac{1}{8} \times \text{total thickness}$
- the spacing of RIF is limited considering crack control
Main Bar ~~spacing~~ $\nless \text{min. } (3d, 300 \text{ mm})$
Distribution bar spacing $\nless \text{min. } (5d, 300 \text{ mm})$
- the deflection in two way slab $<$ one way slab
- Distribution bars are placed above main RIF to ensure higher eff. depth
- In two way slab both RIF are main RIF. However Higher load transfer will be along the shorter span. and hence keep the RIF along the shorter span closer to the Tensile Surface to ensure higher eff. depth.

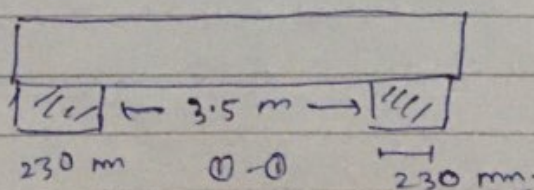
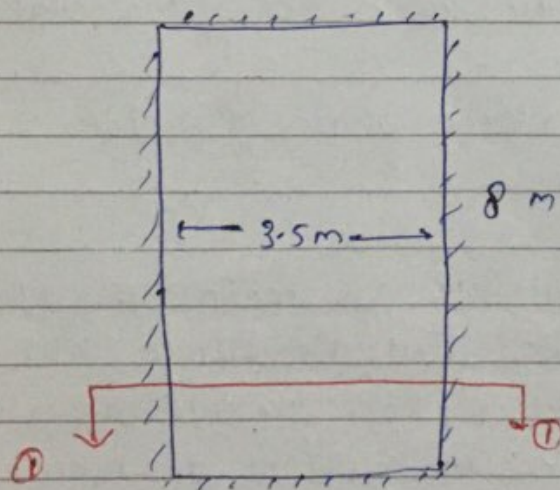
Typical R/F Detail (Simply supported slab)



Que. A slab has a clear span of $8\text{ m} \times 3.5\text{ m}$.

It is supported on all sides on a brick wall of 230 mm width. The LL on the slab is 5 kN/m^2 including surface finish. Consider M20, Fe415.

- i) Design BM
- ii) Depth required from deflection & Bending criteria
- iii) Main R/F for max. BM & spacing
- iv) Design shear force value.



As it is supported on brick wall the slab shall be treated as simply supported

$$\frac{l_y}{l_n} \rightarrow d_{eff.}$$

$$l_y = \min \begin{cases} L_c + d \xrightarrow{8} \\ L_c + a \rightarrow 230 \text{ mm} \end{cases}$$

$$l_n = \min \begin{cases} L_c + d \\ L_c + a \rightarrow 0.23 \end{cases} \xrightarrow{3.5}$$

$$\frac{l_y}{l_n} = \frac{\min (8.23, L_c + d) \xrightarrow{8}}{\min (3.73, L_c + d) \xrightarrow{3.5}} = \frac{8}{3.73} = 2.14 > 2$$

One way slab.

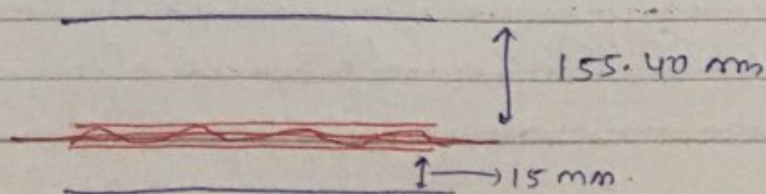
one way slab are governed by deflection criteria and hence depth shall be calculated considering

$\frac{l}{d}$ ratio and K_1

$$d_{act} \geq \frac{l}{\left(\frac{l}{d}\right)_{lim} K_1 K_2 K_3}$$

$$d_{act} \geq \frac{3.73 \times 10^3}{20 \times 1.2 \times 1 \times 1}$$

$$d_{act} > 155.40 \text{ mm}$$



$$D = 155.40 + \frac{\phi_m}{2} + 15 = 175.40 \text{ mm.}$$

$$D = 180 \text{ mm.}$$

$$d = 180 - 20 = 160 \text{ mm.}$$

$$D.L = 25 \times 0.18 \times 1 = 4.5 \text{ kN/m.}$$

$$L.L = 5 \text{ kN/m}^2 \times 1 = 5 \text{ kN/m.}$$

$$W_u = 1.5(D.L + L.L) = 14.25 \text{ kN/m.}$$

$$i) \text{ BM} = \frac{W_u l_{\text{eff}}^2}{8} = \frac{14.25 \times 3.66^2}{8} = 23.86 \frac{\text{kNm}}{\text{m.}}$$

$$l_{\text{eff}} = \min \left\{ \begin{array}{l} L_c + d \\ L_c + a \end{array} \right\} = 3.66 \text{ m.}$$

$$ii) d_{\text{requ.}} = 160 \text{ mm (from deflection)}$$

$$d_{\text{req.}} = \sqrt{\frac{M_u}{R_{\text{w}} b}} = \sqrt{\frac{23.86 \times 10^6}{0.138 \times 20 \times 1000}} = 92.6 \text{ mm}$$

from bending.

$$iii) A_{\text{st}} = \frac{f_{\text{cu}}}{2f_y} \left[\sqrt{1 - \frac{4.6 M}{f_{\text{cu}} b d^2}} \right] b d$$

$$= 438.13 \text{ mm}^2$$

$$A_{\text{st, min}} = \frac{0.12}{100} \times b D = 216 \text{ mm}^2 < A_{\text{st, req.}}$$

$$\text{Spacing, } \left(\frac{1000}{\left(\frac{A_{\text{st}}}{\pi/4 \phi^2} \right)} \right) = \left(\frac{1000}{\frac{438.13}{78.5}} \right) = 179.17 \text{ mm}$$

Max spacing limit $\rightarrow$ for crack
 Min $\rightarrow$ Rupture (workability)

$$\text{spacing} \nlessgtr \min(3d, 300 \text{ mm})$$

$\downarrow$
160

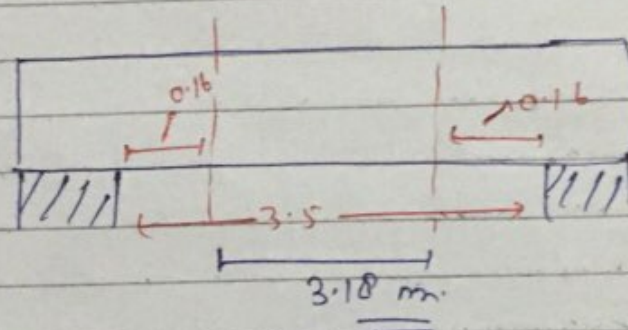
provide 10 mm dia @ 175 mm c/c

spacing for distribution R/F assuming dia = 8 mm dia

$$\text{spacing} = \frac{1000}{\frac{216}{50}} = 231.48 \nlessgtr \min(5d, 300)$$

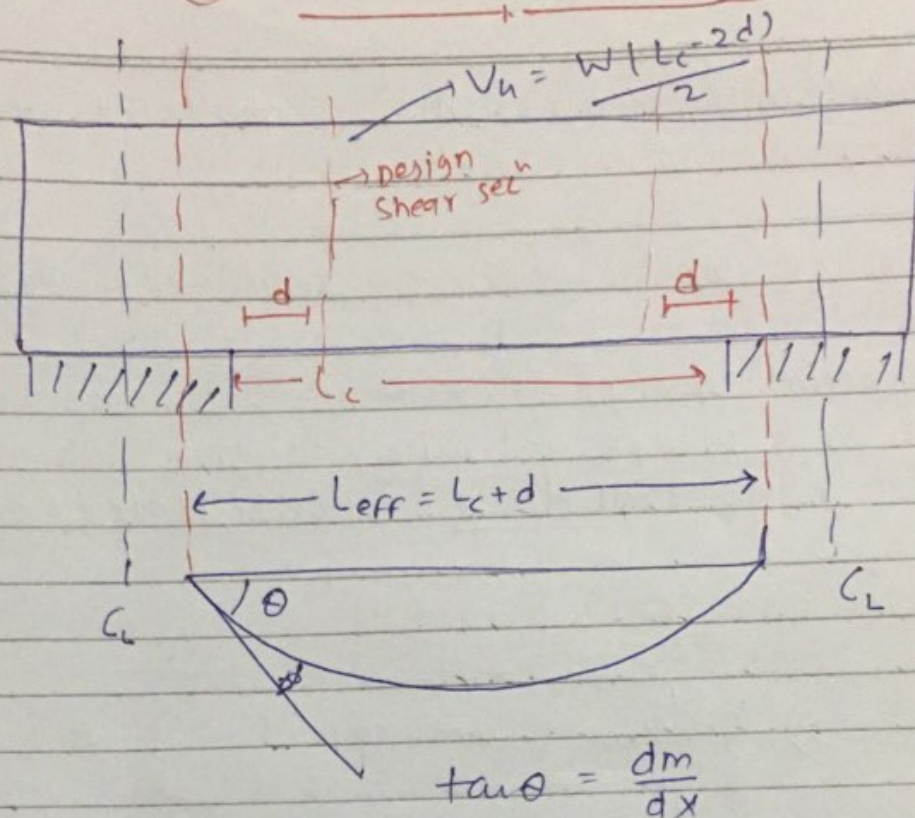
$\approx$

(iv)



$$V_u = \frac{W_u \cdot l_{eff}}{2} = \frac{14.25 \times 3.18}{2} = 22.65 \text{ KN}$$

(II) TWO-WAY SLAB



$$L_d \leq 1.3 \frac{M}{V} + l_o$$

S.F @ point of zero BM

$$V = \frac{W L_{eff}}{2}$$

Generally 4 sides supported and $\frac{L_y}{L_x} \leq 2 \rightarrow$ two way slab.

- Twisting moment develops in a two way slab and it is significant along the slab diagonal. due to this outer strips bend as well as Twist.
- Due to Twisting the corners of a two way slab lift up IF it is not allowed to lift up, the slab is called restrain two way slab and if it can freely lift up then it is called unrestrained two way slab / freely supported slab.

- thickness of two way slab is calculated considering deflection criteria
- same (l/d) can be taken as considered in Beam
- In special case of two way slab, for shorter span ~~of~~ upto 3.5 m with loading class upto 3.5 kN/m^2 , the total depth is calculated as follows.

For Fe250

$$D \geq \begin{cases} l/40 & \rightarrow \text{continuous} \\ l/35 & \rightarrow \text{simply supported} \end{cases}$$

Fe 415

$$D \geq \begin{cases} l/32 & \rightarrow \text{continuous} \\ l_n/28 & \rightarrow \text{simply supported.} \end{cases}$$

(140×0.8)
 (35×0.8)

the shorter span is considered for deflection calculation

Type of R/F :-

Based on amount of R/F provided along l_x & l_y .

- (i) Isotropically R/F :- same R/F in both dir^t that is $MOR_x = MOR_y$
- ii) orthotropically R/F :- Diff. R/F along l_x & l_y
 $MOR_x \neq MOR_y$

Method of Analysis :-

Highly statically indeterminate structure both elastic & inelastic method are available for structural analysis.

Elastic Method is used for unrestrained simply supported slab and inelastic method is used for restrained two way slab.

Elastic Analysis :-

Applicable for simply supported slab in which corners are not restrained and do not have adequate provision to resist torsion or to prevent the corners from lifting up.

The Max. BM per unit width $M_x = \alpha_x W l_x^2$
 $M_y = \alpha_y W l_y^2$ } Both l_x

$M_x = \text{BM along } l_x \text{ (RIF along } l_y)$

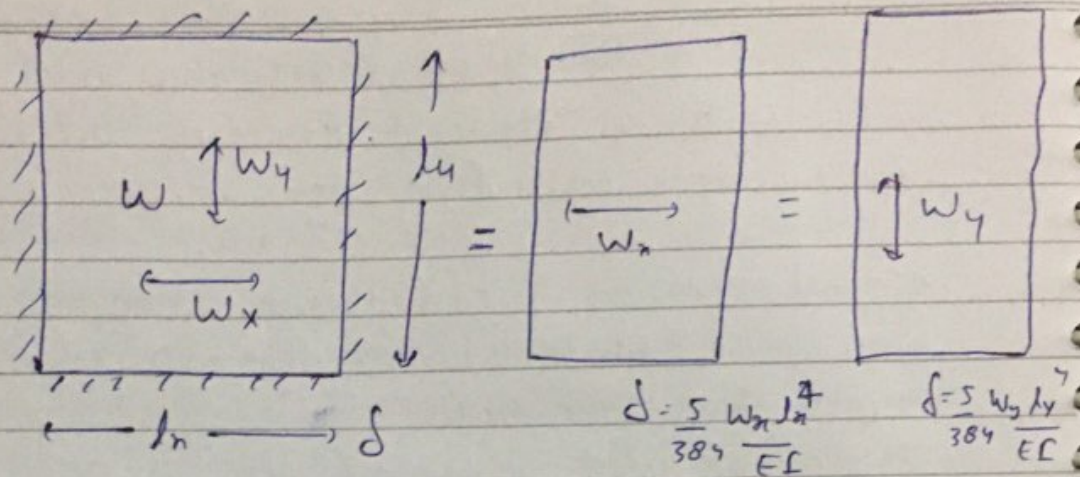
$M_y = \text{BM along } l_y \text{ (RIF along } l_x)$

α_x, α_y are coefficients

The elastic analysis is based on Rankine-Grashoff's theory.

In this theory assumptions are -

- 1) Torsion has been neglected and influence of torsion on either side of slab element has been neglected.
- 2) The deflection from both the sides are equated that is compatibility condition is used.



$$W = W_x + W_y$$

$$\frac{5 W_x l_n^4}{384 EI} = \frac{5 W_y l_y^4}{384 EI}$$

$$\frac{W_x}{W_y} = \left(\frac{l_y}{l_n} \right)^4 = r^4$$

$$W = r^4 \cdot W_y + W_y$$

$$W_y = \frac{W}{1+r^4} ; W_x = \frac{W r^4}{1+r^4}$$

$$M_x = \frac{W_x l_n^2}{8} , M_y = \frac{W_y l_y^2}{8}$$

$$M_x = \frac{W r^4 l_n^2}{(1+r^4) 8} = \frac{r^4}{8(1+r^4)} W l_n^2 \rightarrow \alpha_x$$

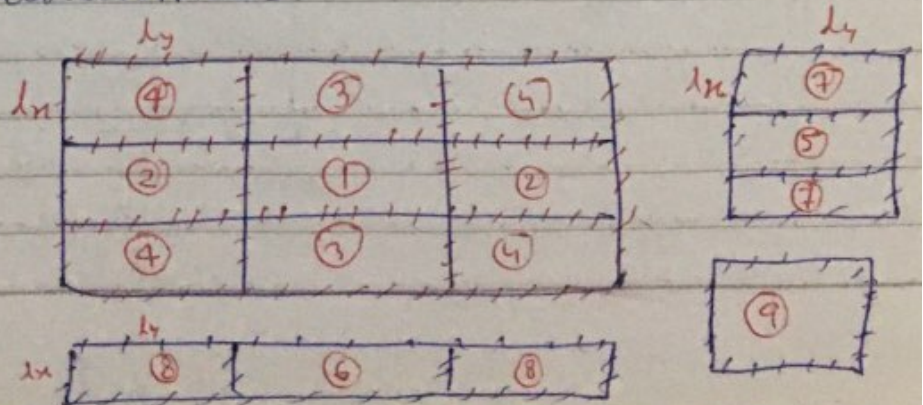
$$= \alpha_x W l_n^2$$

$$M_y = \frac{W_y l_y^2}{8} = \frac{W}{(1+r^4)} \cdot \frac{l_y^2}{8} = \frac{W l_n^2 l_y^2}{l_n^2 8(1+r^4)}$$

$$= \frac{r^2}{8(1+r^4)} W \cdot l_n^2 = \alpha_y W l_n^2$$

Inelastic Method:-

- valid for uniformly loaded restrain rectangular slab
- Analysed using yield line concept (inelastic method)
- The corners are held down and Torsion RIF provided to reduce crack
- The edges may be continuous, fixed or discontinuous (simply supported) However corners not allowed to uplift) with 9 diff support conditions.
- When using the coefficients of this Method for BM calculation, moment redistribution is not allowed
- The 9 different support cond. are -
 - 1) Interior panel
 - 2) one short edge discontinuous
 - 3) one long edge discontinuous
 - 4) Two adjacent edges discontinuous
 - 5) Two short edges discontinuous
 - 6) Two long edges discontinuous
 - 7) Three edges discontinuous / one long edge continuous
 - 8) Three edges discontinuous / one short edge continuous
 - 9) All edges discontinuous

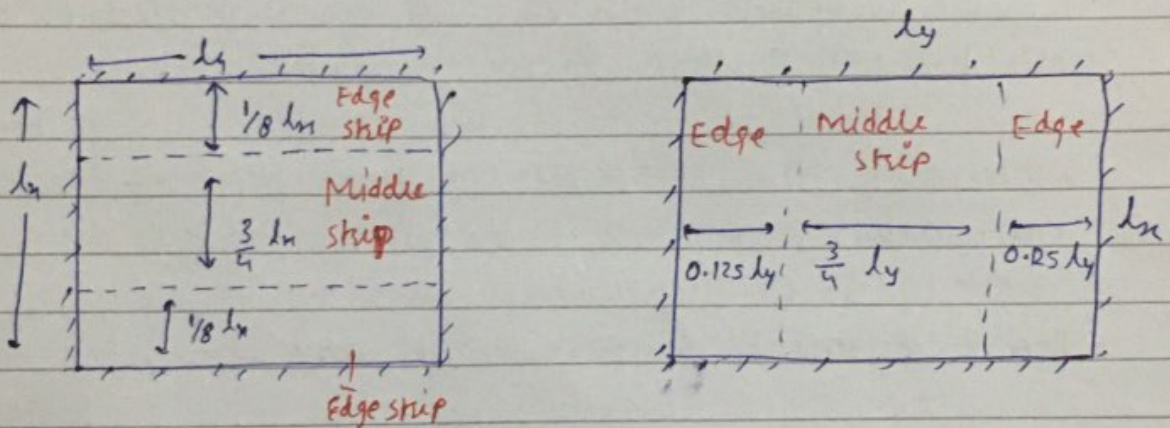


for each case moment coefficient are provided at mid span and at edges

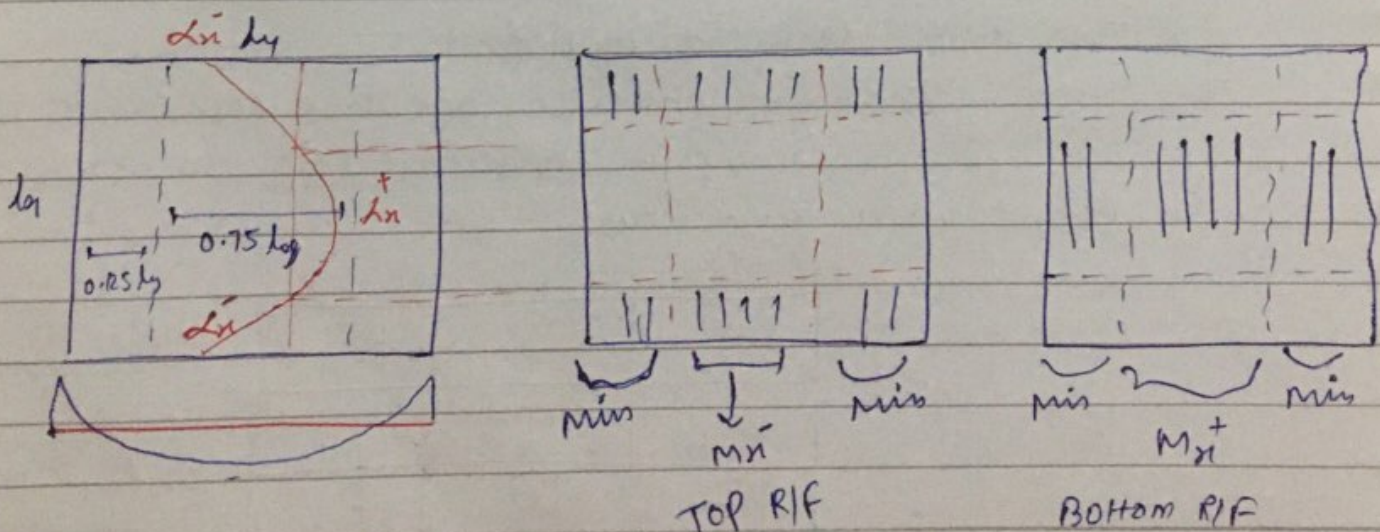
$$M_x = B_x w l_x^2$$

$$M_y = B_y w l_y^2$$

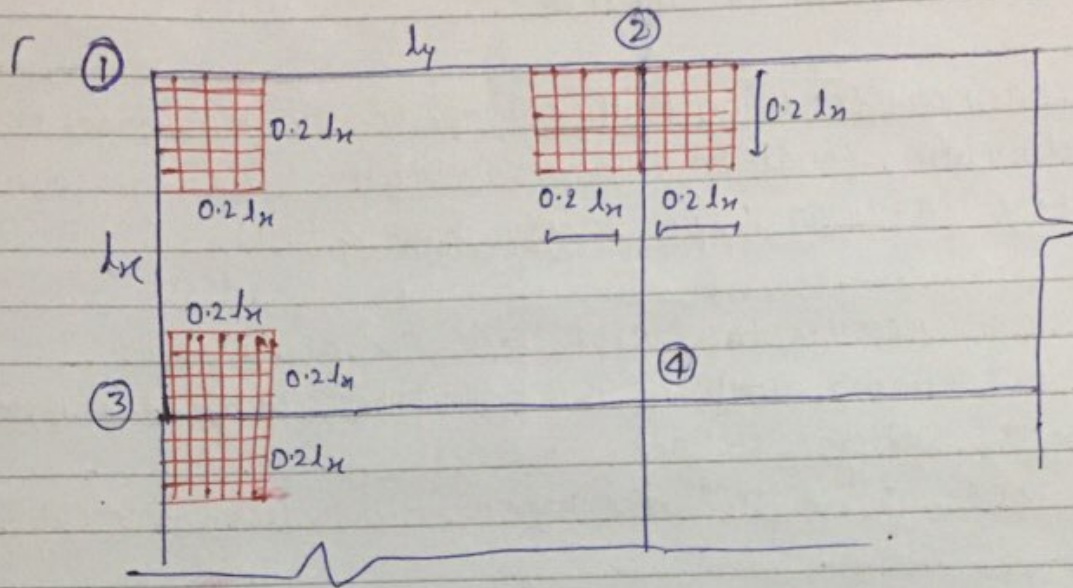
The above moment calculated shall be used for Middle strip and R/F shall be distributed across the Middle strip.



R/F in edge strip, parallel to that edge shall be provided as per minimum R/F criteria that is
 $0.12\% \rightarrow$ HYSD, $0.15\% \rightarrow$ MS



Torsion RIF / Corner RIF :- (for restrain slab)



- ① $\rightarrow$ 4 layers RIF, each layer = $0.75 A_{sr, x}^+$
- ② & ③ $\rightarrow$ 4 layers RIF, each layer = $0.375 A_{sr, x}^+$
- ④ $\rightarrow$ No torsion RIF

$A_{sr, x}^+ = \text{Max}^m +ve \text{ moment RIF along } l_n$

Yield Line Analysis (crack line) :-

- valid for Two way (Restrained slab)
- The coefficient given in IS code for restrained slab that is β_x & β_y are obtained by using inelastic analysis that is yield line analysis
- yield line Analysis is carried out to economise the structure
- The attainment of the ultimate flexural strength at one c/s doesn't necessarily leads to collapse of the slab

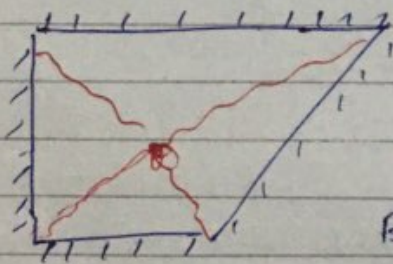
The following conditions need to be satisfied during the impending state of collapse

- (i) Mechanism Condition (sufficient nos of plastic hinges)
- (ii) equilibrium condition (static equilibrium valid in all cases)
- (iii) Safety condition (for safety $MOR \geq BM$)

It is not possible to satisfy all the above three conditions during analysis. We pick two cond. at a time and try to satisfy 3rd one.

When cond. Ist & IInd are taken, the Method is Yield Line Method

It gives higher collapse load during the analysis than the actual collapse load. Hence it is called Upper Bound theorem. This Method is simpler! However chances of failure are there due to higher calculated collapse load during the analysis.



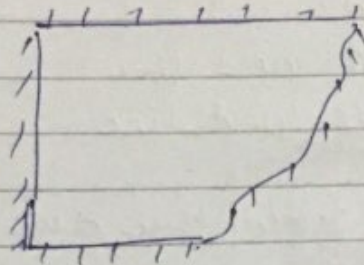
Actual Failure $\leq 80 \text{ kNm}$

B.M = (80 kNm, 100, 130, ----)

Yield line is an upper bound theorem valid for UR slab as yielding will occur only in UR.

In order to minimise the error in upper bound theorem we calculate all possible collapse load considering different yield line patterns and report the minimum one.

When we combine cond. 2 & 3 the theorem is called lower bound theorem. The result obtained in this case from analysis will be lower than the actual collapse load



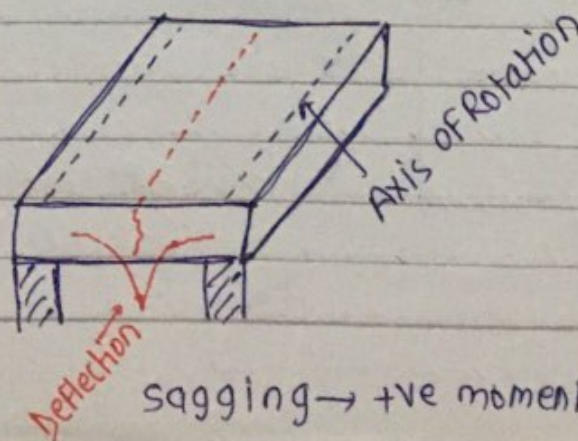
Actual failure $> 130 \text{ kNm}$

B.M = (80, 100, 130 kNm --)

Yield line assumption

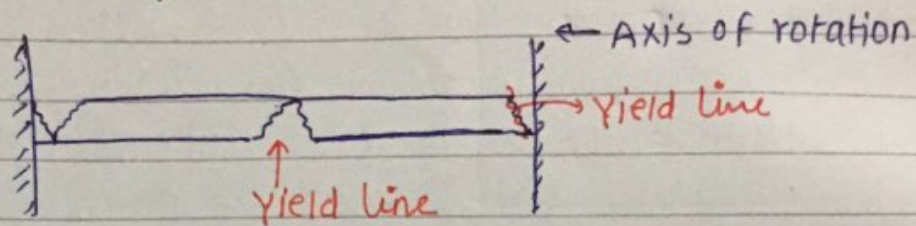
- The R/F steel is fully yielded along the yield line at failure
- The slab separates into segment by the yield lines :
- The individual segment behaves almost elastically after formation of yield line
- Elastic deformation are negligible compared to plastic deformation i.e; deflection before failure and can be ignored compared to deflection @ failure
- The B.M and twisting moment are uniformly distributed along the yield line and have the Max^m Value provided by the ultimate moment capacity

Location of yield line { Guidelines }



Thick slab

Fixed support
(Axis of rotation = yield line)



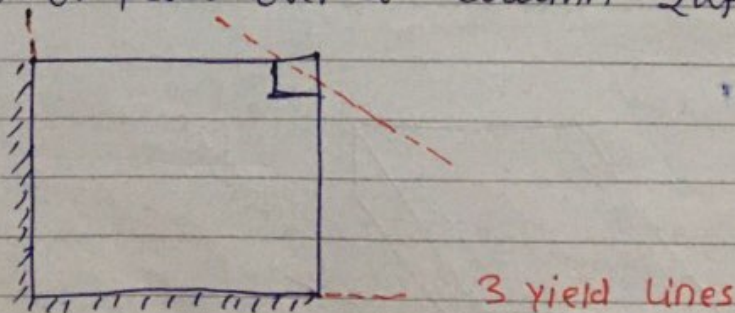
Sagging moment $\rightarrow$ +ve yield line

Hogging moment $\rightarrow$ -ve yield line

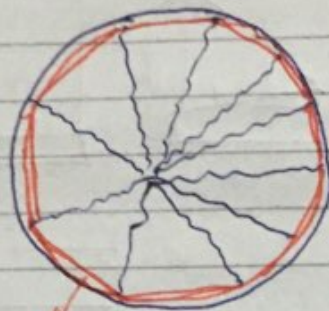
- In simple supported slab, yield line due to sagging BM @ mid span (bottom surface)
- In fixed supported, cantilever slab, -ve yield line @ support and +ve yield line at mid span in continuous slab
- In two way slab, the individual slab segment separated at yield line undergoes rigid body rotation under compatible deformation condition.
- Axis of rotation is not always the yield line

Guideline to draw yield line pattern:-

- All Guidelines may ~~be~~ not be valid for every yield line
- The Axis of rotation lies along the lines of support or pass over a column support

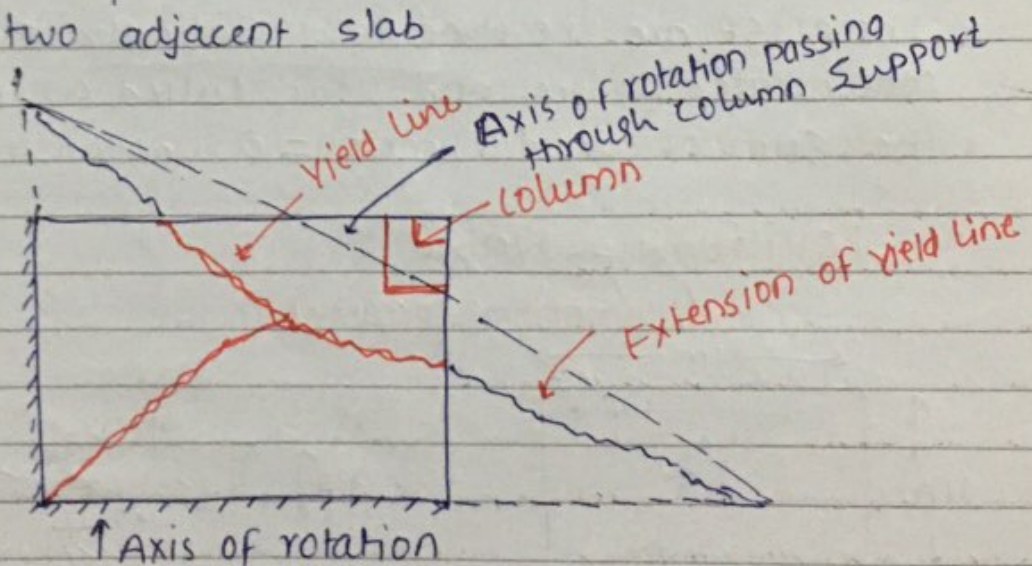


- yield line must end @ slab boundary or at another yield line
- yield line propagates outward from a point load
- The yield lines are generally straight lines.
- Continuous Support repel and simple Support attract +ve or sagging yield lines.



Almost straight

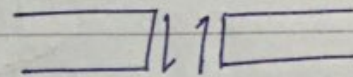
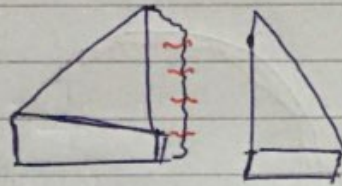
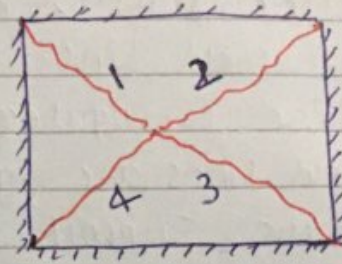
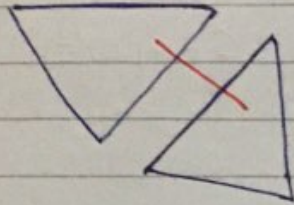
- A yield line or its extension b/w two slab segments passes through the intersection of axis of rotation of the two adjacent slab



Methods of Analysis

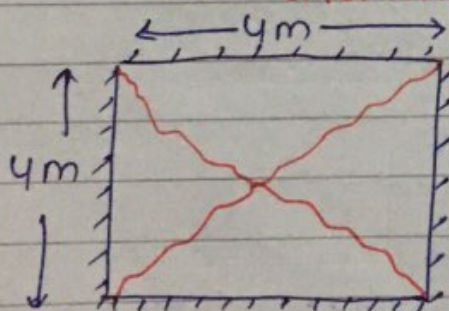
- 1) Virtual Work Method
- 2) Equilibrium Method

Equilibrium Method (Nodal forces)



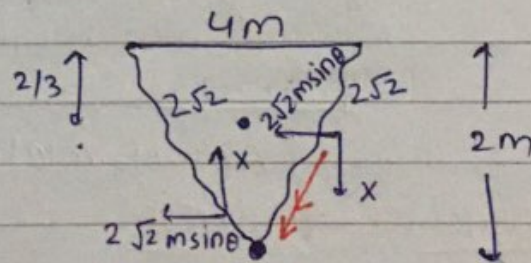
- Slab 1, 2, 3 and 4 are independent in $E\alpha^m$.
- Shear forces exist along yield lines (statically indeterminate)
- These SF are resolved into statically equivalent concentrated force and are called as nodal forces
- The sum of nodal forces = 0, along the yield lines.

Equilibrium Method (Moment capacity)



$$LL = 6 \text{ kN/m}^2$$

$$D = 120 \text{ mm}$$



$$\begin{aligned} \Sigma H &= 2 \times 2\sqrt{2} \sin \theta m \\ &= 4\sqrt{2} m \sin \theta \\ &= 4m \end{aligned}$$

$$DL = (0.5 \times 4 \times 2) \times 0.12 \times 25 = 12 \text{ kN}$$

$$LL = (10.5 \times 4 \times 2 \times 6) = 24 \text{ kN}$$

$$\text{Total Factored Load} = 1.5 (24 + 12) = 54 \text{ kN}$$

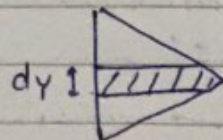
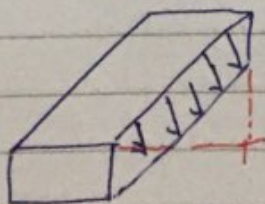
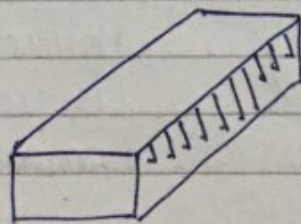
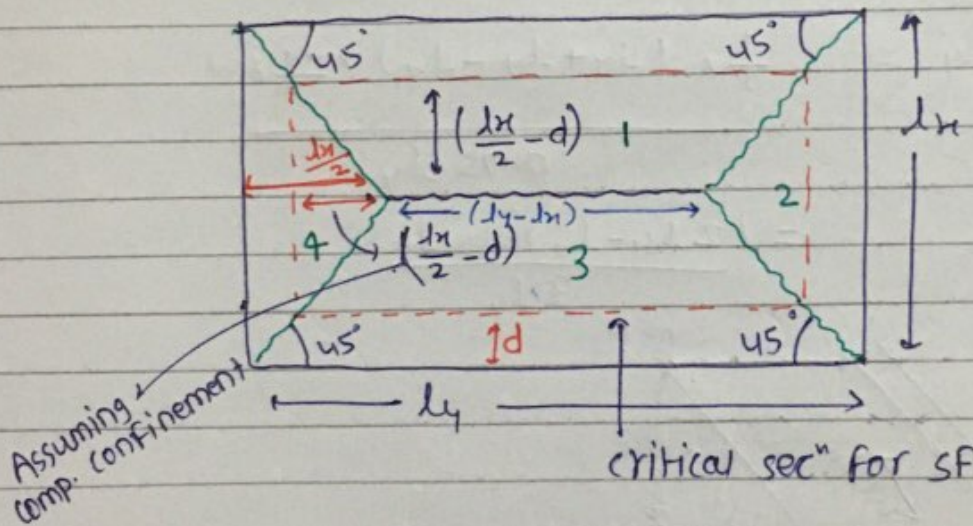
$$\text{moment about axis of rotation} = 54 \times \frac{2}{3} = 36 \text{ kN-m}$$

$$MOR = M \text{ per meter length} = 2\sqrt{2} m \sin \theta \times 2 = 4m$$

$$4m = 36$$

$$M = 9 \text{ kN-m}$$

Max^m SF in a slab



$$A = \left(\frac{l_x}{2} - d \right) dy$$

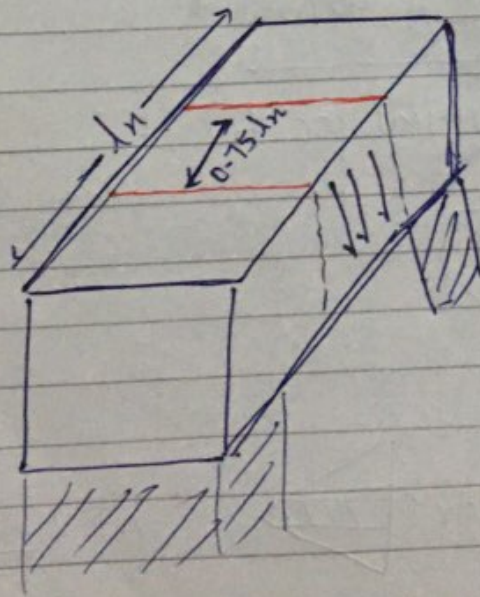
$$\tau_{y, \max} = \frac{\left(\frac{l_x}{2} - d \right) dy \cdot w_u}{dy \cdot d}$$

- The above yield line pattern is Valid For Shear stress calculation
- The Max^m Shear stress in a Slab will be Same along l_n and l_y .

Max^m SF per unit length along the edge.
(lie in Beam)

$$\begin{aligned} \text{Along } l_n &= \frac{\frac{1}{2} \times l_n \times \frac{l_n}{2} \times w}{0.75 l_n} \\ &= \frac{1}{3} w l_n \text{ (KN/m)} \end{aligned}$$

$$\begin{aligned} \text{Along } l_y &= \frac{\frac{1}{2} \times (l_y + l_y - l_n) \times \frac{l_n}{2} \times w}{0.75 l_y} \\ &= \frac{(2 l_y - l_n) \times w l_n}{3 l_y} \end{aligned}$$



Thickness of slab
is very less as
compared to Stiffness.

COLUMN

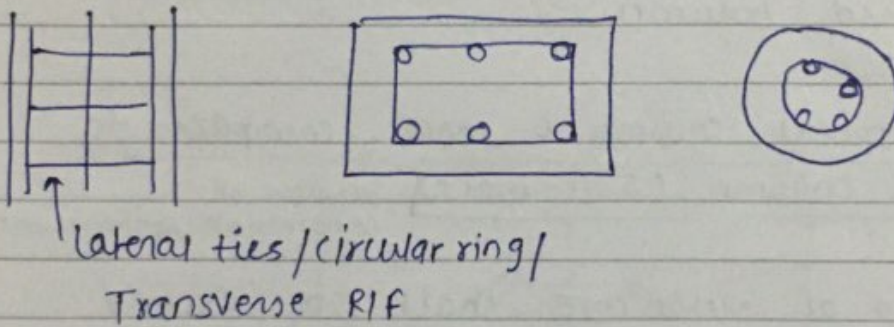
It is a compression member predominantly subjected to axial force. The eff. length of column exceeds the 3 times the least lateral dimension

If eff. length is less than 3 times the least lateral dimension the member is called pedestal.

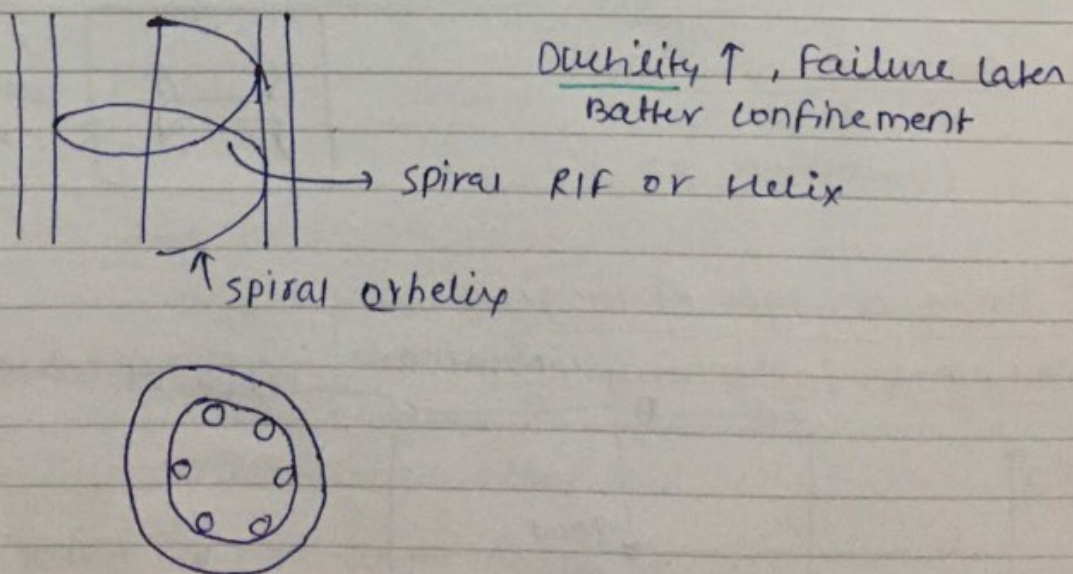
Classification of columns :-

1) Based on type of RIF :-

a) Tied columns

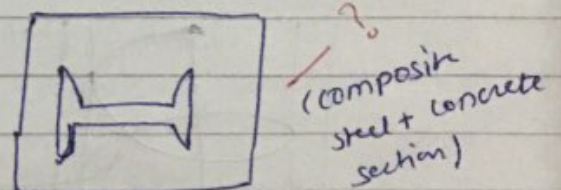


b) spiral / helical columns



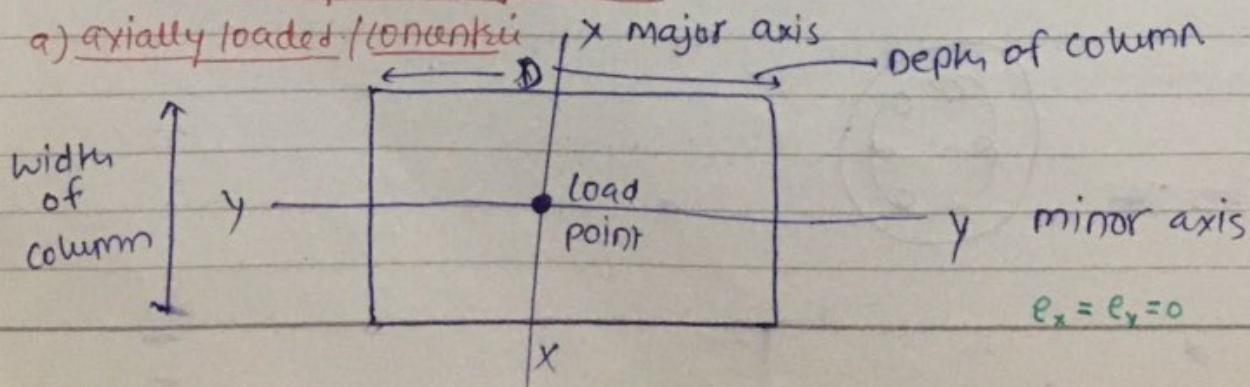
- In tied column, main RIF is enclosed within closely spaced lateral ties whereas in spiral column main RIF is induced within the spiral
- Generally spiral RIF is provided for circular column, square column or octagonal column. However, it can be provided in any type of c/s
- In case of spiral column, the core concrete is subjected to triaxial state of stress, spiral is better and it gives higher ductility compare to tied column
- Capacity of spiral column is more compared to that of tied column (15% more) because of compression confinement
- The c/s area of metal core, shall not exceed 20% of Gross area in case of composite column

③ Composite column:

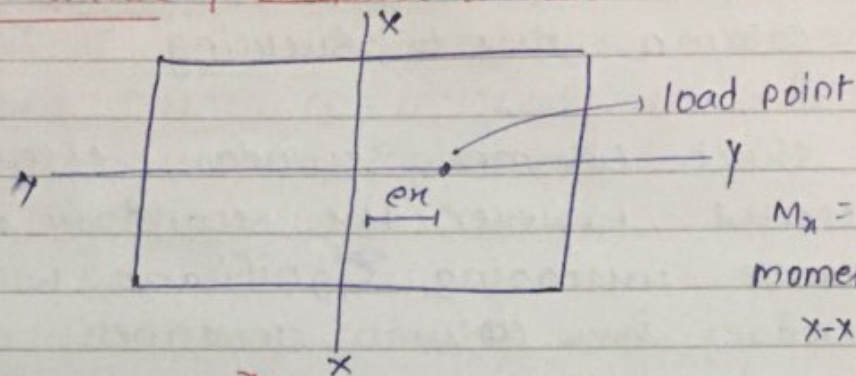


2) Based on type of loading :-

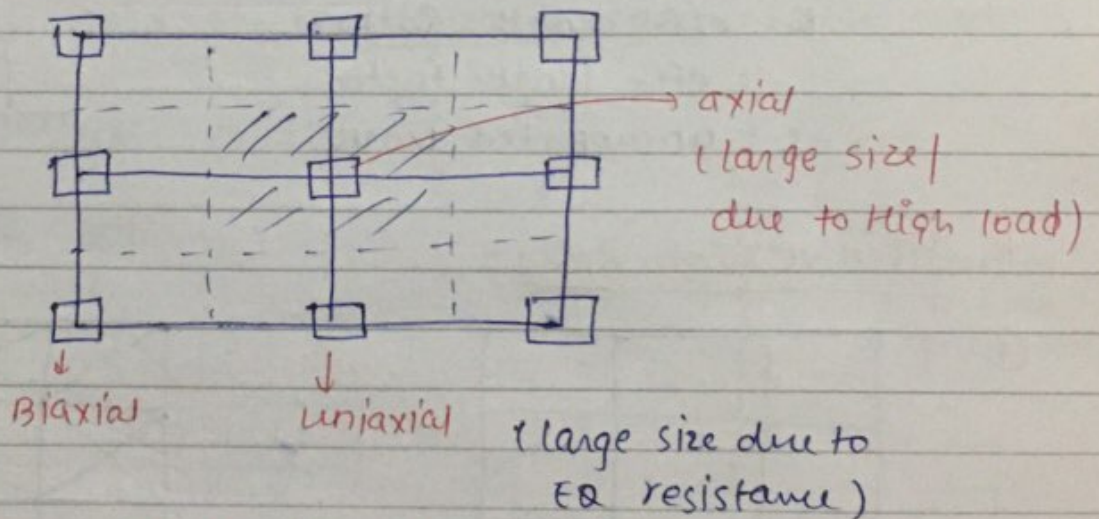
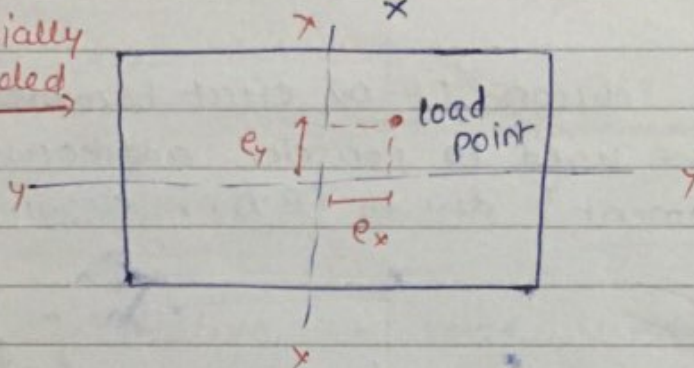
a) axially loaded / concentric



b) uniaxially loaded / ~~concentric~~ loaded :-



c) Biaxially loaded



iii) Based on slenderness Ratio ⇒

① Short column ⇒

Both $\frac{l_{ex}}{D}$ and $\frac{l_{ey}}{b} \leq 12$ or else large column.

l_{ex} → eff. length about x-x

l_{ey} → eff. length about y-y

Short column fails due to crushing and
long column due to Buckling.

→ In short column, secondary effects are neglected, However the secondary moment becomes ~~because~~ increasing significant with increases in column slenderness.

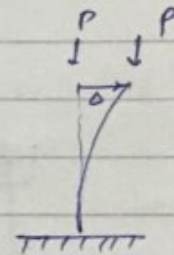
- Generally long column (P-Δ) effect becomes prominent and we used to consider additional moment (secondary Moment) due to (P-Δ) in designing of long column.

Effective length $\Rightarrow K L = l_{eff}$

K = eff. length Ratio

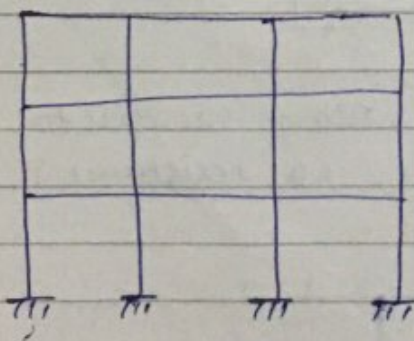
$\cdot$ = eff. length factor

L = unsupported length.

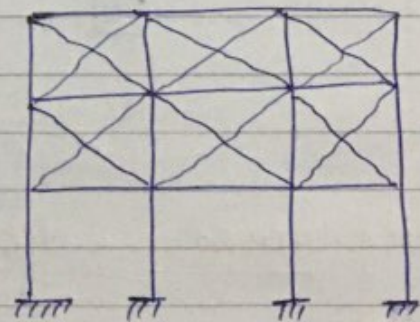


K (effective length Ratio)

①



unbraced frame

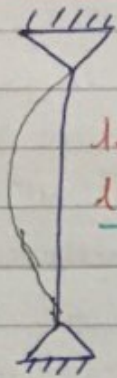


Braced frame

- Generally building with shear walls, masonry walls or Infill walls are considered as Braced frame.

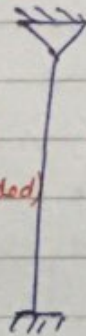
When the relative transverse displacement b/w upper & lower ends of column is prevented, the frame is called braced frame or frame braced against side sway.

→ The k value for braced frame are



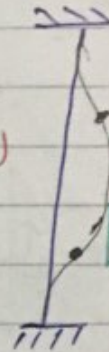
$$l_{eff} = l \text{ (Theory)}$$

$$l_{eff} = l \text{ (recommended)}$$



$$l_{eff} = \frac{l}{\sqrt{2}} \text{ (theory)}$$

$$l_{eff} = 0.8l \text{ (recommended)}$$

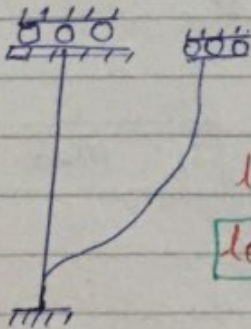


$$l_{eff} = 0.5l \text{ (theory)}$$

$$l_{eff} = 0.65l \text{ (recommended)}$$

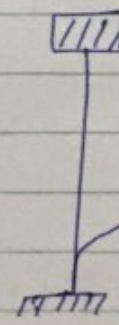
When the relative transverse displacement b/w upper and lower ends of column is not prevented, the frame is said to be unbraced against side sway.

K values for this conditions are -



$$l_{eff} = l \text{ (theory)}$$

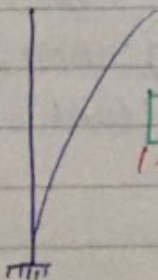
$$l_{eff} = 1.2l \text{ (recommended)}$$



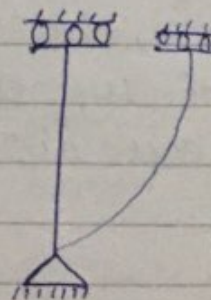
partial restrain against rotation

$$l_{eff} = \text{---}$$

$$l_{eff} = 1.5l \text{ (recommended)}$$



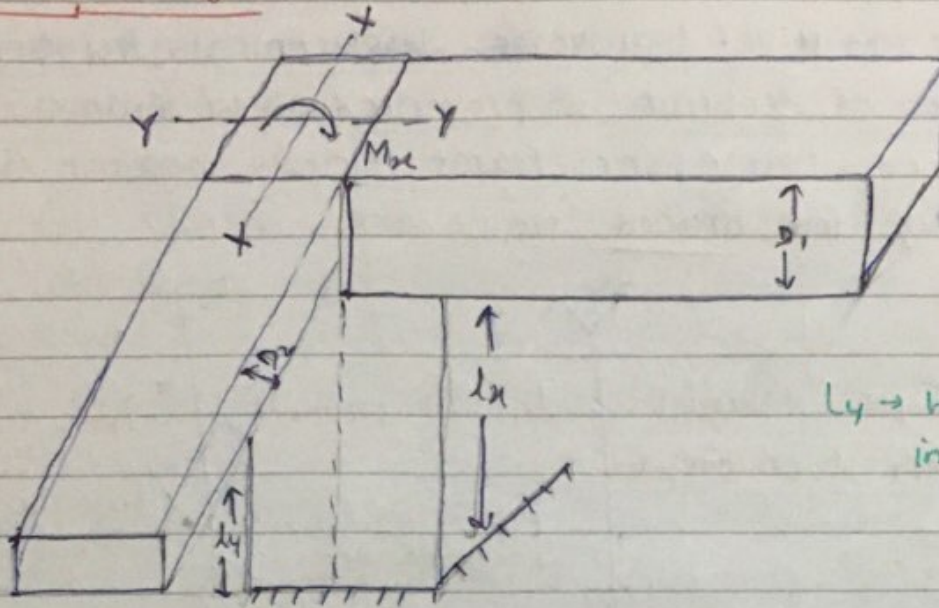
$$l_{eff} = 2l \text{ (theory \& recommended)}$$



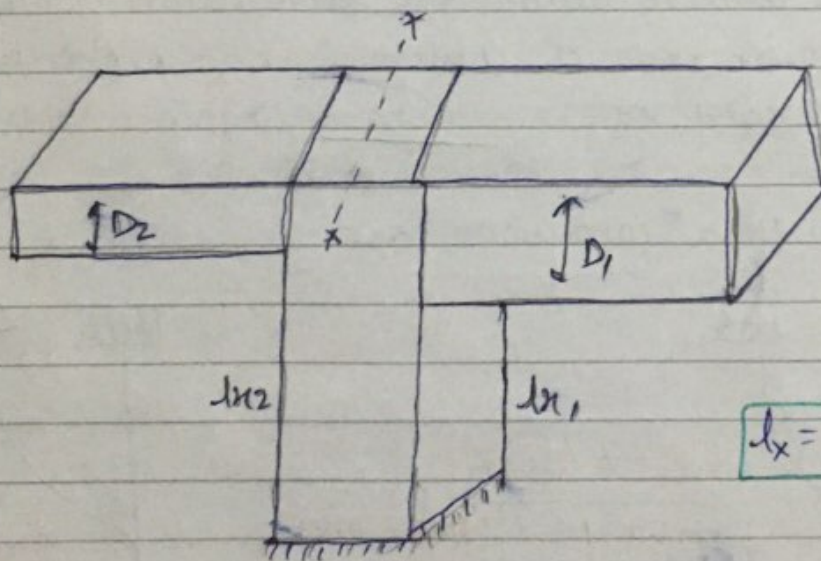
$$l_{eff} = 2l \text{ (theory)}$$

$$l_{eff} = 2l \text{ (recommended)}$$

UNSUPPORTED length :-



$l_y \rightarrow$ When beam is in y dir $\rightarrow$ longitudinally



$$l_x = \max. (l_{x1}, l_{x2})$$

unsupported length is the clear distance b/w the floor and underside of the shallower beam framing into the column in each dirⁿ at the next higher level.

Code Recommendations :-

1. Slenderness ratio :-

unsupported length b/w two end restrain shall not be greater than 60 times least lateral dimension

$$(l \nless 60b)$$

for cantilever column

$$\left(l \nless \frac{100b^2}{D} \right)$$

2. Minimum eccentricity :-

Generally it is not possible to apply the load perfectly at the centroid and also some construction error can create eccentricity in the loading

To account for this, IS code suggest to design axially loaded column also for some Minimum eccentricity.

$$e_{x,min} = \text{Max.} \begin{cases} \frac{l_x}{500} + \frac{D}{30} \\ 20 \text{ mm} \end{cases} *$$

$$e_{y,min} = \text{Max.} \begin{cases} \frac{l_y}{500} + \frac{b}{30} \\ 20 \text{ mm} \end{cases} *$$

l_x & l_y are unsupported length.

3. Longitudinal R/F :-

C/S Area of longitudinal R/F shall not be less than 0.8% and more than 6% of Gross C/S

When lapping is done the Max. % shall be limited to

4%

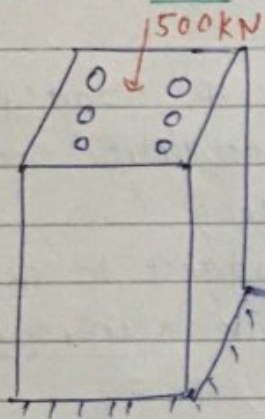
For strength!

Do prevent buckling!

* Calculate hypothetical Area Based upon load and provide R/F for that Area not Gross Area!

- if the column has larger c/s Area then that required to support axial load, Minimum % of R/F shall be based on c/s required to resist the direct load and not upon the actual area provided.

- Due to creep, the strain in both steel & concrete $\uparrow$
However stress in concrete $\downarrow$ and in R/F $\uparrow$.
This is strong.



R/F $\rightarrow$ 200 kN
concrete $\rightarrow$ 300 kN } before creep

R/F $\rightarrow$ 280 kN
concrete $\rightarrow$ 220 kN } after creep

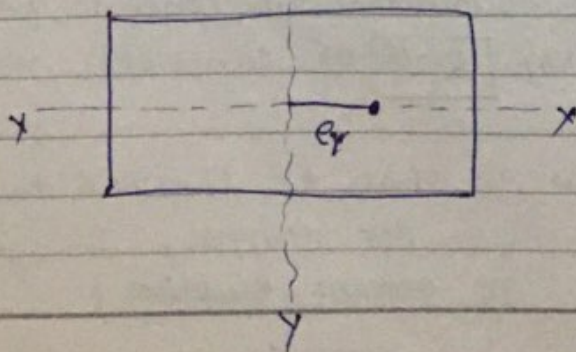
- The Min R/F criteria for column is based on creep consideration

- Minimum no. of longitudinal bars in rectangular column shall be 4 and shall be 6 in circular column or column with helical R/F

- for Non rectangular c/s provide at least one R/F at each corner

$$M_n = p e_y$$

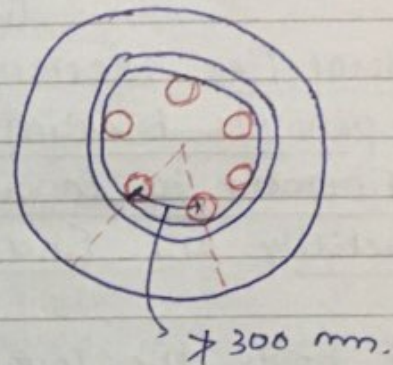
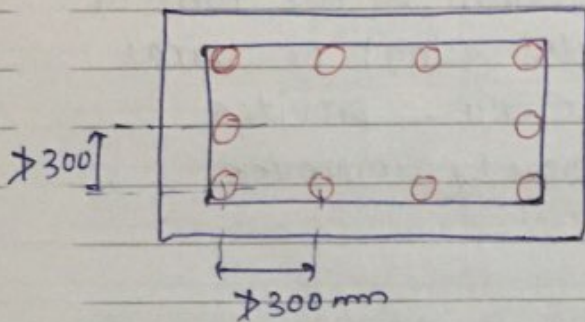
(opposite in sign)
in sign $e_x \rightarrow$ eccentricity along x.



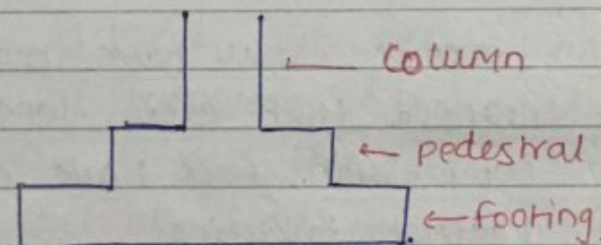
- longitudinal bar dia ϕ 12 mm

- spacing of longitudinal bars measured along periphery of column shall not exceed 300 mm

(because if spacing exceeds 300, then ties will sag.)



- In case of pedestal, Nominal longitudinal R/F not less than 0.15% of the c/s area of pedestal shall be provided. However pedestal is designed as a pcc member and R/F is not considered in strength.



④ Cover to R/F :-

A min. clear cover of 40 mm or bar dia whichever is greater to the column ties is recommended in general

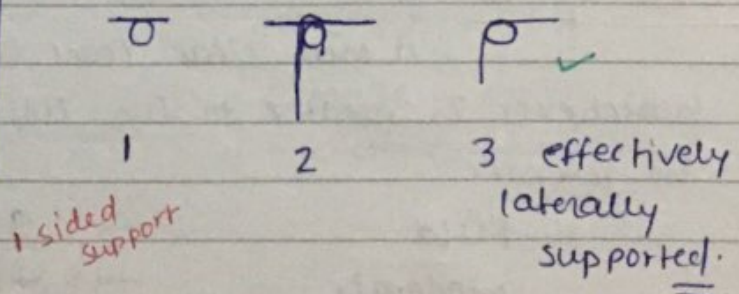
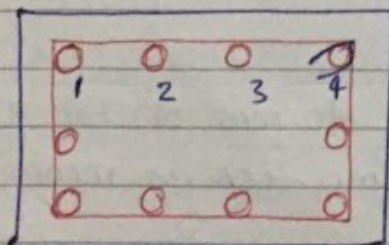
Mild	→	40	} more due to <u>fire resistance</u> !
Moderate	→	40	
Severe	→	45	
Very severe	→	50	
Extreme severe	→	75	

- However the cover can be reduced to 25 mm for small size column ($D \leq 200 \text{ mm}$) and Main R/F dia $\leq 12 \text{ mm}$.

⑤ Transverse R/F :-

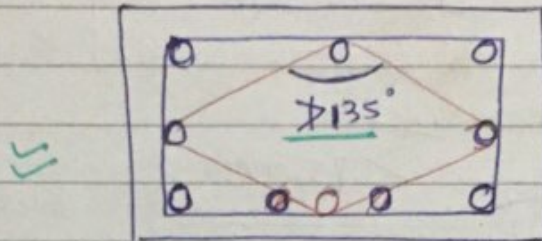
These are provided in the form of lateral ties (Rectangular / Circular ring) or spiral prevents buckling or column R/F, provides confinement to concrete and thereby improves Ductility and Strength.

- It holds the longitudinal bars in position during construction and provides resistance against shear and Torsion.
- Generally column is not critical in shear as axial compressive load increases the shear strength { compression confinement
- RCC Compression member shall have transverse or helical R/F so arranged that every longitudinal bar nearest to the compression face have effective lateral support against buckling.



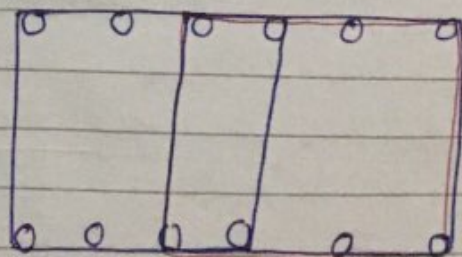
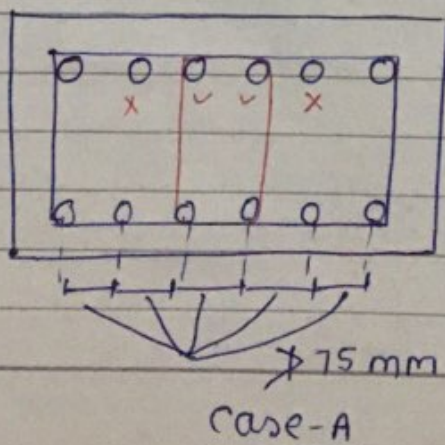
Efficiency $\rightarrow 1 < 2 < 3$

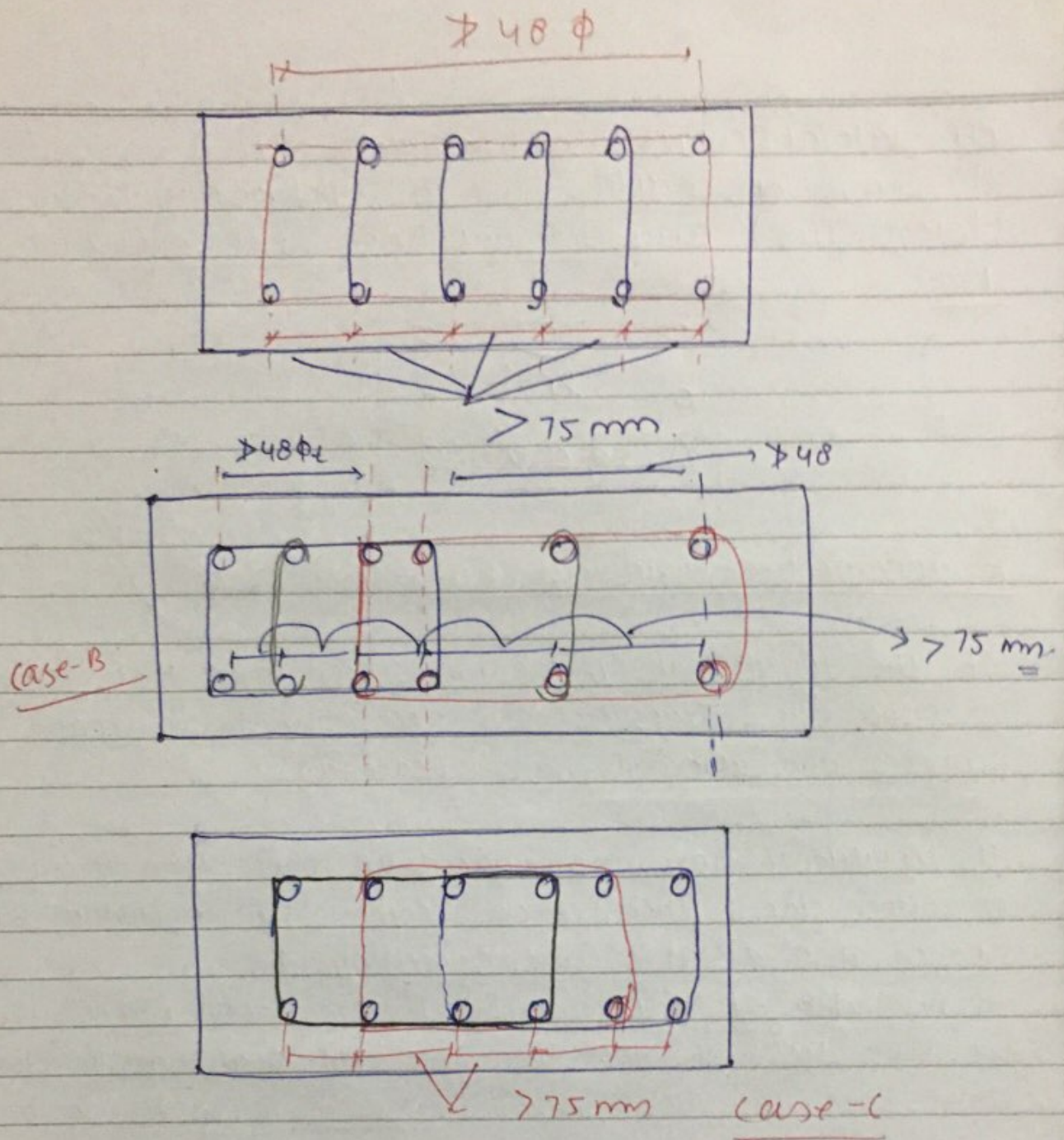
- eff. lateral support to longitudinal R/F is given in the form of circulen ring or polygonal link (lateral Ties) with internal angle not exceeding 135°



Arrangement of Transverse R/F :-

- If the longitudinal R/F are not spaced more than 75 mm on either side, Transverse R/F need only to go round corners and alternate bars. (Case-a)
- If longitudinal bars are spaced at more than 75 mm on either side, then check whether the longitudinal bars spaced at a distance not exceeding $48\phi_t$ ($\phi_t \rightarrow$ dia of transverse R/F) are effectively tied in two dirⁿ. If yes then the additional bars in b/w these bars can be tied in one dirⁿ by open ties only (Case-b)
- If NO then all the bars shall be effectively supported from both sides (Case-c) $> 75\text{ mm}, > 48\phi_t$





Diameter of lateral ties and spiral :-

$$\text{diameter} \geq \max. \begin{cases} 6 \text{ mm} \\ \frac{1}{4} (\phi_{\text{long}})_{\text{max.}} \end{cases}$$

Spacing of lateral ties $\Rightarrow$

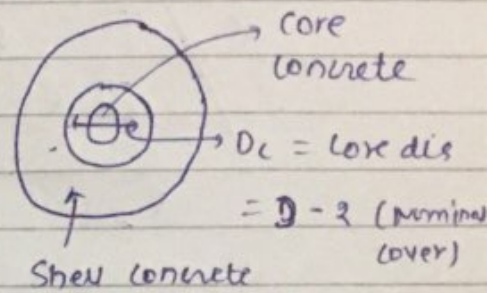
$$\text{spacing} \leq \min \begin{cases} \text{least lateral dimension of column} \\ 300 \text{ mm} \\ 16 (\phi_{\text{long}})_{\text{min}} \end{cases}$$

spacing (pitch of spiral)

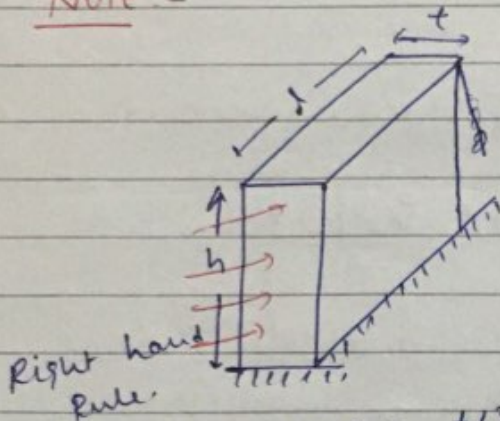
core dia = $D - 2 \times \text{Nominal cover}$

$$p \leq \min \begin{cases} 75 \text{ mm} \\ \frac{1}{6} \text{ core dia} \end{cases}$$

$$\geq \text{Max.} \begin{cases} 25 \text{ mm} \\ 3 \phi \text{ spiral} \end{cases}$$

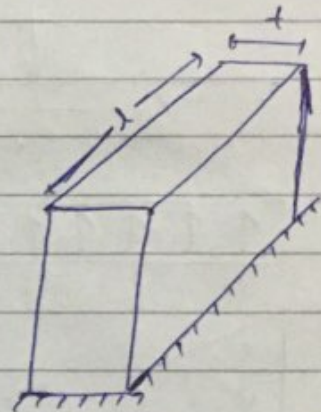


NOTE :-



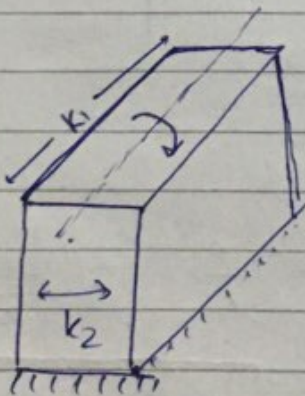
$$M.I = \frac{t l^3}{12}$$

load in plane of wall
Strong plane loading

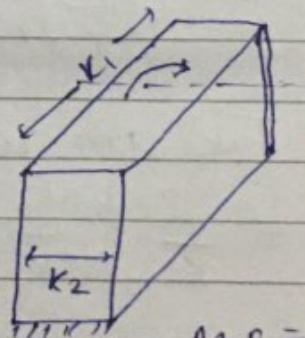


$$M.I = \frac{l t^3}{12}$$

load out of plane
Weak plane loading



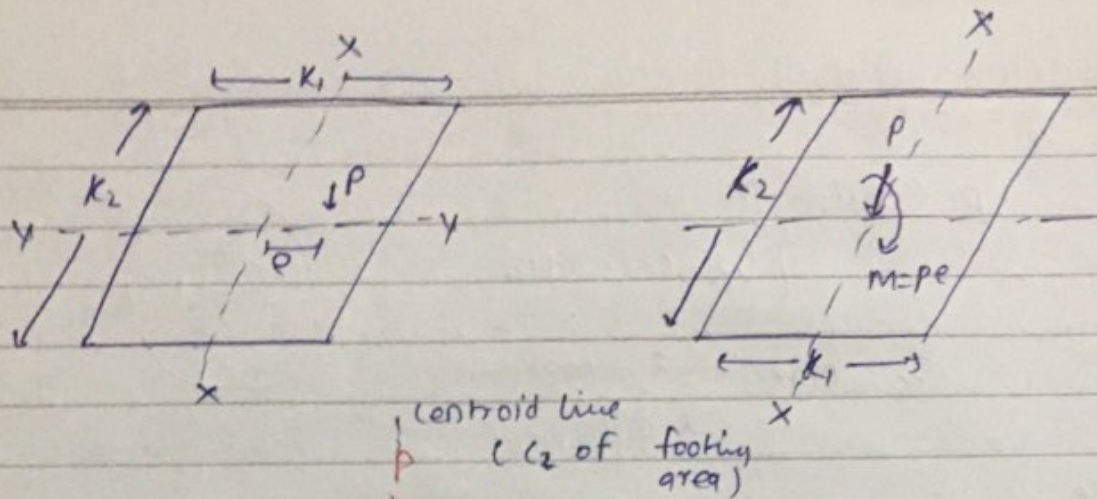
$$M.I = \frac{k_1 k_2^3}{12}$$



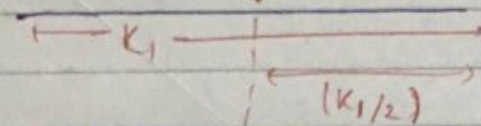
Right hand
thumb
Rule.

$$M.I = \frac{k_2 k_1^3}{12}$$

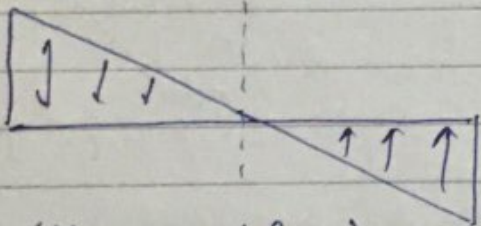
Bending Moment की दि० जिस dimension को मालूम है
उसका cube करना है।



Centroid line
(C₂ of footing area)



$$\left[\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \right] \quad P/A = P/k_1 k_2$$



$$\frac{M}{Z} = \frac{M}{\left(\frac{k_2 k_1^2}{6} \right)} = \frac{6M}{k_2 k_1^2} = \frac{6Pe}{k_2 k_1^2}$$

$$\left(-\frac{M}{Z} = -\frac{6M}{k_2 k_1^2} = -\frac{6Pe}{k_2 k_1^2} \right)$$

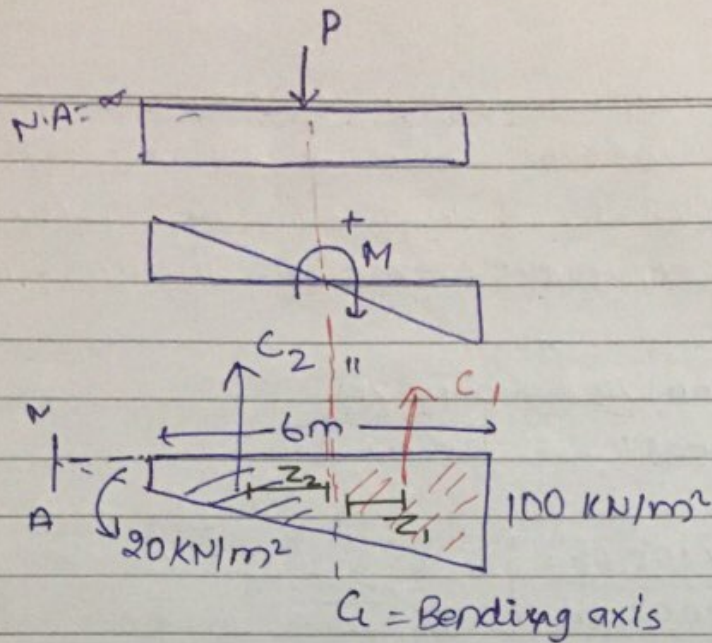
$$\frac{P}{A} \pm \frac{M}{Z} = \frac{P}{k_1 k_2} \pm \frac{6Pe}{k_2 k_1^2}$$

Take 1m width = $k_2 = 1$

$$\frac{P}{A} \pm \frac{M}{Z} = \frac{P}{k_1} \pm \frac{6Pe}{k_1^2} = \frac{P}{k_1} \left(1 \pm \frac{6e}{k_1} \right)$$

$$= \boxed{\frac{W}{b} \left(1 \pm \frac{6e}{b} \right)}^*$$

(b is length)



$$i) \frac{W}{b} \left(1 + \frac{6e}{b} \right) = 100$$

$\rightarrow 6$

$$\frac{W}{b} \left(1 - \frac{6e}{b} \right) = 20$$

Best one!

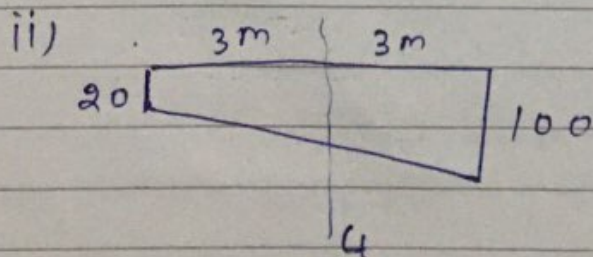
$$W(1+e) = 600$$

$$W(1-e) = 120$$

$$W = 360 \text{ kN}$$

$$e = \frac{2}{3} \text{ m}$$

$$M = W \cdot e = 360 \times \frac{2}{3} = 240 \text{ kNm}$$



$$W = \frac{(100+20)}{2} \times 6 = 360 \text{ kN}$$

For combined stress due to axial force + BM the applied moment can be calculated by taking moment of the stress diagram about centroidal Axis of the Base Area

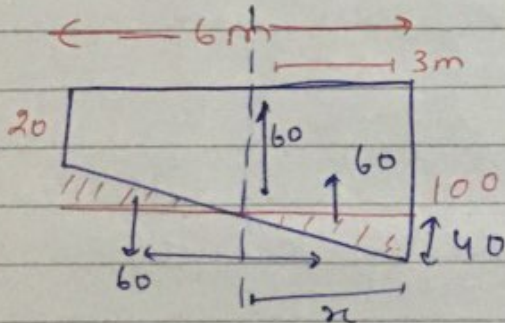
$$C_1 = 0.5 (60 + 100) \times 3 = 240 \text{ kN}$$

$$C_2 = 0.5 (20 + 60) \times 3 = 120 \text{ kN}$$

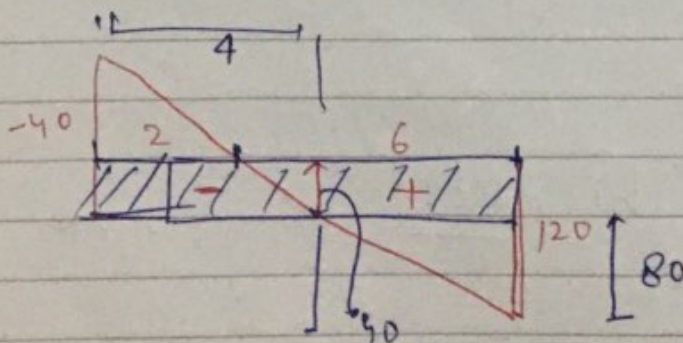
$$Z_1 = \frac{\frac{3}{3} \left(\frac{2 \times 100 + 60}{100 + 60} \right)}{\frac{260}{160}} = \frac{13}{8} \text{ m}$$

$$Z_2 = \frac{\frac{3}{3} (20 \times 2 + 60)}{(20 + 60)} = \frac{100}{80} = \frac{10}{8} \text{ m}$$

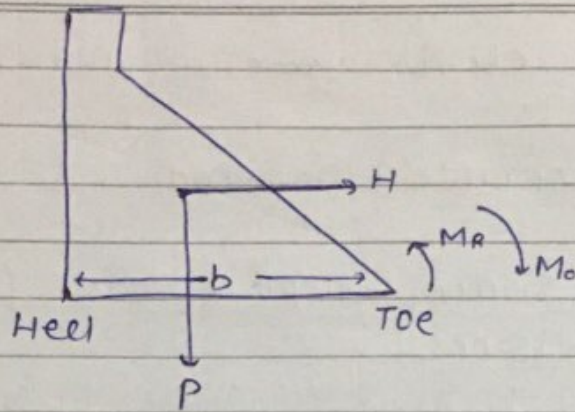
$$M = C_1 Z_1 - C_2 Z_2 = 240 \times \frac{13}{8} - 120 \times \frac{10}{8} = 390 - 150 = 240 \text{ kNm}$$



$$60 \times \frac{4 \times 4}{3} = 240 \text{ kNm}$$



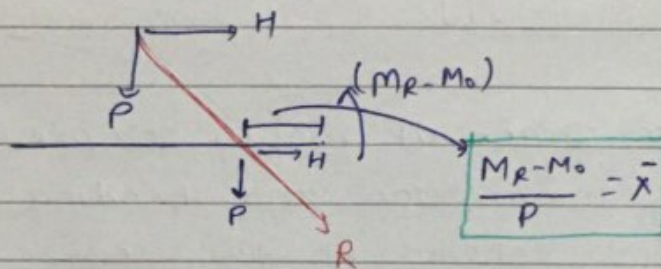
$$160 \times \frac{4 \times 4}{3}$$



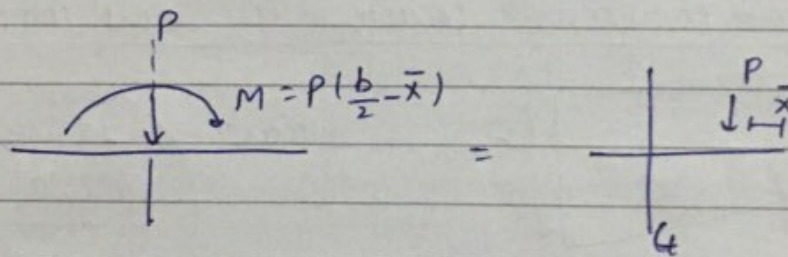
$$FOS = \frac{MR}{M_o} > 1$$

$$\sigma_{toe} = \frac{W}{b} \left(1 + \frac{6e}{b} \right)$$

$$W = P \quad e = \frac{b}{2} - \frac{(MR - M_o)}{P}$$

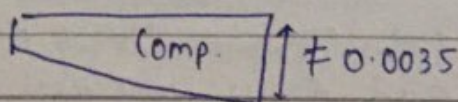
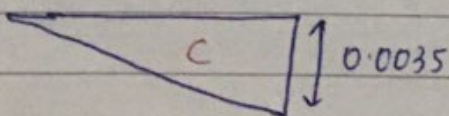


Resultant Moment
Axial force (vertical) = $\bar{x}$



Assumptions :-

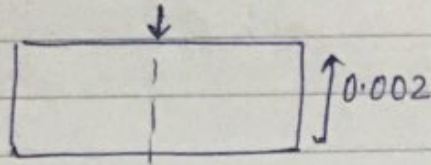
- ① plane section before bending remains plane after bending
- ② Max. compressive strain in concrete at extreme compression fibre is 0.0035 when there is at least some tension in the c/s (or just 0 Tension)



③ σ - ϵ curve for concrete is same as that in Flexure

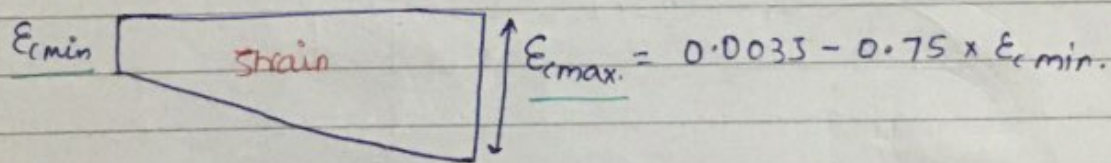
④ Tensile strength of concrete is ignored

⑤ Max. compressive strain in pure Axial (concentric loading) is limited to 0.002



NO chances of redistribution
All fibers will fail
simultaneously.

⑥ Max. compressive strain at highly compressed fibre subjected to axial compression and bending. When there is no Tension on the section shall be $0.0035 - 0.75 \times \text{compressive strain in the least compressed fibre}$



Short axially loaded column:-

A short column with axial force and generally no applied BM

However all columns shall be designed for some minimum eccentricity

if the eccentricity due to external BM is greater, design the column for external axial force and BM

However if the applied eccentricity is less than e_{min} then design for e_{min}

When the min eccentricity does not exceed 0.05 times the respective lateral dimension, the member may be call an axially loaded column and shall be designed

using the following formula.

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

with minimum eccentricity.

$A_c \rightarrow$ Area of concrete = $bD - A_{sc} = A_c$

$A_{sc} \rightarrow$ Area of steel in compression

The above formula is valid only when

* (i) $\frac{l_{ex}}{D} \leq \frac{l_{ey}}{b} < 12$

* (ii) $e_{x,min} = \max \left\{ \frac{l_x}{500} + \frac{D}{30}, 20 \text{ mm} \right\} \leq 0.05 D$

$$e_{y,min} = \max \left\{ \frac{l_y}{500} + \frac{b}{30}, 20 \text{ mm} \right\} \leq 0.05 b$$

To be called as a axially loaded column min. dimension of a column $\geq 400 \text{ mm}$

for a purely axial loaded column that is no minimum eccentricity also $P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

At 0.002 strain

Fe 250 $0.87 f_y$

Fe 415 $0.79 f_y$

Fe 500 $0.745 f_y$ ✓

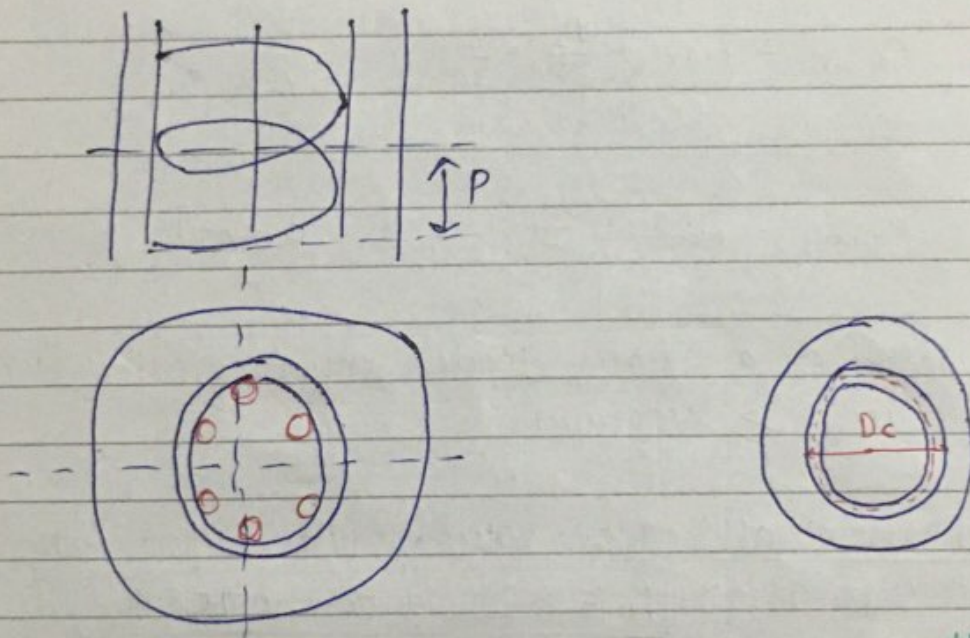
the above axial capacity is reduced by around 10% to account for the BM Generated due to e_{min}

for spiral column the Axial force capacity can be considered 5% higher than that of tied column provided the following condition is satisfied.

$$P_u = 1.05 (0.4 f_{uc} A_c + 0.67 f_y A_{sc})$$

Volume of spiral R/F in one pitch length 0.36 $\left(\frac{A_s - 1}{A_c}\right) \frac{f_{uc}}{f_y}$
Volume of core concrete in one pitch length.

Spiral R/F



HOW?

Volume of spiral R/F in one pitch length = $\frac{\pi}{4} \phi_s^2 \times \pi (D_c - \phi_s)$

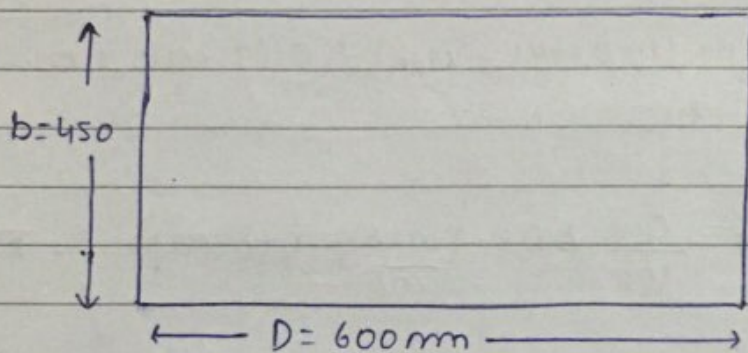
Volume of core concrete in one pitch length = $\frac{\pi}{4} D_c^2 \cdot P$

$f_y \rightarrow$ characteristic strength of spiral ≥ 415 MPa

Que: A RCC column of size $450 \text{ mm} \times 600 \text{ mm}$ is subjected to Axial load of 2000 kN under service load.

unsupported length of column is 3 m . The column is restrained against lateral displacement at both ends take M20 and Fe 415

- i) calculate A_{sc} to be provided
- ii) Detail the C/s



$P = 2000 \text{ kN}$, $L_{\text{supported}} = 3 \text{ m}$
M20 , Fe 415

Considering a pinned column

$$l_{\text{eff}} = l = 3 \text{ m} \quad \checkmark$$

$$\Rightarrow \text{slenderness ratio} = \frac{l_{\text{eff}}}{D} = \frac{3000}{600} = 5 < 12$$
$$= \frac{l_{\text{eff}}}{b} = \frac{3000}{450} = 6.67 < 12 \quad \checkmark$$

Short column.

$$e_{x, \min} = \max \left\{ \begin{array}{l} \frac{l_n}{500} + \frac{D}{30} \\ 20 \end{array} \right. = 26 \text{ mm}$$

$$0.05 D = 30 \text{ mm}$$

$$e_{x, \min} \leq 0.05 D \quad \checkmark$$

$$e_{y, \min} = \max. \begin{cases} \frac{ly}{500} + \frac{b}{30} = 21 \text{ mm} \\ 20 \text{ mm} \end{cases}$$

$$0.05b = 22.5 \text{ mm}$$

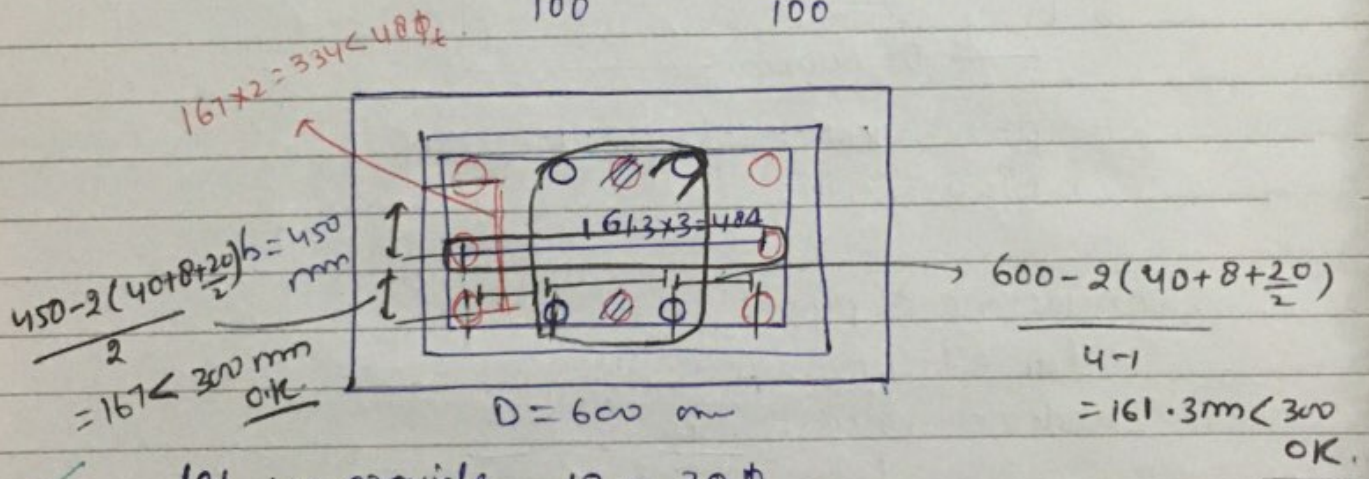
$$e_{y, \min} < 0.05b \quad \checkmark$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$1.5 \times 2000 \times 10^3 = 0.4 \times 20 \times (450 \times 600 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$A_{sc} = 3110.53 \text{ mm}^2$$

$$\checkmark A_{sc, \min} = \frac{0.8}{100} bD = \frac{0.8}{100} \times 450 \times 600 = 2160 \text{ mm}^2$$



let us provide 10 - 20 ϕ

$$A_{sc, \text{provided}} = 10 \times 314 = 3140 \text{ mm}^2$$

$$\phi_t \geq \max. \begin{cases} 6 \text{ mm} \\ \frac{1}{4} (\phi_{\text{long}})_{\max} = 06 \end{cases} = 6 \text{ mm}$$

$$\text{take } \phi_t = 8 \text{ mm}$$

$$\text{spacing} \leq \min \begin{cases} \text{least lateral dimension} = 450 \\ 300 \\ 16(\phi_{\text{long}})_{\min} = 320 \end{cases} = 300 \text{ mm}$$

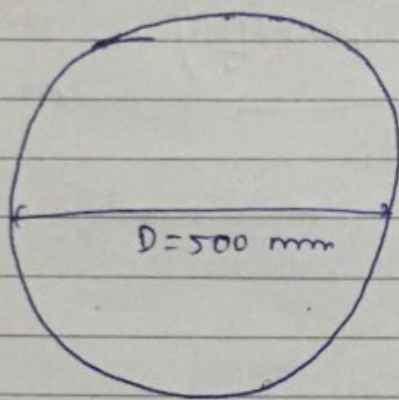
$$\text{spacing} = 300 \text{ mm} \quad \checkmark$$

$$48 \phi_t = 384 \text{ mm}$$

$$\underline{404 > 384} \quad \underline{\text{O.K.}}$$

Que. A short axially loaded spiral column of 500 mm dia is subjected to a factored load of 1500 kN
M25 & Fe415

- ① Calculate A_{sc} to be provided
- ② Arrange the main RIF in the c/s
- ③ calculate dia and pitch of spiral



$$P_u = 1500 \text{ kN}$$

M25, Fe415

$$\underline{D = 500 \text{ mm}}$$

$$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

$$1500 \times 10^3 = 1.05 (0.4 \times 25 (\frac{\pi}{4} \times 500^2 - A_{sc}) + 0.67 \times 415 \times A_{sc})$$

$$\underline{A_{sc} = -1995.60 \text{ mm}^2}$$

-ve implies c/s size is more than sufficient to take care of load without RIF

Hypothetical column size (pcc) to resist the direct load

$$= \frac{P_u}{1.05 \times 0.4 f_{ck}} = 142857 \text{ mm}^2$$

spiral
RIF

$$\frac{\pi}{4} \times 500^2 = 196349 \text{ mm}^2$$

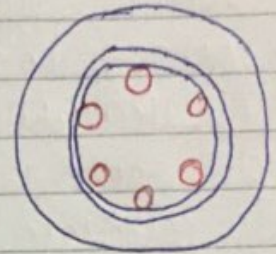
provide for this Area

$$A_{sc, min} = \frac{0.8}{100} \times 142857 = 1142.85 \text{ mm}^2$$

Let us provide 6 no of 16 mm dia

$$A_{sc, provided} = 6 \times 201 = 1206 \text{ mm}^2$$

$$(ii) \quad \phi_s \geq \max. \begin{cases} 6 \text{ mm} \\ \frac{1}{4} (\phi_{long})_{max} \rightarrow 16 = 6 \text{ mm} \end{cases}$$



provide $\phi_s = 8 \text{ mm}$ ✓

$$\text{spacing} = \frac{\pi (D - 2(40 + 8 + \frac{16}{2}))}{6} = 203.10 \text{ mm} < 300 \text{ mm}$$

o.k. ✓

$$(iii) \quad \phi_s = 8 \text{ mm}.$$

$$P \leq \min \begin{cases} 75 \text{ mm} \\ \frac{1}{6} \text{ core dia} \rightarrow \frac{420}{6} = 70 \text{ mm} \end{cases} = 70 \text{ mm}$$

$$\geq \max. \begin{cases} 25 \text{ mm} \\ 3\phi_s = 24 \text{ mm} \end{cases} = 25 \text{ mm}$$

$$D_c = \text{core dia} = 500 - 2 \times 40 = 420 \text{ mm}$$

$$\frac{\text{Volume of spiral R/F in one pitch length}}{\text{Volume of core concrete in one}} \geq 0.36 \left(\frac{A_s}{A_c} - 1 \right) \frac{f_{sc}}{f_y}$$

$$\frac{\frac{\pi}{4} \phi_s^2 \times \pi (D_c - \phi_s)}{\frac{\pi}{4} D_c^2 \times P} \geq 0.36 \left(\frac{\frac{\pi}{4} D_c^2}{\frac{\pi}{4} D_c^2} - 1 \right) \frac{25}{415}$$

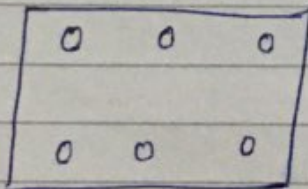
$$P \leq 51 \text{ mm}$$

provide 8mm dia spiral @ 50 mm pitch.

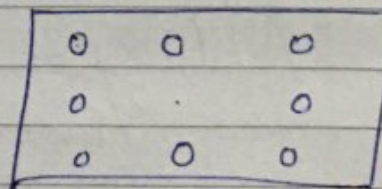
Short column with uniaxial loading :- (interaction Diagram)

Design of column under axial loading and moment is done using interaction diagram because the Normal calculation involves huge no. of Trial & error

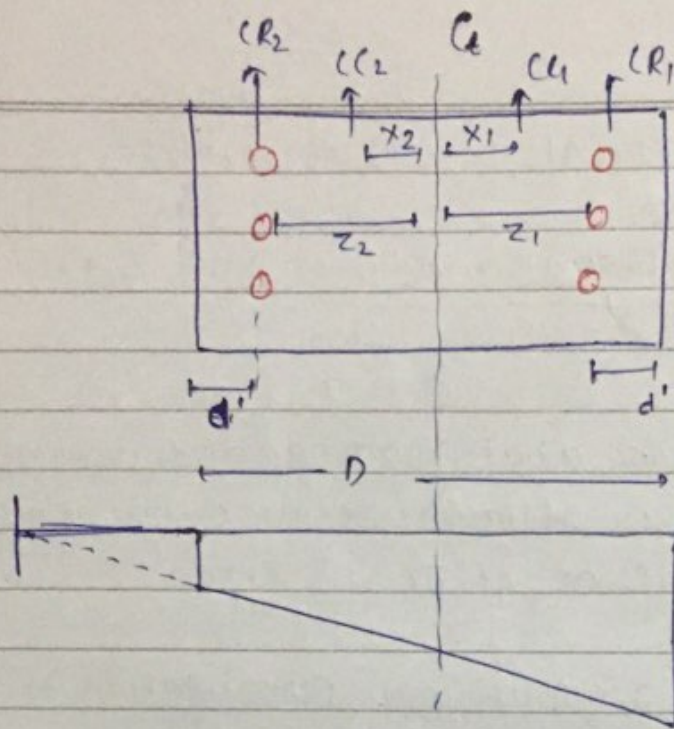
- Interaction curve is a graphical representation of design strength of uniaxially loaded column
- The interaction curve are available for,
1) different $\frac{d'}{d}$ value like 0.05, 0.10, 0.15, 0.20
2) different type of cls like rectangular, circular etc.
3) Arrangement of R/F i.e. distributed equally on both sides or distributed equally on all sides



(distributed
equally on both
sides)

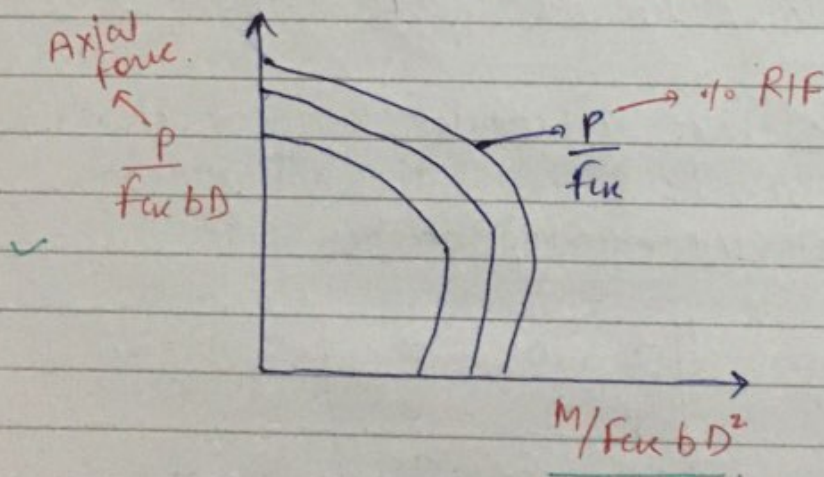


(distributed equally
on all sides)



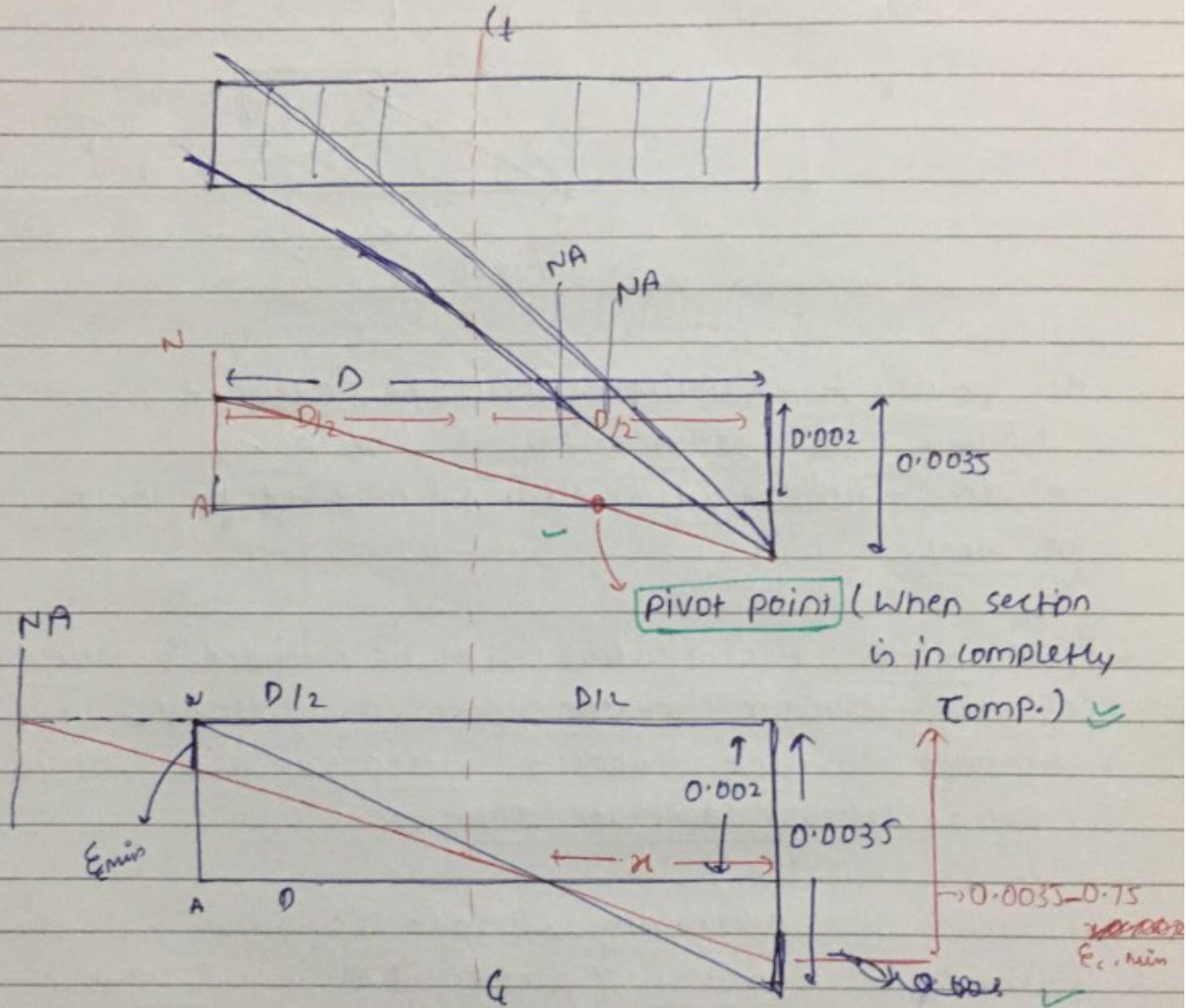
$$C = CR_1 + CL_1 + CR_2 + CL_2$$

$$M = \underline{CR_1 z_1} + \underline{CL_1 x_1} + \underline{CR_2 z_2} + \underline{CL_2 x_2}$$



- from the interaction diagram we'll get the value of $\frac{P}{f_{ck}}$ ($P = \% \text{ steel}$) for a particular M_u & P_u
- Each point under curve represents the possible combination of M_u & P_u which will make the column just safe.

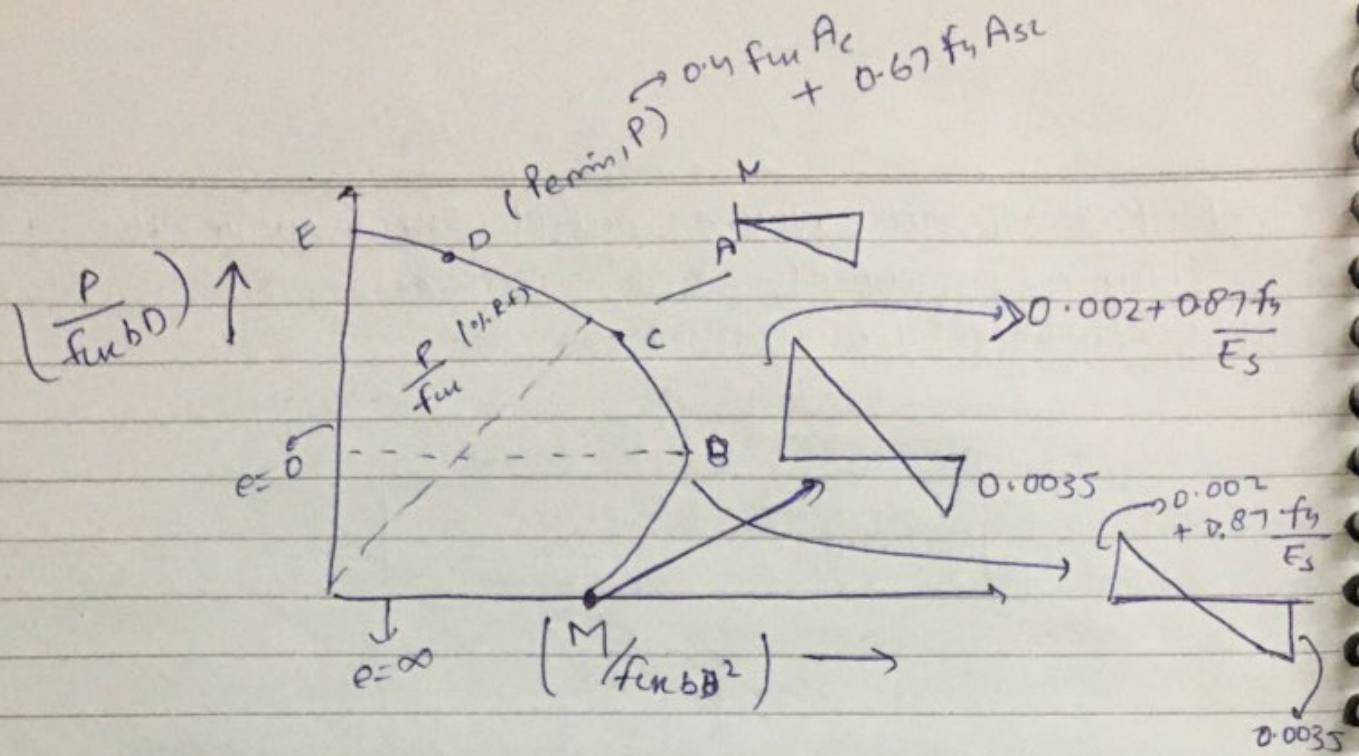
gf P_u & M_u gets plotted outside the curve the column is unsafe and if it gets plotted inside the curve, the column is safe.



$$ABC \sim ADO$$

$$\frac{x}{D} = \frac{0.0015}{0.0035} = \frac{3}{7}$$

$$\Rightarrow \boxed{x = \frac{3}{7} D}$$



at point A, column shall be designed as a Doubly UR section ($P_u = 0$, M_u , $e = \infty$)
at this point failure will be initiated by yielding of steel.

from A $\rightarrow$ B, the failure will be initiated by yielding of steel. as we increase the axial load, the moment carrying capacity increases. this zone is called Tension controlled zone

At point B, Eccentricity of load is such that the failure is Balance failure that is $e_c = 0.0035$

* $e_{ft} = 0.002 + 0.87 f_y / E_s$

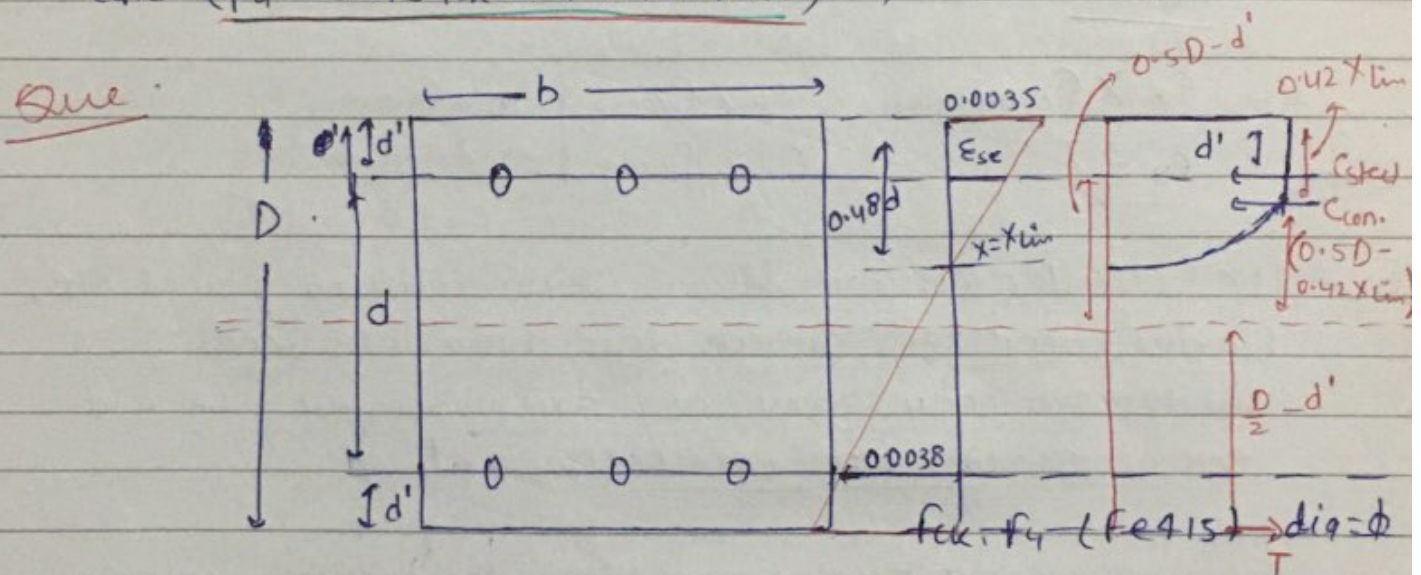
from B $\rightarrow$ E, the failure will be due to crushing of concrete. so when we increase the axial load, in order to make the column safe the Applied BM shall be reduced. This zone is called compression controlled zone.

at point C the eccentricity of load is such that the NA lies at one edge of the section

from C $\rightarrow$ E, the cs will be in compression
 point D corresponds to axial force and moment
 due to minimum eccentricity.

We don't design the column for eccentricity
less than E_{min} .

point E corresponds to concentric axial loading
 and $(P_u = 0.45 f_{ck} A_c + 0.75 f_y A_{sc})$,



Derive the eqⁿ for total axial force and moment
 when eccentricity is such that $x = x_{lim}$

Also write the eqⁿ of max. moment that can be
 applied on the given secⁿ :-?

$$P = C_{steel} + C_{con} - T$$

$$= f_{sc} \times 3 \times \frac{\pi}{4} \phi^2 + 0.36 f_{ck} b x_{lim} - 0.87 f_y \times 3 \times \frac{\pi}{4} \phi^2$$

$$M = f_{sc} \times 3 \times \frac{\pi}{4} \phi^2 \times (0.5D - d') + 0.36 f_{ck} b x_{lim} (0.5D - 0.42 x_{lim})$$

$$+ 0.87 f_y \times 3 \times \frac{\pi}{4} \phi^2 (0.5D - d')$$

The above obtain moment corresponds to $x = x_{lim}$
 i.e. Balance condition
 so Max. capacity of column for $BM = M$
 as calculated above.

Short column with biaxial loading:-

When biaxial bending is considered it is ~~crossi~~ sufficient that eccentricity exceeds the min about one axis at a time.

UNI

$$e_x > e_{x,min}$$

$$e_y = 0$$

Bi

$$e_x > e_{x,min}$$

$$e_y > e_{y,min}$$

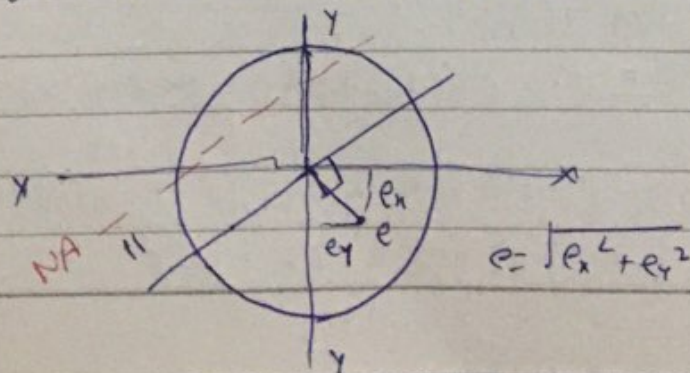
$$e_x > e_{x,min}$$

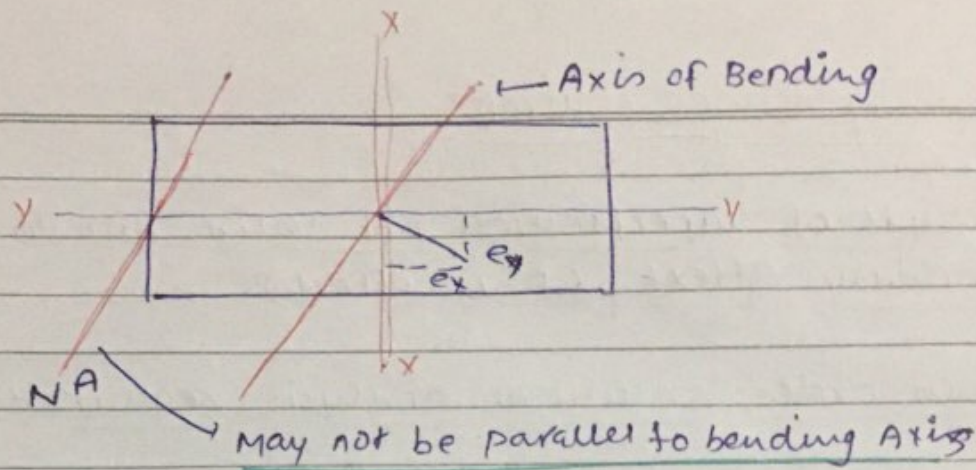
$$e_y < e_{y,min}$$

$$e_y \neq 0$$

- for circular column with symmetrically placed R/F, biaxial bending converts into uniaxial bending case due to symmetry about any axis. here the NA and axis of bending are parallel
- for Non-symmetrical axis like rectangular column or unsymmetrically placed column, NA is generally not parallel to the resultant axis of bending.
NA can be determined by trial and error procedure.

- Interaction dia for biaxial bending is a 3D plot





IS Code procedure for Biaxially loaded short column:-

Based on Bresler's formulation

→ The resistance of the member is subjected to axial force and biaxial Bending shall be obtained on the basis of assumptions given in compression with NA so decided to satisfy force eqⁿ & moment eqⁿ.

OR Such member can be designed by following eqⁿ

$$\left[\frac{M_{ux}}{M_{uxl}} \right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}} \right]^{\alpha_n} \leq 1.0$$

P_u , M_{ux} and M_{uy} are the axial force and moment about respective axis for the applied load.

M_{ux1} , M_{uy1} → moment capacity in uniaxial loading about respective axis

α_n → constant Based on $\frac{P_u}{P_{u2}}$

$P_{u2} \rightarrow 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$

Long column / slender column :-

- principle of superposition not valid due to higher secondary effect (P-D effect)
- second order structural analysis required.
- As per IS code either we can design a long column with reduced material strength (in WSM) or taking some additional moment / secondary moment (in LSM)
- Strength reduction coefficient

$$C_r = 1.25 - \frac{l_{eff}}{48b} \rightarrow \text{WSM}$$

$b \rightarrow$ least lateral dimension of column
for Helical R/F column, b is dia of core

In LSM, if the effect of deflections are not considered, the column shall be designed for Actual moment + some additional moment

$$M_{ax} = \frac{P_u D}{2000} \left(\frac{l_{ex}}{D} \right)^2$$

$$M_{ay} = \frac{P_u b}{2000} \left(\frac{l_{ey}}{b} \right)^2$$

Design moment

$$\begin{aligned} & M_{ux} + M_{ax} \\ & \& M_{uy} + M_{ay} \end{aligned}$$

$M_{ux}, M_{uy} \rightarrow$ Actual Moment

Footings

purpose of footings and foundations are :-

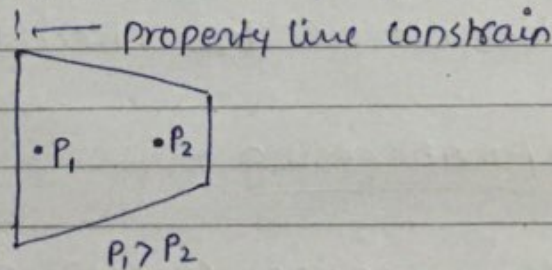
- 1) Transfer the applied load without exceeding SBC.
- 2) Ensuring settlement within the permissible limit.

Types of footing :-

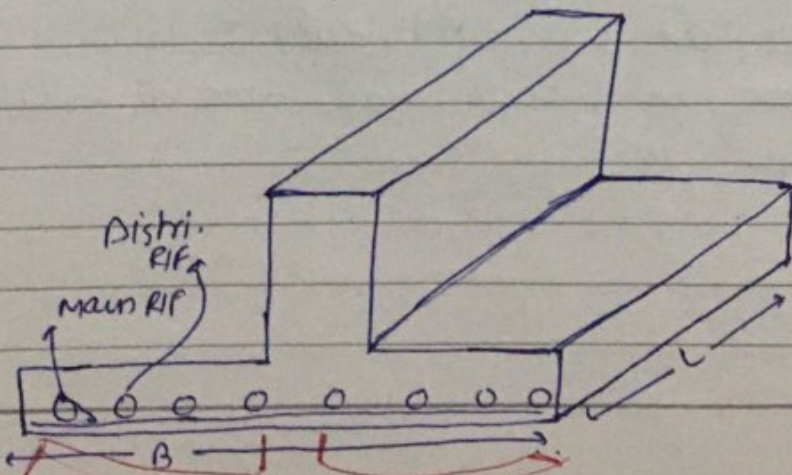
① Isolated footing below column :-

② combined footing :- When isolated footing of one column overlaps with other then we provide combined footing.

Also the combined footings are required to obtain Base pressure in case of property line constrain.

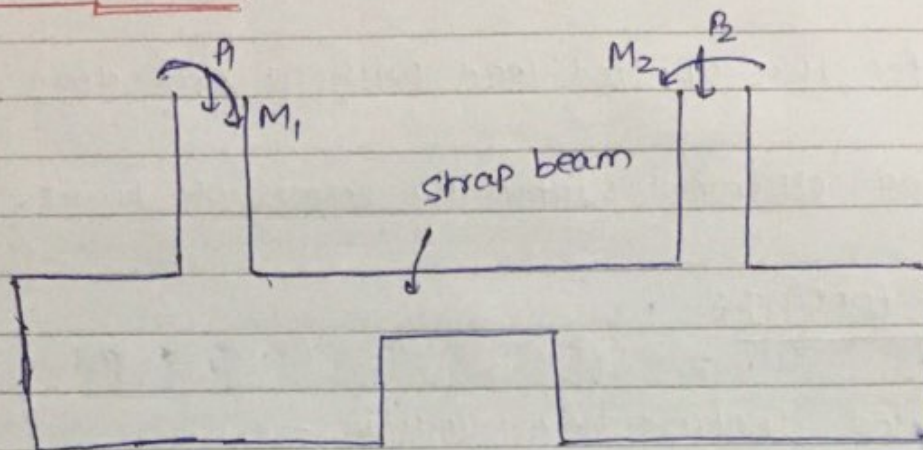


③ Strip footing / Wall footing :- provided for closely spaced columns or pcc or masonry wall



for strip footing.
 $L > 5B$

(iv) Strap footing :-



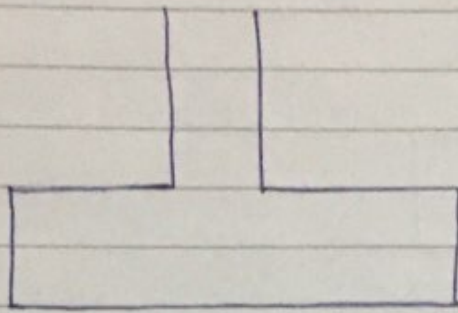
- for columns placed @ large distances, strap footing can be provided to control differential settlement by shearing the load b/w the columns.
- Strap footing is economical than combined footing.
- Strap beam does not transfer direct load to the soil.

(v) RAFT footing / Mat footing :-

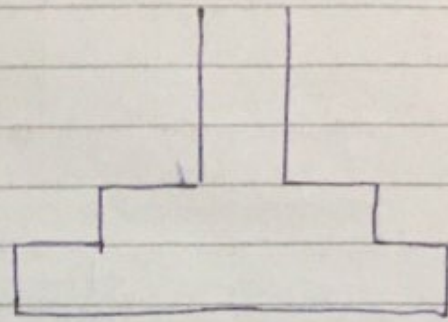
provided when, bearing capacity of soil is less, load applied is high, chances of differential settlement is more or ground water table is Very High.

Also raft is provided when individual isolated footings covers more than 50% plan Area of building.

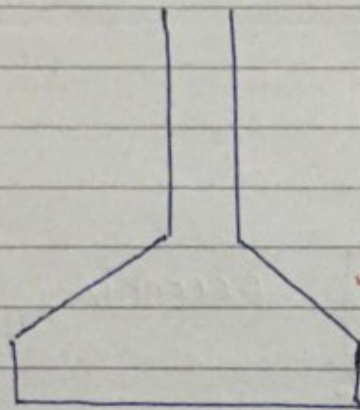
Footing Based on thickness :-



footing with constant thickness



stepped footing.

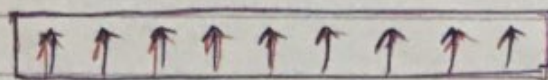
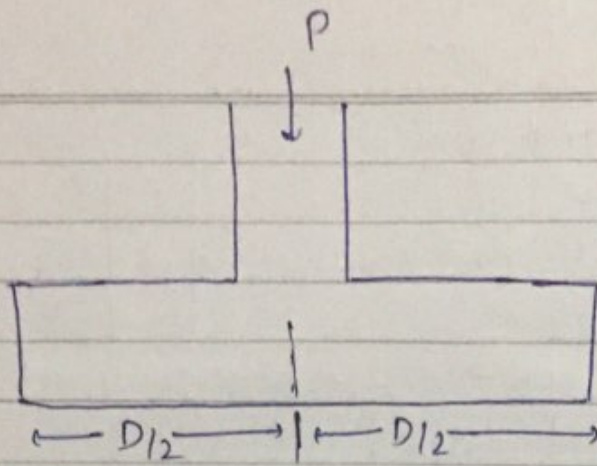


sloped footing

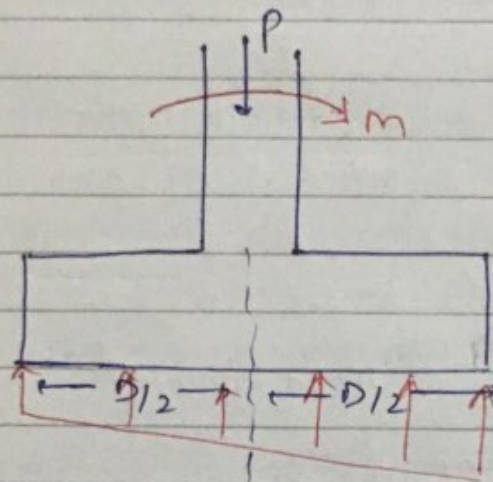
only to economise

Base pressure distribution :-

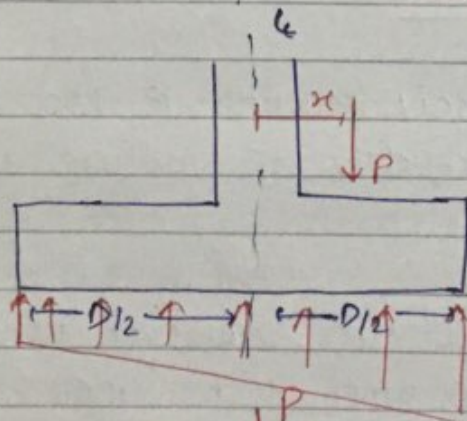
- ① The distribution of soil reaction @ the base of footing depends on rigidity of footing and properties of soil.
- ② It is generally non-linear, however in normal design practice it is assumed to be linear. We consider soil as linear elastic and footing as rigid.
- ③ Types of loading on a footing can be
 - a) concentric
 - b) eccentric



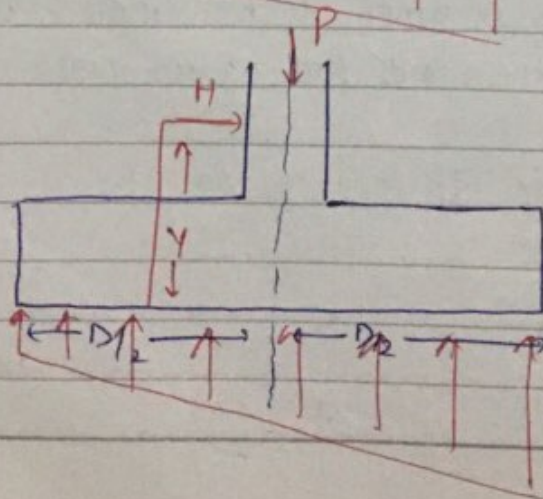
concentric
→ soil reaction



Eccentric

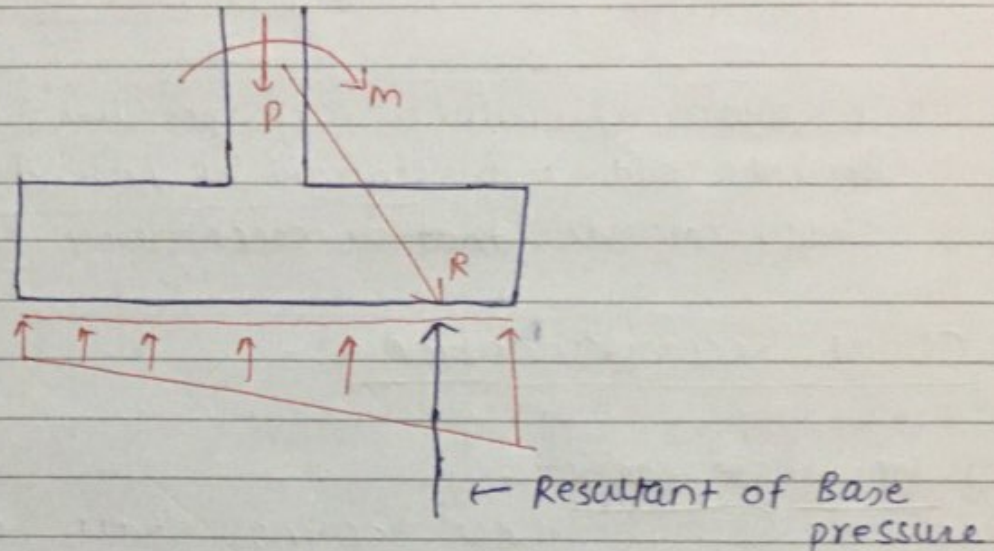


Eccentric



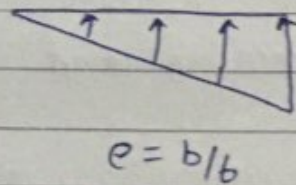
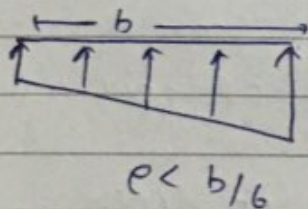
Eccentric

NOTE :- ① Whatever maybe the base pressure distribution, the resultant of base pressure will be co-linear with the line of resultant applied load (static equilibrium condition)

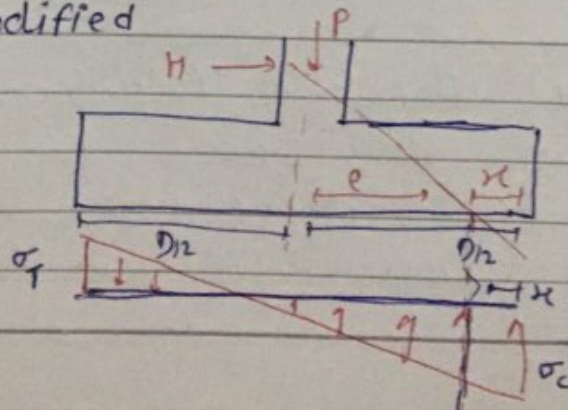


② If eccentricity of resultant loading $\leq B/6$,

There will be NO TENSION @ footing Base.



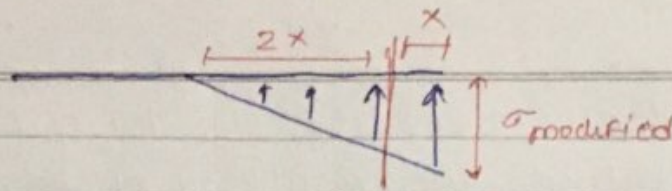
③ If $e > b/6$, some portion of Base width will experience Tension. However as soil can not resist Tension at the interface, Base pressure has to be modified



$$x = \frac{M_R - M_0}{P}$$

$$e = b/2 - x$$

$$\sigma_{\text{modified}} > \sigma_c$$



$$\frac{1}{2} \times 3x \times \sigma_{\text{modified}} = P$$

$$\sigma_{\text{modified}} = \frac{2P}{3x}$$

- ④ To obtain a uniform Base pressure, centroid of applied load and centroid of base area of footing shall coincide. that is eccentricity $e=0$

Codal Recommendations :-

① Depth of footing:-

- All footings shall be located at a min depth of 0.5 m below the ground surface
- The depth is primarily governed by availability of bearing capacity, Minimum seasonal variation like swelling and shrinkage of soil
- for a preliminary estimate, Min depth is given by Rankine formula.

$$D_f = \frac{q}{\gamma} \left[\frac{1 - \sin \phi}{1 + \sin \phi} \right]^2$$

$D_f \rightarrow$ depth of footing

$q \rightarrow$ Gross SBC

$\gamma \rightarrow$ Density of soil

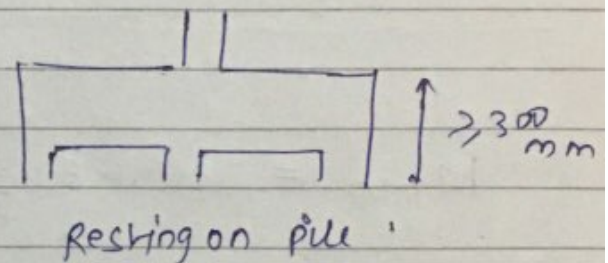
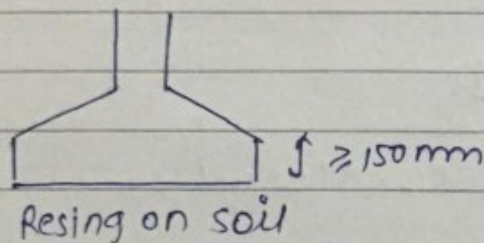
$\phi \rightarrow$ angle of internal friction.

2. Cover to RIF :-

A minimum cover of 50 mm to all RIF

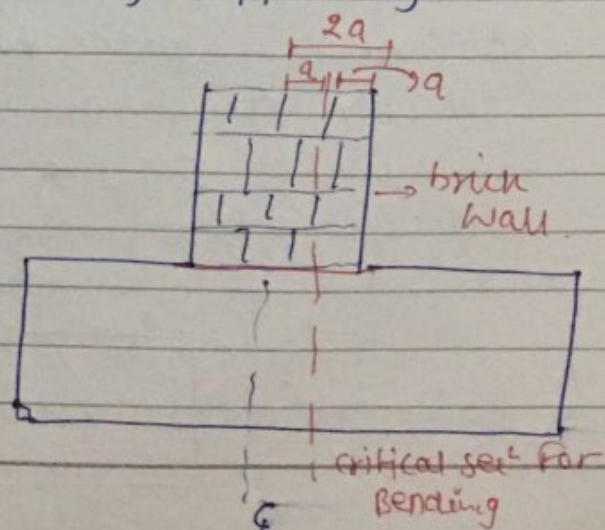
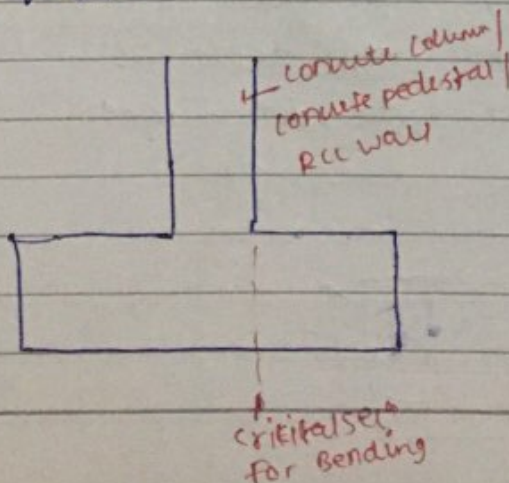
3. Thickness of footing :-

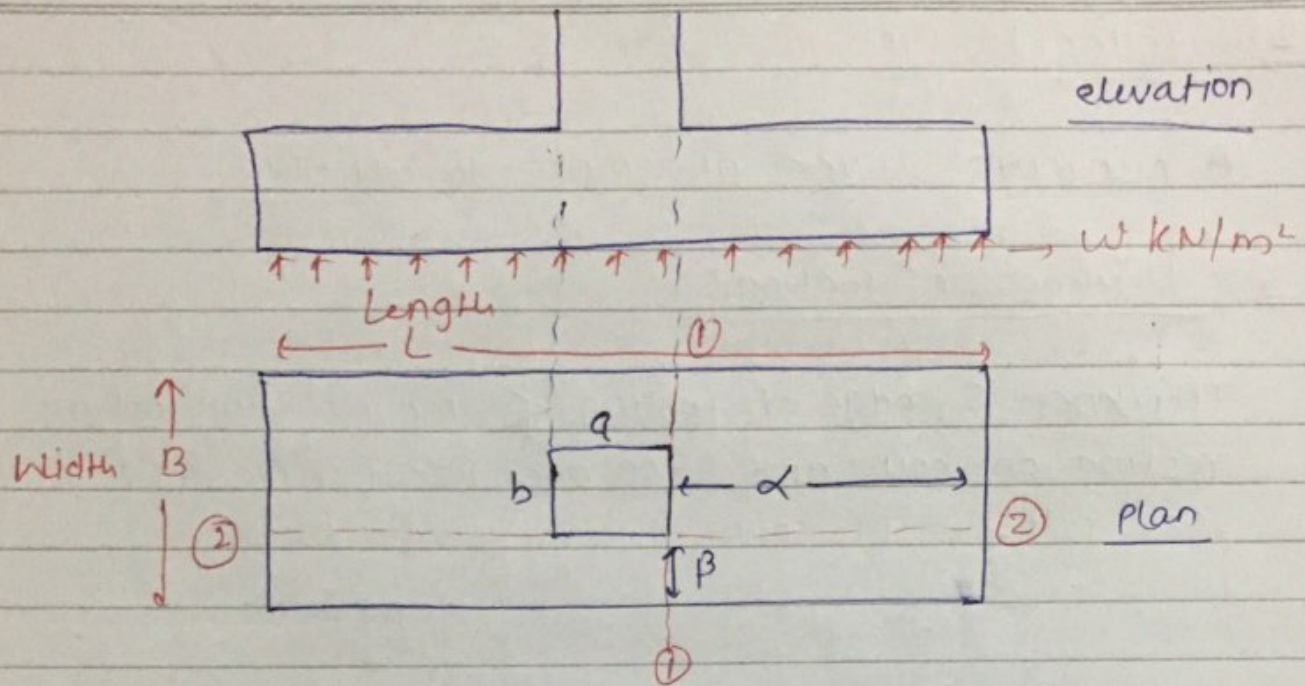
Thickness @ edge of footing ≤ 150 mm for footing resting on soil and 300 mm for footing resting on piles. (To ensure the min. rigidity !)



4. critical section for Bending :-

critical section for Bending for Isolated concrete footing which supports column, pedestal or wall shall be 1) @ Face of column, pedestal or wall $\rightarrow$ for footing supporting concrete column, concrete pedestal, or RCC wall 2) @ Half way b/w the center line and edge of wall $\rightarrow$ for footing supporting masonry wall.





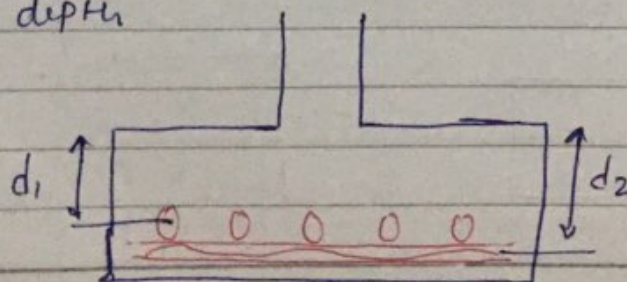
$$M_{1-1} = w \cdot \alpha \cdot B \cdot \frac{\alpha}{2} \quad (\text{RIF along } L) \quad \text{Moment about } 1-1$$

$$M_{2-2} = w \cdot B \cdot L \cdot \frac{B}{2} \quad (\text{RIF along } B)$$

$$A_{st} = \frac{f_{ck}}{2f_y} \left[1 - \sqrt{1 - \frac{4 \cdot b M}{f_{ck} B d^2}} \right] B d$$

The RIF with higher BM shall be placed close to the Tensile Surface in order to get higher Lever Arm

- In case of square footing, calculate the RIF quantity considering smaller eff. depth as this will be on the safer side for higher effective depth



critical section of shear:-

In case of footing shear governs the thickness
However we don't provide shear R/F generally
and if required then increase the depth or fix

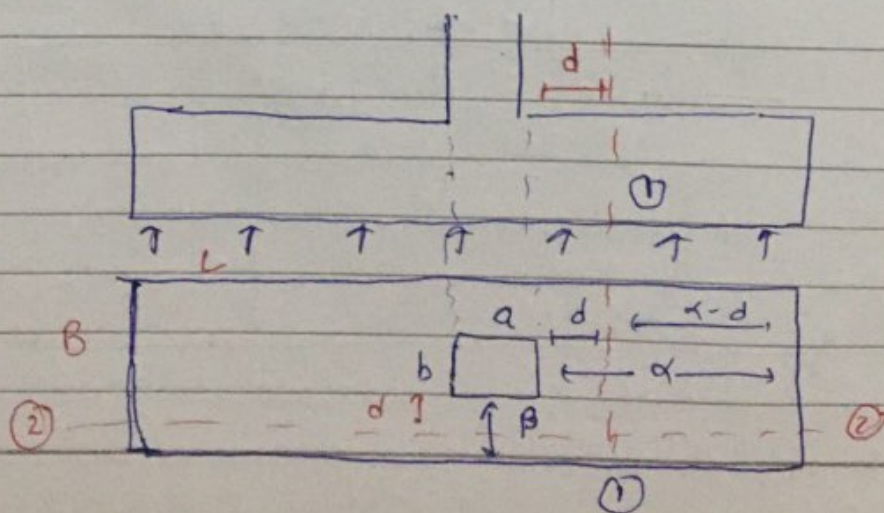
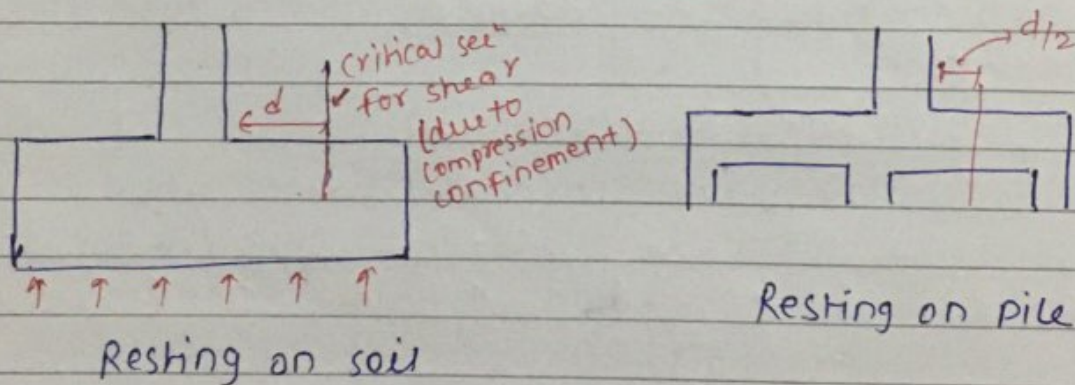
Two types of shear

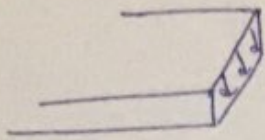
- 1) One way shear / Flexural shear
- 2) Two way shear / punching shear

① one-way shear:-

The critical secⁿ for shear is

- a) at a distance d from the face of wall/column
When the footing is resting on soil
- b) at a distance $d/2$ from the face of wall/column
When the footing is resting on piles





Nominal shear stress $\tau_{v1} = \frac{W(B-d)}{B \cdot d}$

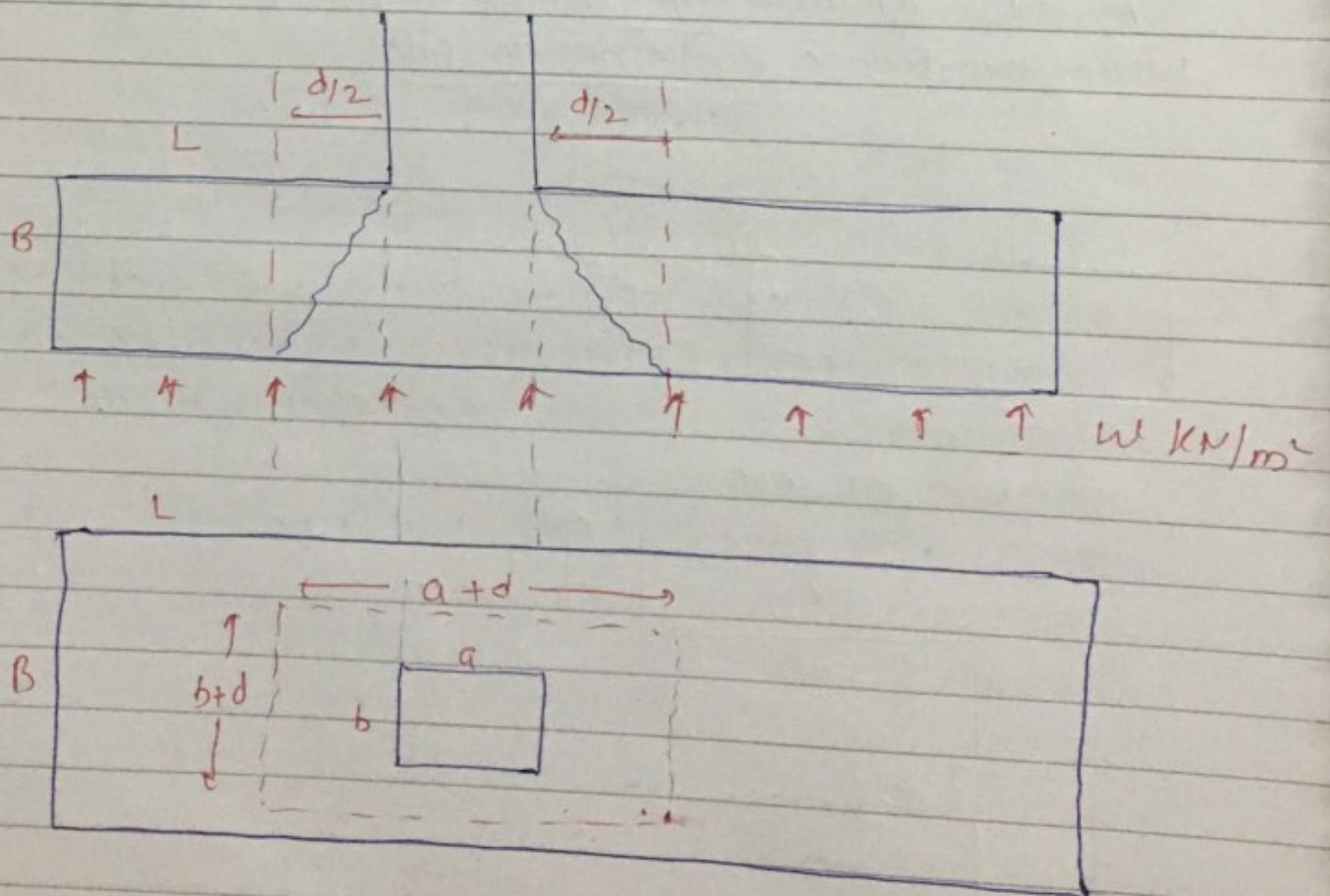
$\tau_v \leq k \tau_c$

$\tau_{v2} = \frac{WL(B-d)}{Ld}$

As footing thickness is governed by shear, for preliminary design take $k=1$ & $\tau_c = 0.32 \text{ MPa}$ corresponding to a P_t value to 0.2 % for both M20 & M25

TWO-WAY shear:-

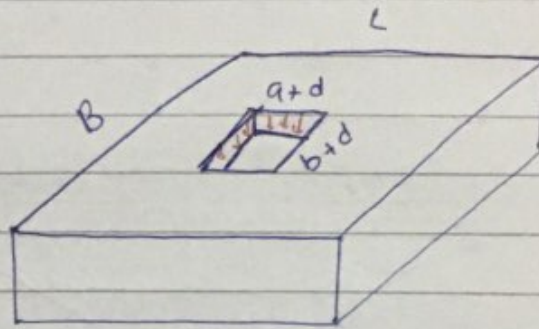
Two way or punching shear shall be checked around the column on a perimeter half the effective depth of footing slab away from the face of column or pedestal



$$\tau_{v(\text{two way})} = \frac{w(LB - (a+d)(b+d))}{2[(a+d) + (b+d)]d}$$

Design shear strength in case of two way shear
 $= K_s \tau_c'$

$$\tau_c' = 0.25 \sqrt{f_{ck}}$$



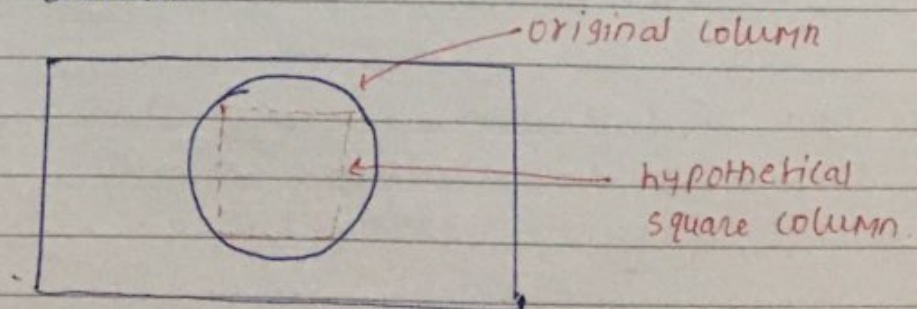
$$K_s = 0.5 + \beta$$

≥ 1.0

$\beta = \frac{\text{shorter dimension of column}}{\text{longer dimension of column}}$

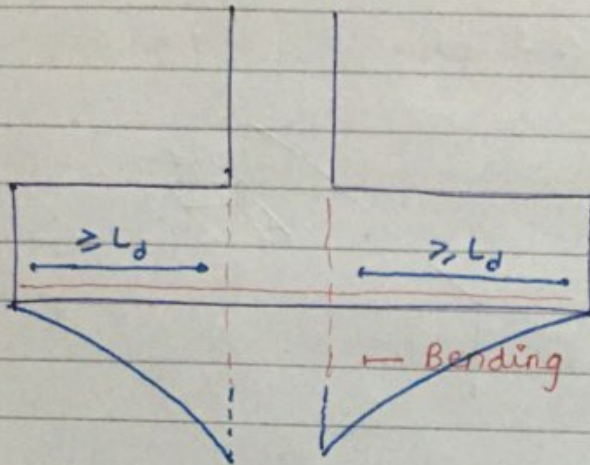
Two way shear strength is not dependent on
 % Tensile R/F

NOTE for the purpose of calculating stress in footing which supports a round or octagonal concrete column or pedestal, the face of column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or rectangular pedestal or column.



Critical secⁿ for Bond :-

The critical secⁿ for Bond i.e. check for development length shall be done @ same section that has been considered for Bending.



Tensile RIF :-

provided in the same way as discussed in slab, all the criteria related to RIF in solid slab are also valid for footing slab.

In one way Bending condition Main RIF is provided in the dirⁿ of Bending (Plane of Bending) and Nominal RIF is provided in the other direction (like in case of strip footing)

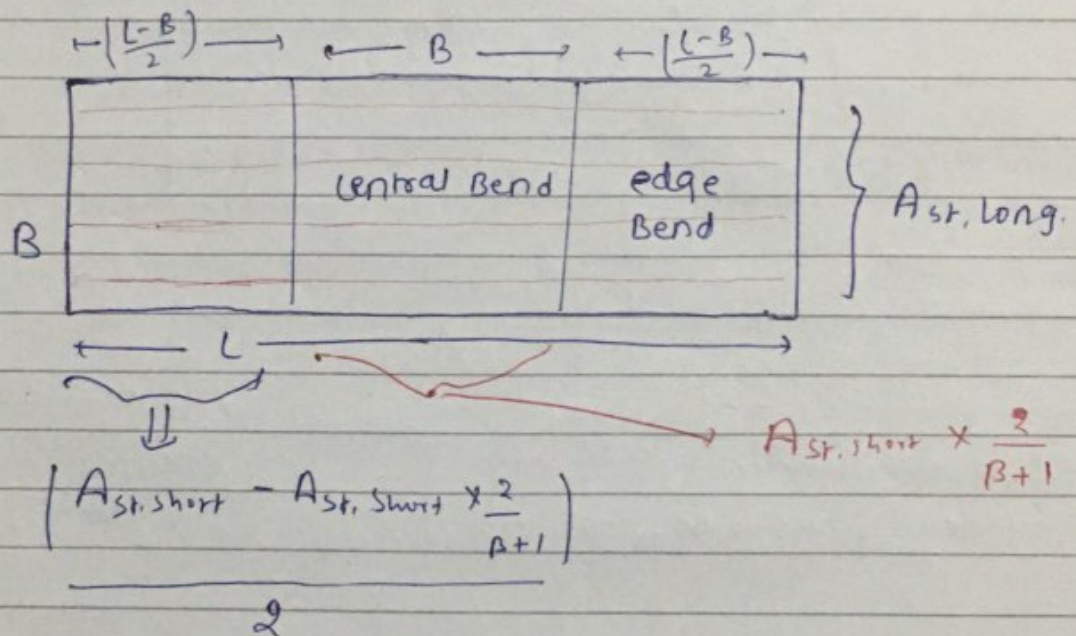
Distribution RIF is provided to take care of Differential settlement, shrinkage and Temp effects.

In case of Two way RIF square footing RIF in each dirⁿ shall be distributed equally.

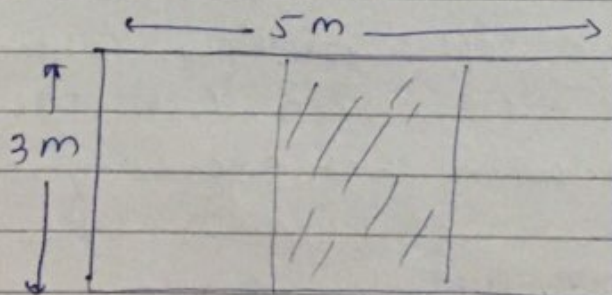
In Two way Rectangular footing, RIF in the longer dirⁿ shall be uniformly distributed across the full width. and in the shorter direction, the total length is divided into central bend and edge bends.

$$\text{The RIF in central bend} = A_{st, short} \times \frac{2}{\beta + 1}$$

$$\beta = \frac{\text{longer dimension of footing}}{\text{shorter dimension of footing}} = \frac{L}{B}$$



Ex



$$A_{st, long} = 2010 \text{ mm}^2$$

$$A_{st, short} = 1340 \text{ mm}^2 \quad , \quad A_{st} \text{ in the central Band} = A_{st, short} \times \frac{2}{\beta + 1}$$

$$= 1005 \text{ mm}^2$$

from unitary method the R/F required is $1340 \times \frac{3}{5}$
 $= 804 \text{ mm}^2$

However from local criteria we need to provide 1005 mm^2

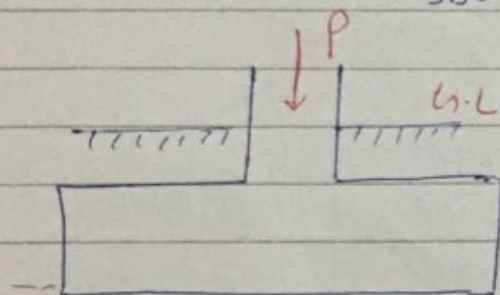
NOTE :: ultimate Bearing capacity (X)

Gross SBC

Net SBC

ultimate bearing capacity.

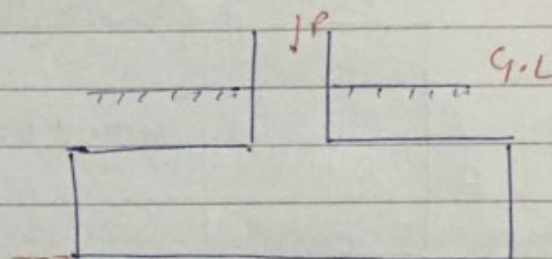
$$\text{SBC} = \frac{\text{ultimate bearing capacity}}{\text{FOS (2.5 to 3)}}$$



300

Gross SBC

(soil overburden + difference in density has to be considered)



280

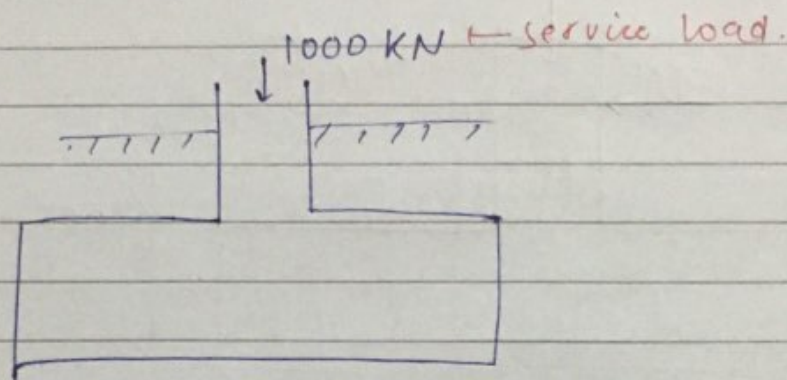
Net SBC

only difference ^{has to be} ~~has to be~~ considered in density (soil & concrete)

To account for the soil overburden or difference in density b/w concrete and soil, multiply column service load by 1.1 - 1.2 when Gross SBC is given and 1.05 - 1.1 when Net SBC is given.

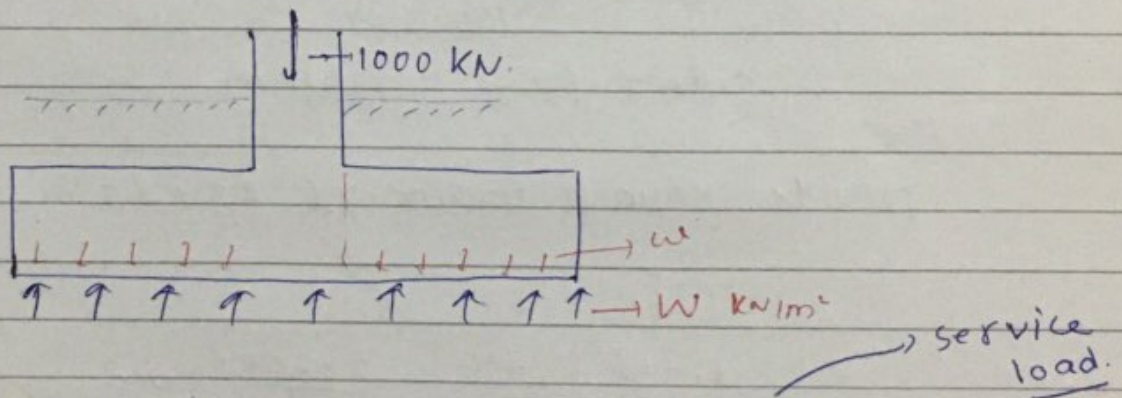
$$\text{Area of footing} = \frac{P \times (1.1)}{\text{Gross SBC}} \quad \text{or} \quad \frac{P \times 1.05}{\text{Net SBC}}$$

plan dimension of footing is calculated Based on service load of column. However structural component like thickness of footing, Amount of R/F etc. are designed based on Net factored upward intensity of soil.



$$\text{Gross SBC} = 100 \text{ kN/m}^2 \rightarrow (\text{ultimate} = 250 \text{ kN/m}^2)$$

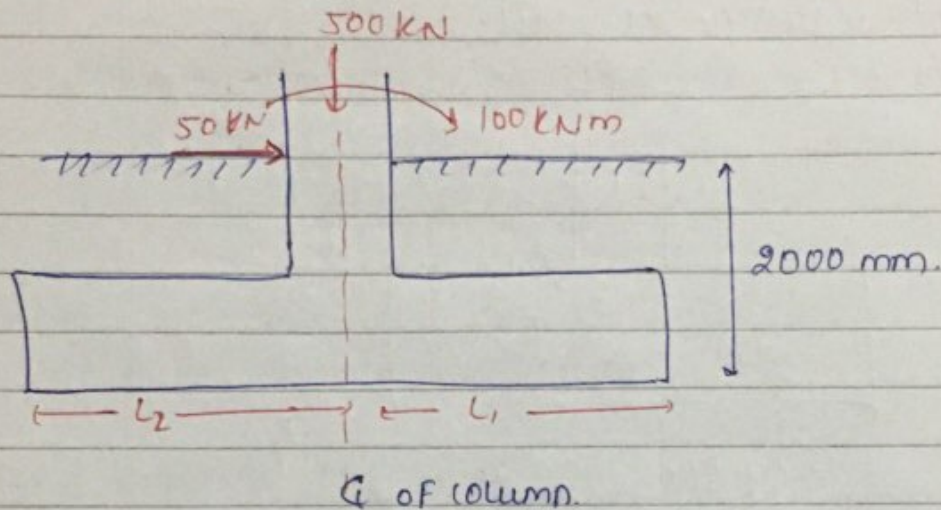
$$\text{Area} = \frac{1000 \times 1.1}{100} = 11 \text{ m}^2$$



$$\text{B.M for (W-W)} \rightarrow \text{Net intensity} = \frac{P \times 1.5}{\text{Area}}$$

Que. An Isolated footing is transferring load from a column as shown in figure. arrange the plan dimension of footing so that there will be uniform soil pressure intensity.
following parameters are given.

- 1) Column size $300 \times 300 \text{ mm}$
- 2) SBC $\rightarrow 100 \text{ kN/m}^2$
- 3) Dead wt. of footing and soil over it may be taken as 10% of vehicle load of column.



$$\text{Area} = \frac{500 \times 1.1}{100} = 5.5 \text{ m}^2$$

$$\text{side} = \sqrt{5.5} = 2.34 \text{ m.}$$

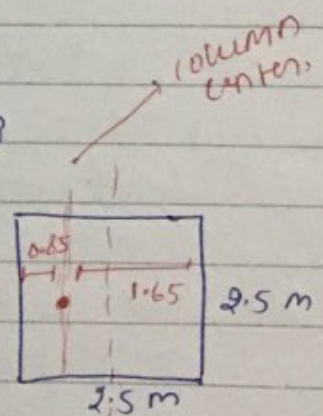
provide square footing of $2.5 \times 2.5 \text{ m}$.

$$L_1 + L_2 = 2.5$$

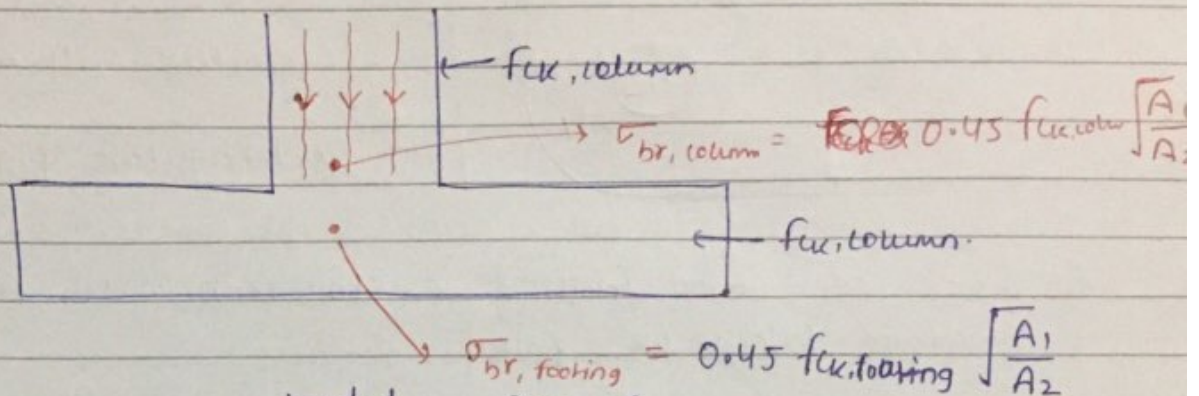
$$L_1 - L_2 = \frac{2M}{P} = \frac{2 \times 200}{500} = 0.8$$

$$\Rightarrow L_1 = 1.65 \text{ m}$$

$$L_2 = 0.85 \text{ m}$$



Transfer of load at Base of Column:-

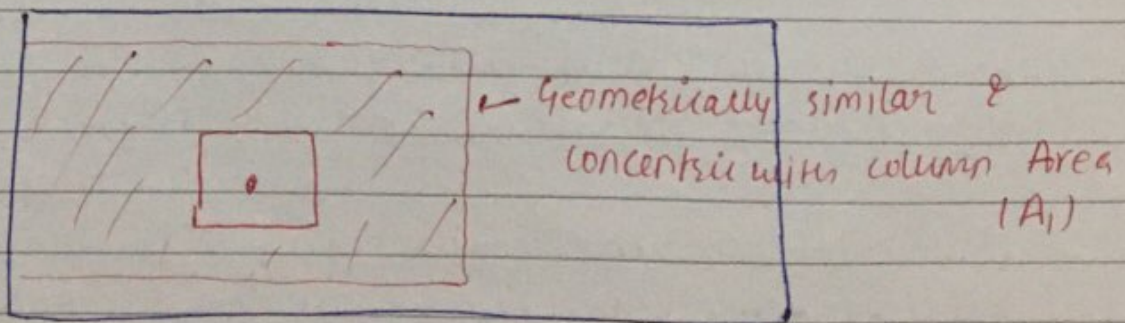
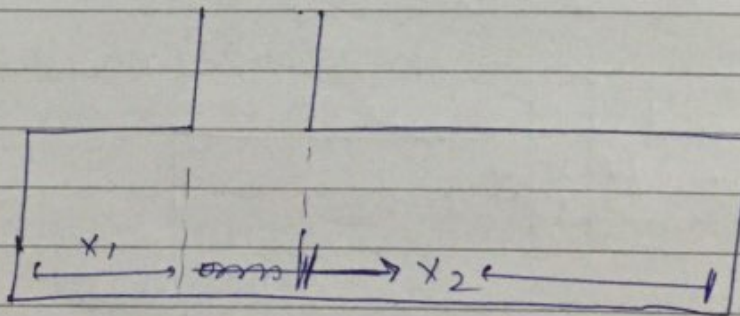


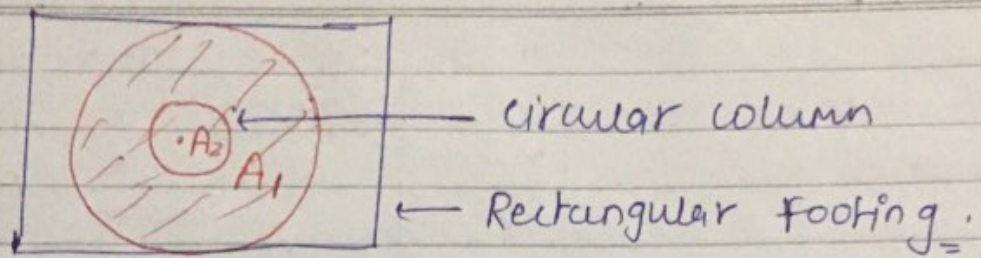
loaded Base of.
 $A_2 \rightarrow$ Area of column

$A_1 \rightarrow$ Area of Max. supporting Area of footing Geometrically similar and concentric with the loaded Area.

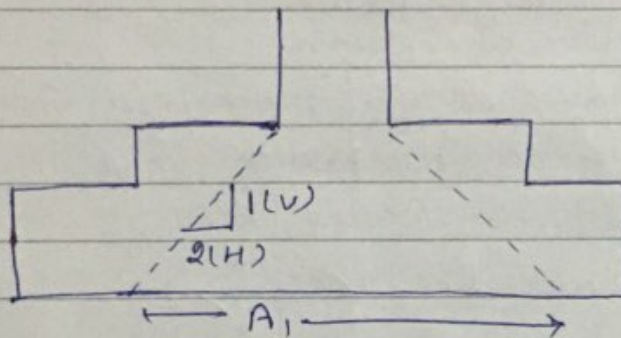
$\sqrt{\frac{A_1}{A_2}}$ can not be taken > 2

In the column side $A_1 =$ Area of column
 and hence $\frac{A_1}{A_2} = 1$

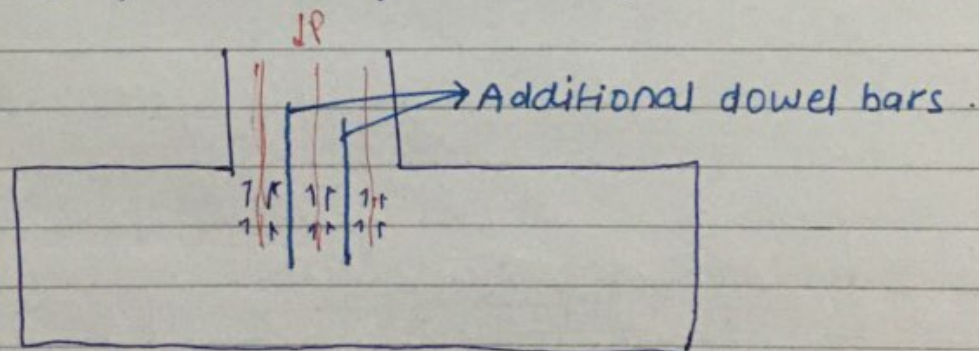




In case of ~~step~~ ^{step} footing or step footing the Area A_1 shall be taken as follows.



If the permissible bearing stress in concrete either in column or in footing is exceeded, R/F is provided to resist the additional force through bond action that is by extending the Bars.

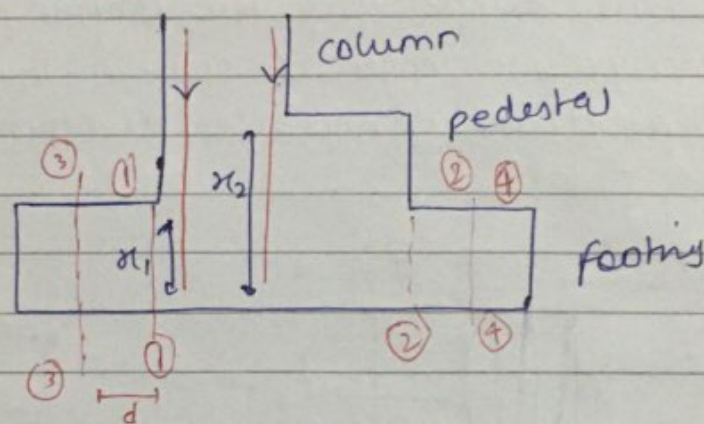


- The additional R/F shall be either in the form of longitudinal R/F of column or additional dowel Bars.
- Sufficient development length shall be available for transferring the additional load above & below the junction of footing & column.

If pedestal is provided then sufficient development length shall be available beyond the junction of column and pedestal.

In all cases the development length shall be calculated considering the permissible stress

NOTE: purpose of pedestal



- ① To increase the distance available for development of stress in column RIF
- ② The critical section for shear changes and Design SF ↓
④ instead of ③
- ③ The critical section for Bending changes and design BM ↓
② instead of ①

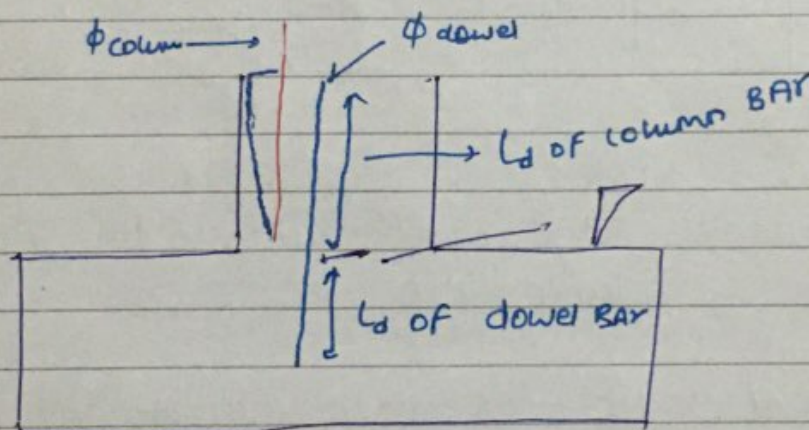
Minimum Area of extended longitudinal RIF or dowel (combined) shall be 0.5% of the supported column or pedestal

Minimum 4 bars shall be provided @ interface.

Where dowel Bars are used, Dia of dowels shall not exceed Dia of column Bar by more than 3 mm.

Column Bars of Dia > 36 mm in compression can only be dowled with Bars of smaller Dia.
(with equivalent Area)

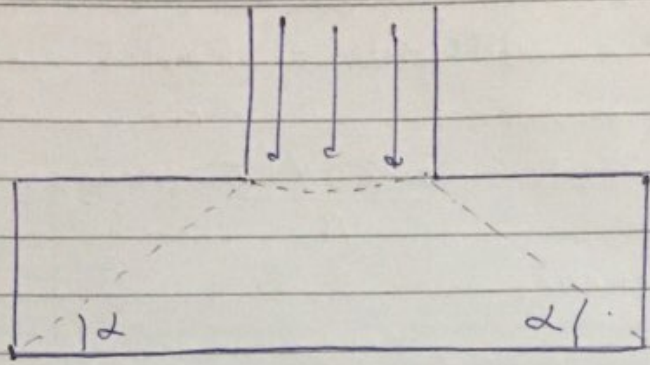
The dowel shall extend into the column a distance equal to development length of column Bar and into the footing a distance equal to development length of dowel Bar.



plan concrete footing :-

When the column is lightly loaded without any bars in Tension and base area of footing is relatively low, plan concrete footing can be provided. However σ_r criteria shall satisfy @ interface.

$$\tan \alpha \leq 0.9 \sqrt{\frac{100 q_a}{f_{cu}} + 1}$$

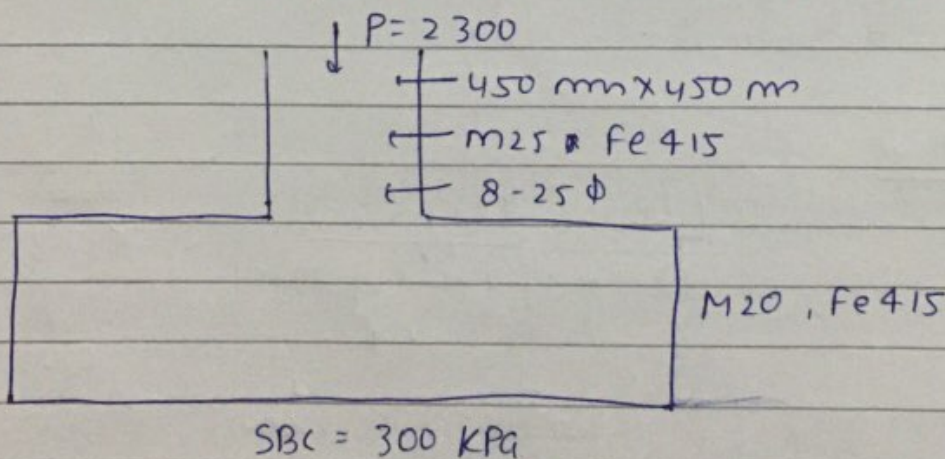


$q_a \rightarrow$ bearing capacity of soil

Que: A column of size 450×450 mm with 8-25 ϕ Bars carry a service load of 2300 kN.

Gross SBC = 300 kPa at a depth of 1.5 m below the Ground Surface. M20, Fe 415 for footing and M25, Fe 415 $\rightarrow$ column.

- 1) Calculate footing Area
- 2) Calculate thickness required
- 3) No. of bars required in each dirⁿ (Take $\phi = 16$ mm)
- 4) Check for L_d ~~and~~
- 5) Check adequacy of Bearing capacity @ interface



$$i) \text{ Area} = \frac{P \times 1.1}{\text{SBC}} = \frac{2300 \times 1.1}{300} = 8.43 \text{ m}^2$$

Design Square Footing of 3×3 m.

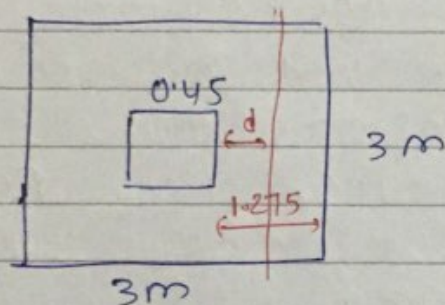
ii) Net factored intensity = $\frac{P \times 1.5}{A}$

$$= \frac{2300 \times 1.5}{9} = 383.33 \text{ kN/m}^2$$

from one way shear.

$$\tau_v \leq K \tau_c \rightarrow 0.32 \text{ N/mm}^2 \quad (P_t \text{ assumed} = 0.2\%)$$

assuming $D \geq 300 \text{ mm}$



$$\frac{2300 \times 3 \times (1.275 - d)}{3 \times d} \leq 0.32 \times 10^3 \times 10^6$$

$$d \geq 0.694 \text{ m}$$

$$d > 694 \text{ mm}$$

Two Way Shear

$$\frac{W(LB - (a+d)(b+d))}{2[(a+d) + (b+d)]d} \leq K_s \tau_c$$

$$383.33 \left(3 \times 3 - (0.45 + \cancel{0.694})(0.45 + \cancel{0.694}) \right)$$

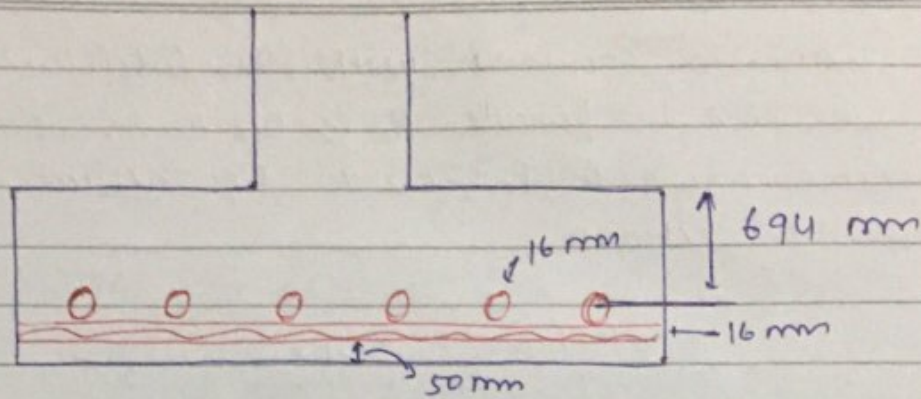
$$2 \times \left[(0.45 + \cancel{0.694}) \frac{d}{d} + (0.45 + \cancel{0.694}) \frac{d}{d} \right] \frac{d}{d}$$

$$\leq 1 \times 1.1180$$

$$d \geq 0.625 \text{ m} \quad (625 \text{ mm})$$

$$0.25 \sqrt{f_{ck}} \leq \tau_c$$

20



$$694 + \frac{16}{2} + 16 + 50 = 768 \text{ mm}$$

$$D = 770 \text{ mm}$$

$$d = 770 - \left(16 + \frac{16}{2} + 50 \right)$$

$$d = 696 \text{ mm}$$

$$M_{1-1} = W \times 1.275 \times 3 \times \frac{1.275}{2} = 934.73 \text{ kNm}$$

$$= 383.33 \times 1.275 \times 3 \times \frac{1.275}{2} = 934.73 \text{ kNm}$$

$$A_{st} = \frac{f_{cu}}{2f_y} \left[1 - \sqrt{1 - \frac{4.6M}{f_{cu} b d^2}} \right] b d$$

$$= \frac{20}{2 \times 415} \left[1 - \sqrt{1 - \frac{4.6 \times 934.73 \times 10^6}{20 \times 3000 \times 696^2}} \right] 3000 \times 696$$

$$A_{st} = 3870.44 \text{ mm}^2$$

$$\tau_c \cdot P_t (0.2 \cdot 1) \rightarrow A_{st} \text{ for } P_t = 0.2 \cdot 1$$

$$= \frac{0.2}{100} \times b d = \frac{0.2}{100} \times 3000 \times 696$$

$$= 4176 \text{ mm}^2$$

$$= 4176$$

$$L_d < \frac{m}{V} + l_0$$

if we want to continue with the calculated depth then we need to provide $P_t \geq 0.2\%$ as we've assumed τ_c corresponding to $P_t = 0.2\%$ (In actual we can revise the section)

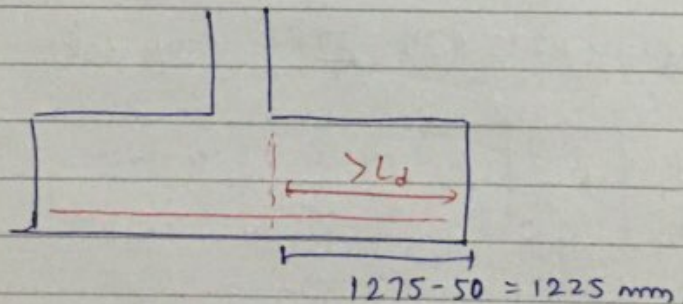
$$\text{No. of bars} = \frac{4177}{\frac{\pi}{4}(16)^2} = 20.7 = \underline{\underline{21 \text{ bars}}}$$

$$\text{spacing} = \frac{3000 - 2(50 + 16/2)}{20} = 145.40 \text{ mm}$$

spacing of main Rf shall not be greater than $(3d, 300) \text{ mm}$

so, Provide 21 No. of bars on each dirⁿ at equal spacing.

(iv)



$$L_d = 47 \phi = 47 \times 16 = 752 \text{ mm} < 1225 \text{ mm} \quad \underline{\underline{O.K.}}$$

$$\text{Total force to be transferred} = 1.5 \times 2300 = 3450 \text{ kN}$$

$$\sigma_{br, column} = 0.45 f_{cu, column} \sqrt{\frac{A_1}{A_2}} = 11.25 \text{ N/mm}^2$$

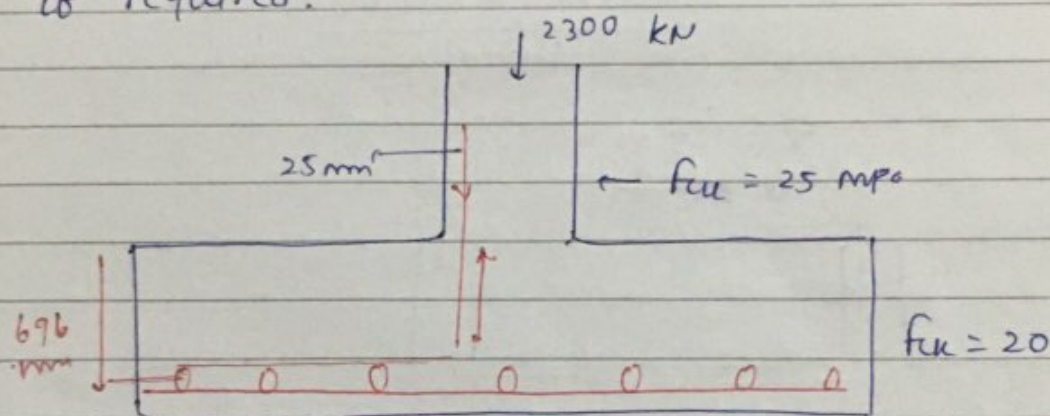
$$\sigma_{br, footing} = 0.45 f_{cu, footing} \sqrt{\frac{A_1}{A_2}} = 18 \text{ N/mm}^2$$

$\times 2.0$

So in bearing column side is critical

Total load that can be transferred through bearing
 $= 11.25 \times 450^2 = 2278 \text{ KN}$

that implies we need to transfer a additional force
 of $3450 - 2278 = 1172 \text{ KN}$ through extension of
 column longitudinal bars + additional dowel bars
 is required.



$$\frac{\pi}{4} \times \phi^2 \cdot f_{sc} = \pi \phi \tau_{bd} \cdot l$$

$$\tau_{bd} = 0.16 (f_{cu})^{2/3} \times 1.25 \times 1.6$$

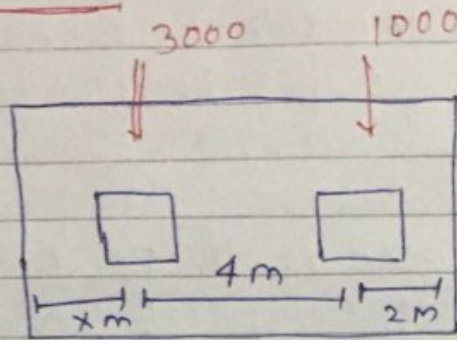
$$\frac{\pi}{4} \times 25^2 \times f_{sc} = 3.14 \times 25 \times 0.16 (20)^{2/3} \times 1.25 \times 1.6 \times (696 - 8)$$

$$f_{sc} = 259.59 \text{ N/mm}^2$$

$$0.67 f_y = 278.05 \text{ N/mm}^2$$

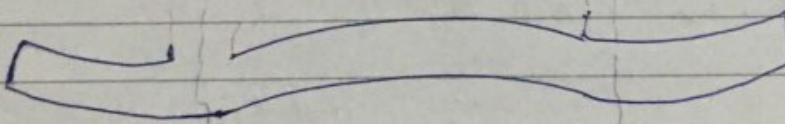
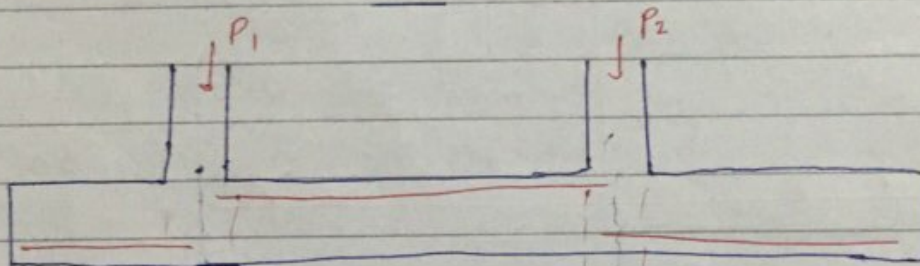
Max. stress that can be developed in a dowel bar/
 Extension of column bar = Min. ($0.67 f_y$, stress that
 can be developed with the available length)

COMBINED FOOTING :-

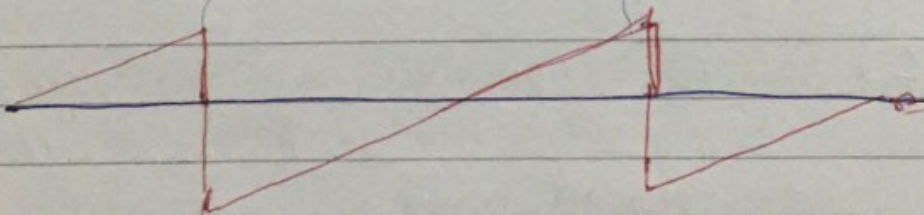


$$\bar{X} = \frac{3000 \times 6 + 1000 \times 2}{4000}$$

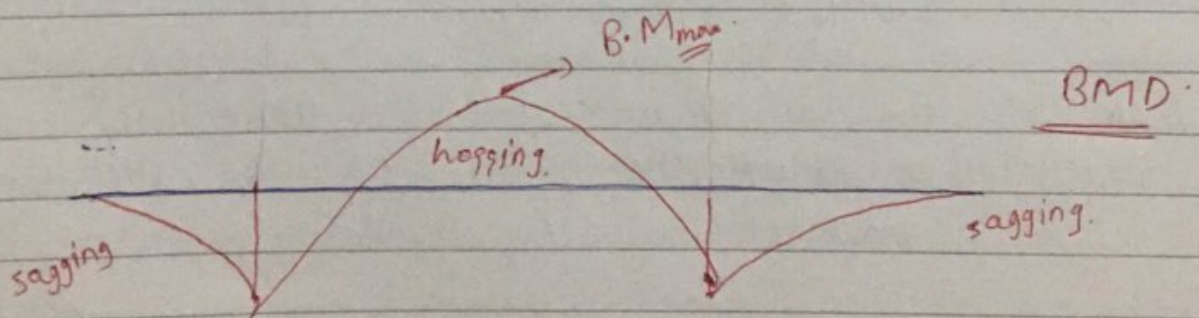
$$\bar{X} = \underline{\underline{5 \text{ m}}}$$



Deflected
shape



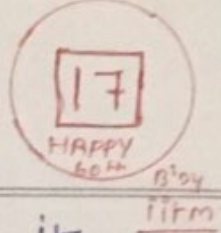
SFD



BMD

- The max. hogging moment occur b/w the two column at a location where SF=0

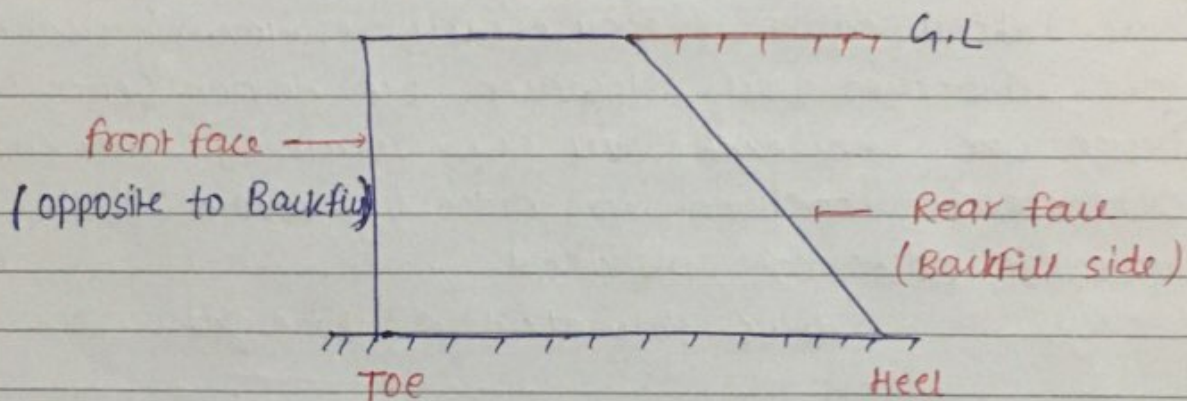
Retaining wall



The walls prevent the soil from taking its natural angle of repose and hence soil applies a lateral pressure

Types of Retaining wall :-

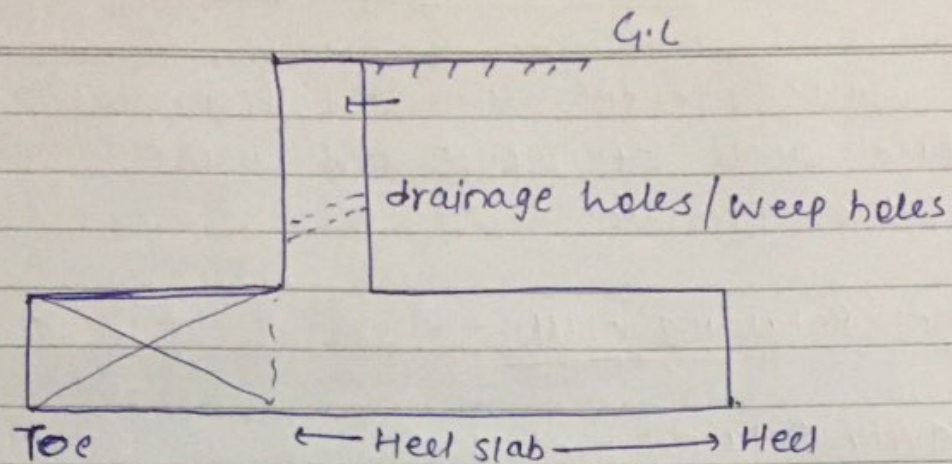
① Gravity Retaining wall :-



- stability by Gravity i.e by it's own wt. can be brick masonry or stone masonry or pcc
- Generally NO Tensile stress allowed on the base ($e \leq B/6$)
- Generally Not used for height $> 3m$ due to economical reason

② Semi-Gravity Retaining wall :-

- (a) cantilever
- (b) counterfort or buttress



NOTE

The lateral earth pressure will be max. in case of submerged soil. However we do not generally design for submerged soil the retaining wall for submergence condition in order to avoid it drainage holes are provided.

dry < saturated < submerged

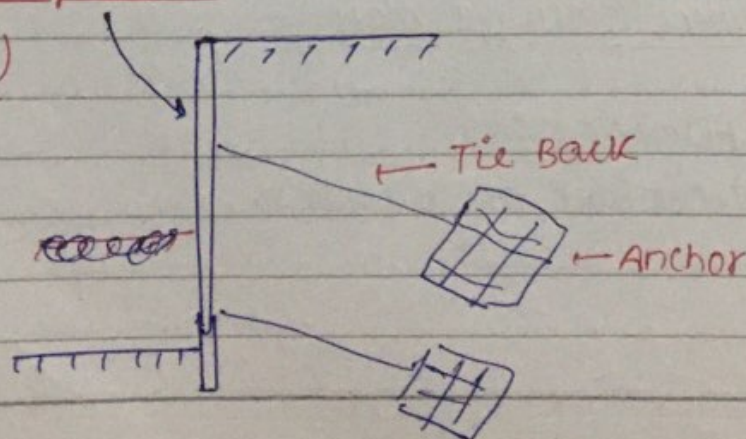
The most common type of retaining wall can be T or L shape.

Stability is maintained by its own wt. + soil above the heel slab.

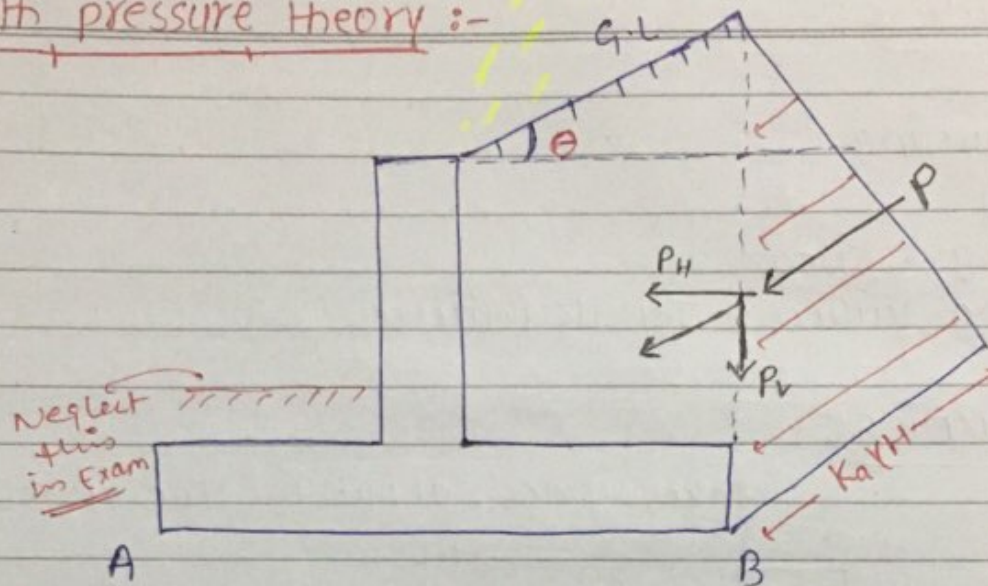
economical upto a height of 8 m

③ Flexible Walls :-

(Sheet pile)



Earth pressure theory :-



$$K_a = \cos \theta \left[\frac{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}} \right]$$

for $\theta = 0$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

ϕ → angle of friction

for the full development of passive pressure it is necessary that the soil is undisturbed. However, during the construction the soil will be disturbed and hence in general we can neglect the passive soil.

Factors to be considered for selection of Type of Retaining Wall :-

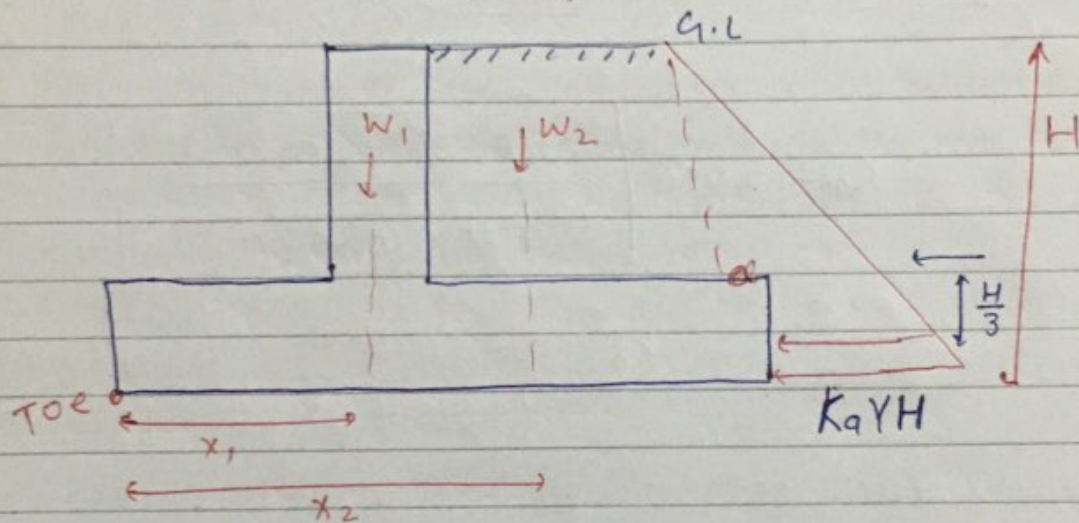
- ① Height
- ② SBC
- ③ space available
- ④ Time available
- ⑤ Ground water table
- ⑥ Material
- ⑦ Economy
- ⑧ Any other factor

Type of Failure :-

- ① Over turning
- ② sliding
- ③ Bending & shear
- ④ Bearing failure & Tensile failure

① Over turning :-

takes place about the Toe of retaining wall



$$FOS_{\text{overturning}} = \frac{\sum M_R}{\sum M_o} = \frac{W_1 x_1 + W_2 x_2}{\frac{1}{2} K_a \gamma H^2 (H/3)}$$

$FOS \geq 2.0$ (for Natural loading)	IS 19458 - Code of retaining wall
$FOS \geq 1.5$ (for EQ loading)	

② sliding :-

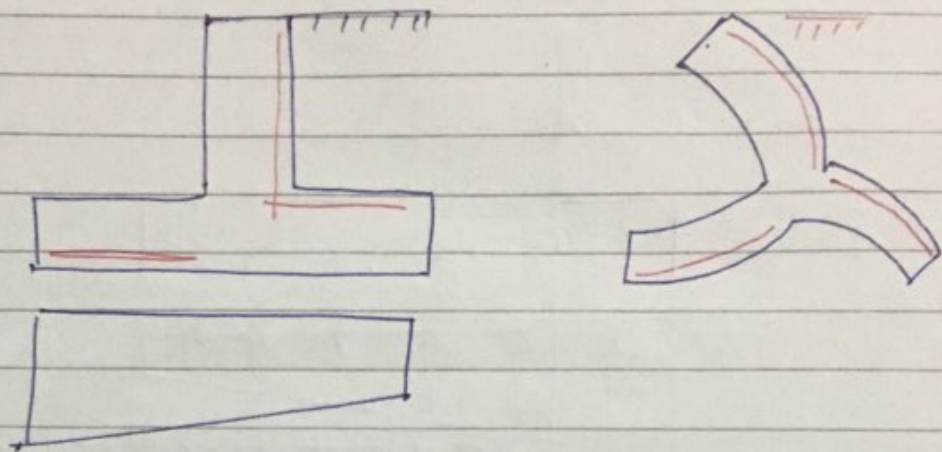
$$FOS_{\text{sliding}} = \frac{UN}{\frac{1}{2} K_a \gamma H^2}$$

$\nearrow (W_1 + W_2)$

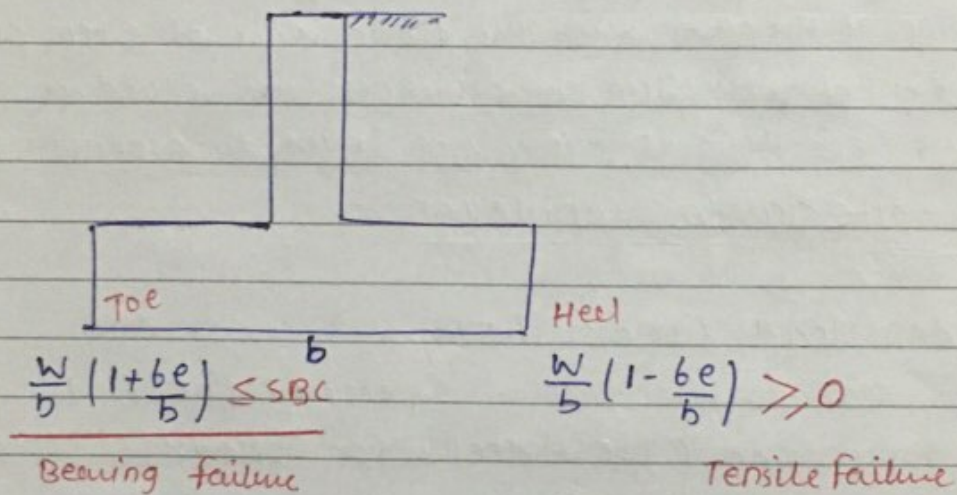
*

$FOS \geq 1.5$ (for Normal loading)
$FOS \geq 1.0$ (EQ)

③ Bending & shear:-

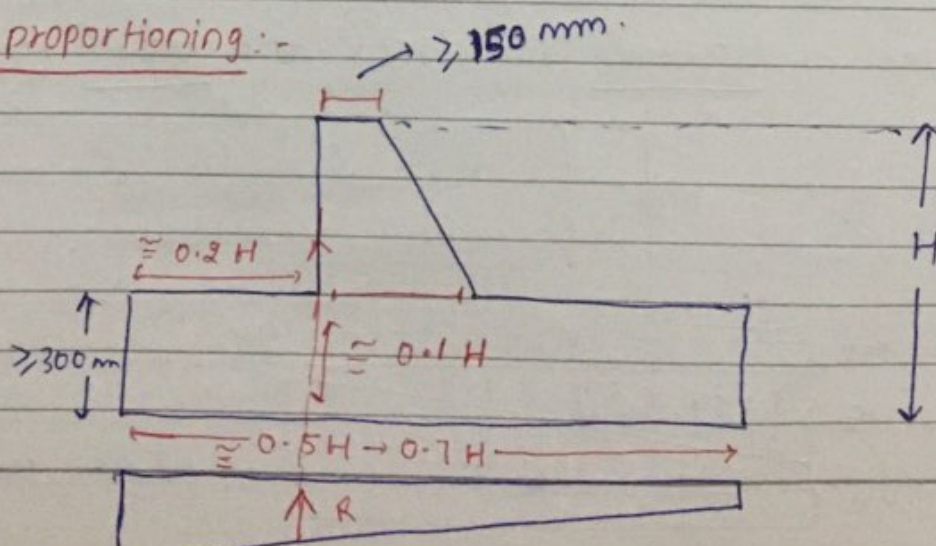


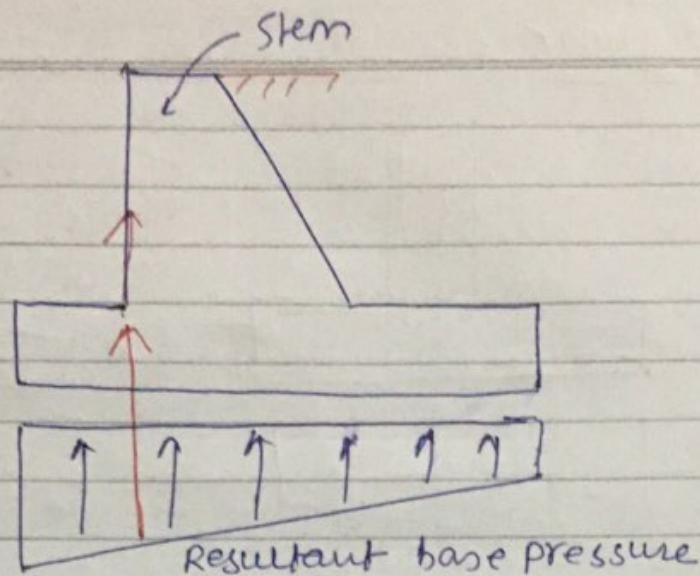
④ Bearing Failure and Tensile failure:-



DESIGN OF CANTILEVER Retaining Wall:-

General proportioning:-





NOTE: An economical design of retaining wall can be obtained by proportioning the base slab so as to align the resultant vertical soil reaction at the base with the front face of Stem slab

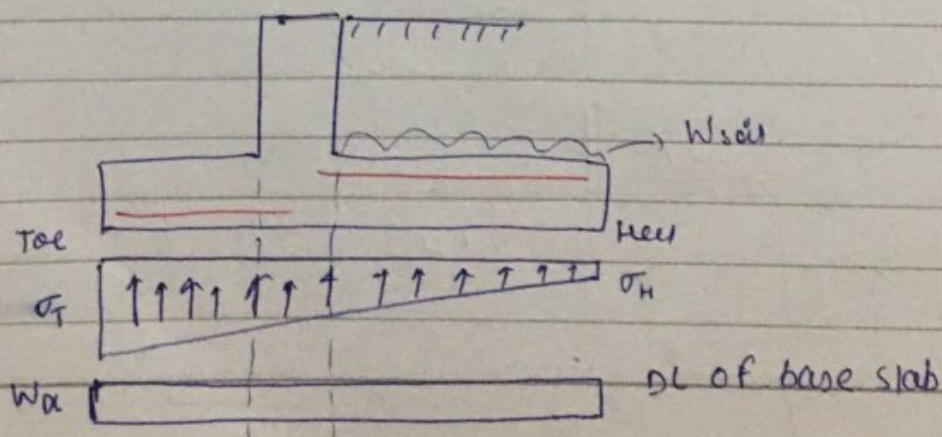
Due to change in BM in the Stem slab, thickness can be reduced if required curtailment of R/F is done at every $\frac{1}{3}$ rd level to economise the structure.

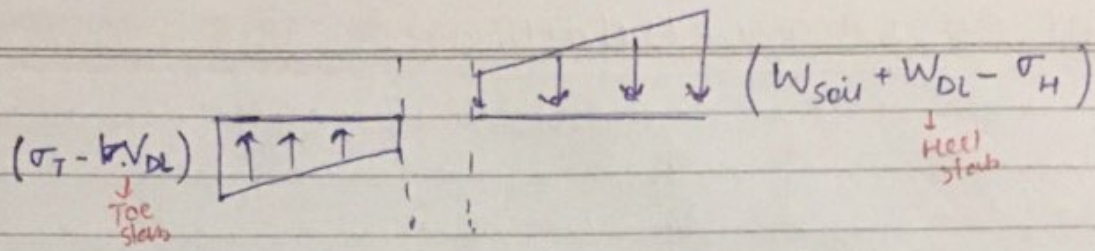
Design criteria for cantilever :-

- Consider load factor 1.5
- Stem slab is Generally Governed for BM and base slab (Toe + Heel) for shear

However we don't provide shear R/F, if required increase the thickness or grade of concrete to ensure $\tau_v \leq K\tau_c$.

- Cover of 50 mm for Stem and 75 mm for Base slab





Stem slab is designed for BM due to lateral pressure and RIF provided near the backfill side

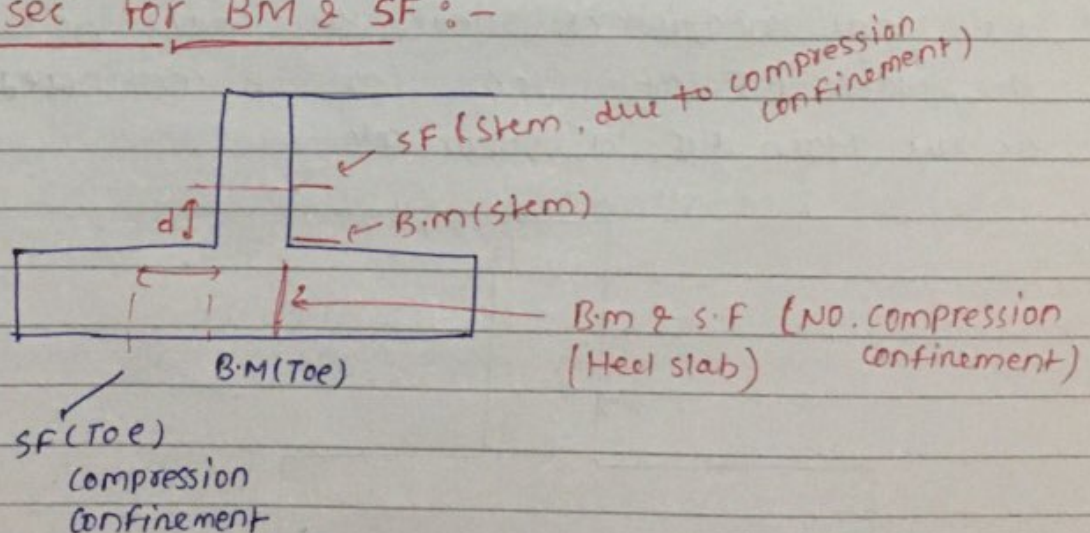
Toe slab is designed for upward soil pressure - DL of Toe slab

RIF provided @ Bottom surface

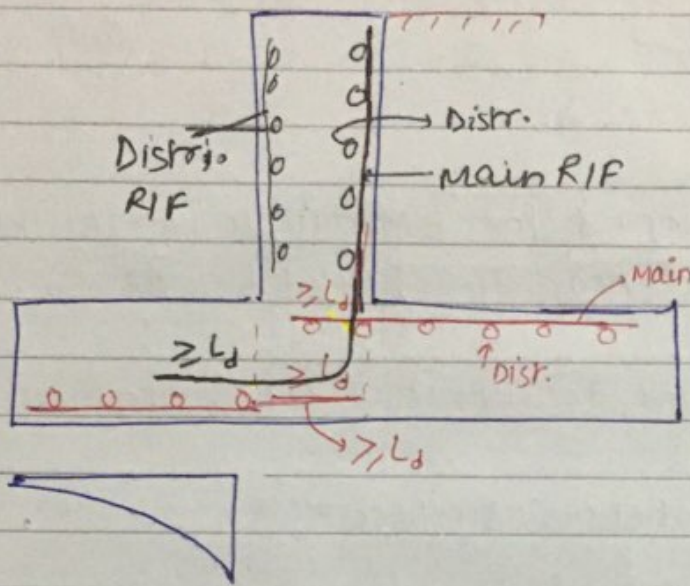
Heel slab is designed for Net downward load ($W_{soil} + W_{DL}$ of Heel slab - upward pressure) and RIF provided @ TOP surface

All the RIF criteria like min. RIF, spacing etc are same as solid slab

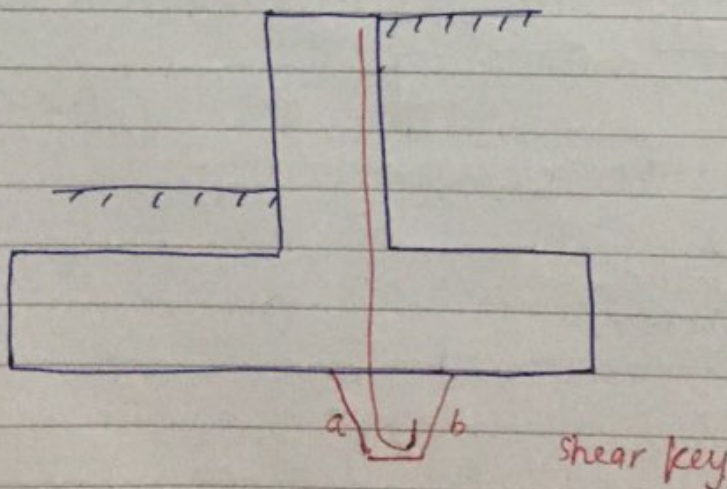
Critical secⁿ for BM & SF :-



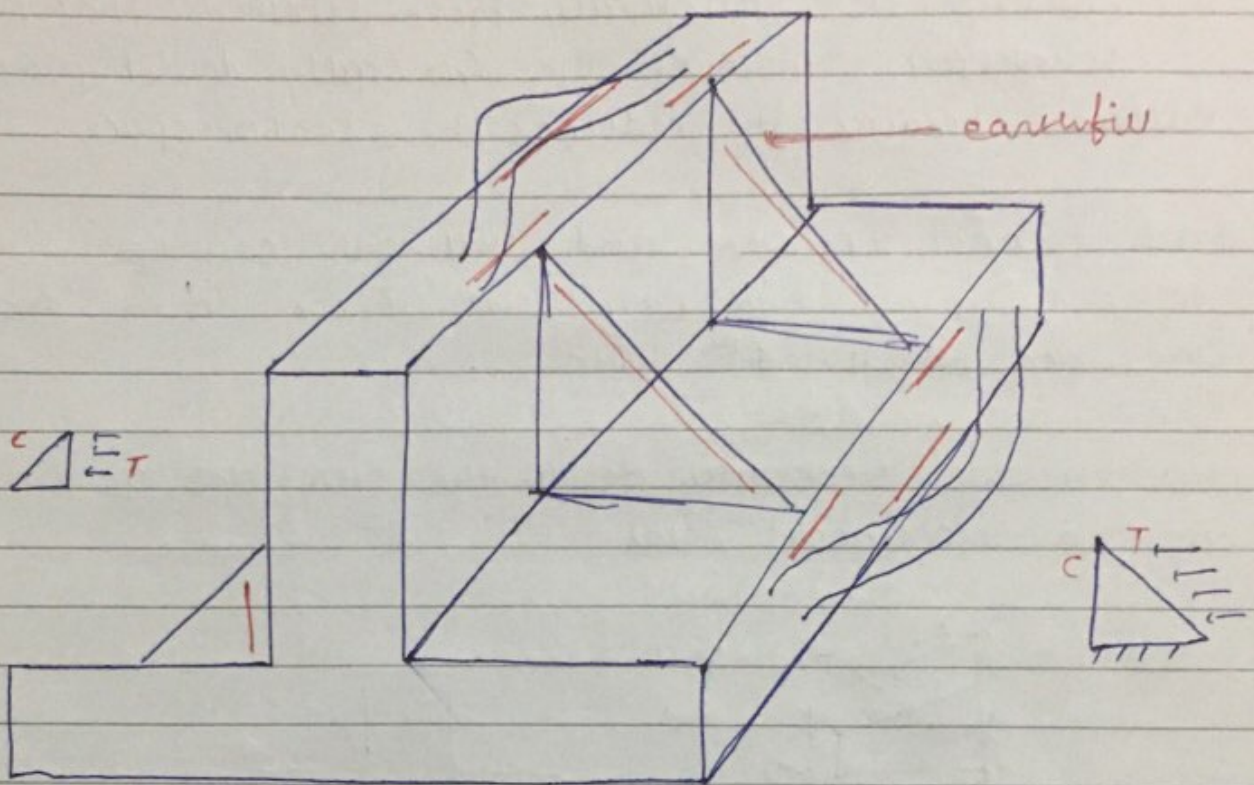
Typical RIF placement (Detailing):-



- Higher amount of Distribution RIF is Generally provided on the front face of stem due to higher variation of Temp.
- To increase the FOS in sliding, shear key is provided
- The ideal location of shear key shall be such that the Main RIF of Stem can be continued as the Main RIF of shear key



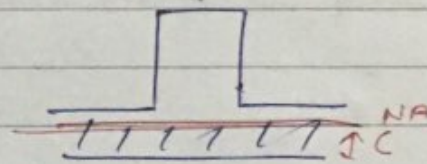
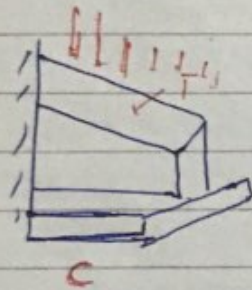
Counterfort and Buttrass :-



- for Higher Height cantilever is uneconomical due to higher BM
- Transverse support called counterforts are provided at a spacing of $\frac{1}{3}$ rd to $\frac{1}{2}$ of Height of wall
- Vertical Bending converted to Horizontal Bending in stem slab
- economical for Height above 8 m
- on soil side $\rightarrow$ counterfort / rear counterfort
on front side $\rightarrow$ Buttrass / front counterfort

NOTE Buttrass is structurally more efficient than counterfort. However we generally do not provide Buttrass because it reduces the usable space.

- Each pannel of stem and Heel slab can be designed as a two way slab fixed on 3 sides and free on the 4th side.
- However we generally design the stem slab as a one way continuous slab.



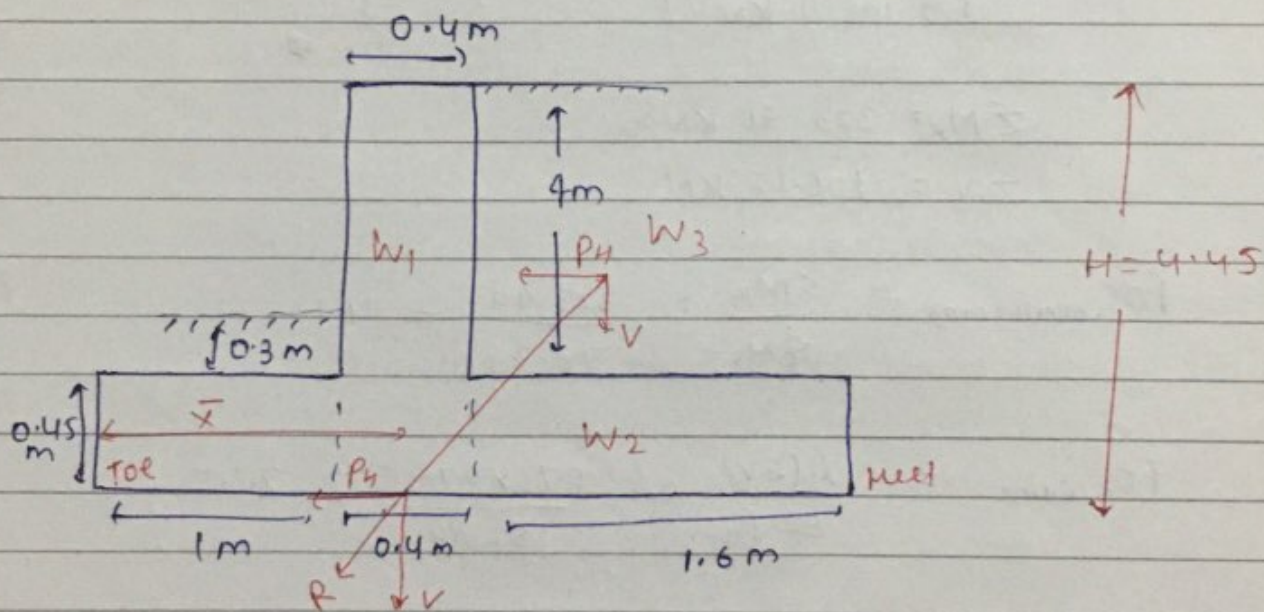
- Near the counterfort, main RIF in stem is closed to rear face and in the mid way the RIF is close to the outer face/ front face.
- In the rear counterfort, the main RIF is along the incline and in the front counterfort the main RIF is along the vertical face (sometimes vertical face is called as bottom face)
- The counterfort can be design as a T-beam of varying depths fix at the base by considering soil load from each side

However to be on the safer side this can be designed as a rectangular beam.

Que. for the retaining wall shown below
 i) find f.o.s against sliding & overturning
 ii) eccentricity associated with the resultant load
 iii) Min and Max stress at the bottom.
 iv) calculate moment about centroid of base slab
 Use the following Data

density of soil $\rightarrow 16 \text{ kN/m}^3$

$\phi = 30^\circ$, SBC $\rightarrow 170 \text{ kN/m}^2$, Coefficient of friction
 b/w concrete and soil $= 0.73$



$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1}{3} \quad \left\{ \phi = 30^\circ \right.$$

~~Horizontal force~~

$$P_H = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times \frac{1}{3} \times 16 \times 4.45^2$$

$$= 52.8 \text{ kN}$$

assuming a 1m length.

$$\sum M_o = P_H \cdot H/3$$

$$= 52.80 \times \frac{4.45}{3} = 78.33 \text{ kN/m}$$

Description	Load	lever arm (m)	Moment (kN-m)
stem slab (W_1)	$0.4 \times 4 \times 1 \times 25$ $= 40 \text{ kN}$	$1 + \frac{0.4}{2} = 1.2$	48
Base slab (W_2)	$0.45 \times 3 \times 1 \times 25$ $= 33.75 \text{ kN}$	$\frac{3}{2} = 1.5$	50.625
soil (W_3)	$1.6 \times 4 \times 1 \times 16$ $= 102.4 \text{ kN}$	$1 + 0.4 + \frac{1.6}{2} = 2.2$	225.28

$$\sum M_R = 323.90 \text{ kNm}$$

$$\sum V = 176.15 \text{ kN}$$

$$(i) \quad FOS_{\text{overturning}} = \frac{\sum M_R}{\sum M_o} = \frac{323.90}{78.33} = 4.135$$

$$FOS_{\text{sliding}} = \frac{H(\sum V)}{\sum P_H} = \frac{0.73 \times 176.15}{52.80} = 2.45$$

(ii)

$$\bar{X} = \frac{\sum M_R - \sum M_o}{\sum V} = 1.39 \text{ m}$$

$$e = \frac{b}{2} - 1.39 = 0.11 \text{ m.}$$

$$\sigma_{\text{Toe}} = \frac{\sum V}{b} \left(1 + \frac{6e}{b} \right) = 71.63 \text{ kN/m}^2$$

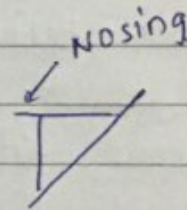
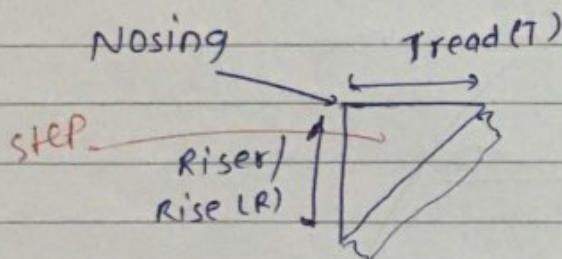
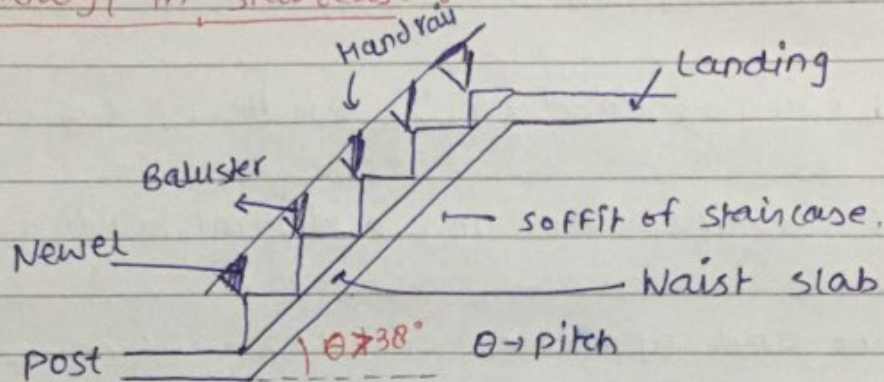
$$\sigma_{\text{heel}} = \frac{\sum V}{b} \left(1 - \frac{6e}{b} \right) = 45.79 \text{ kN/m}^2$$

$$\begin{aligned}\text{Moment about centroid} &= \sum v \cdot e \\ &= 19.37 \text{ kNm}\end{aligned}$$

Staircase

From structural point of view staircases consist of slab and beam.

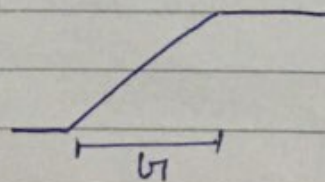
Terminology in staircase :-



① Flight:-

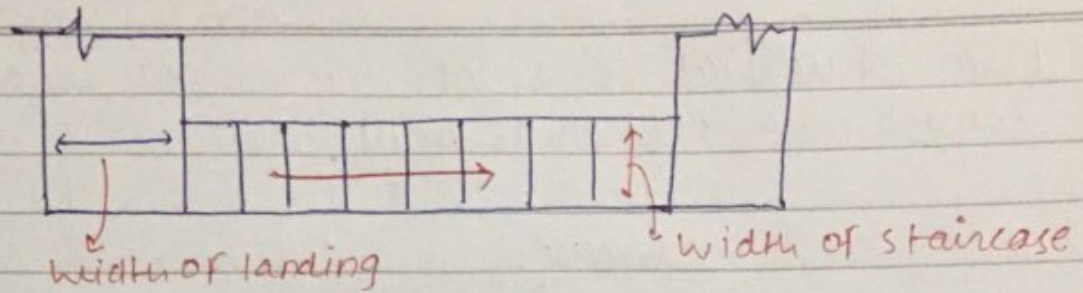
A series of steps b/w two landing

② Going:- (G) :- Horizontal projection of flight



③ Headroom :- The vertical distance of a line connecting the Nosing of all trades and the soffit. The Minimum distance shall be 2 m.

④ Winders:- steps used to change dirⁿ without landing. These are generally Triangular in plan (Like staircase in a overhead water tank)

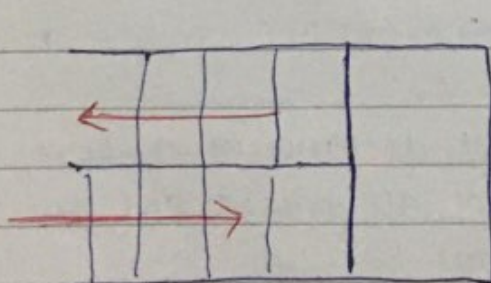


width of landing $>$ width of ~~staircase~~ staircase

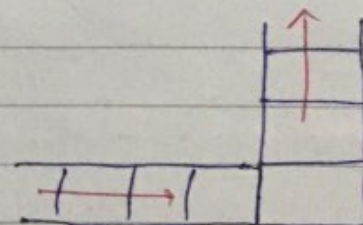
- width of service stairs or residential building = $1 - 1.2$ m.
- for office and apartment = $1.2 - 1.6$ m
- for public building = $1.2 - 2.0$ m.
- NO. OF STEPS shall be b/w 3-12
- Tread = $200 - 300$ mm.
- Riser = $150 - 200$ mm

$$2R + T = 580 \rightarrow 600 \text{ mm}$$

$$RT = 40000 \rightarrow 50000 \text{ mm}^2$$



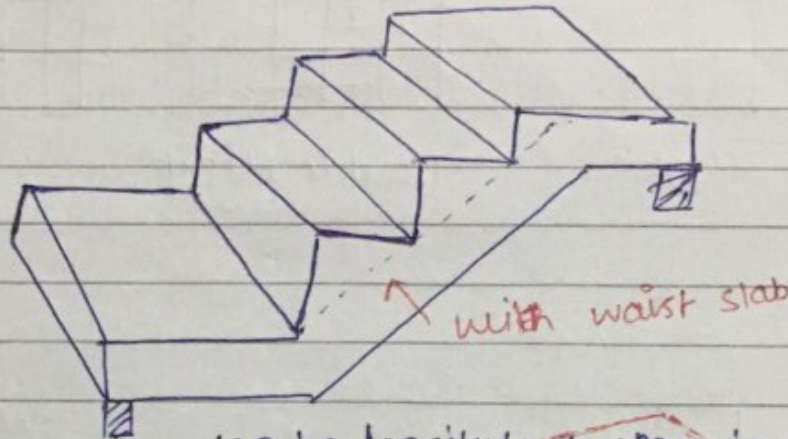
half landing (180° change)



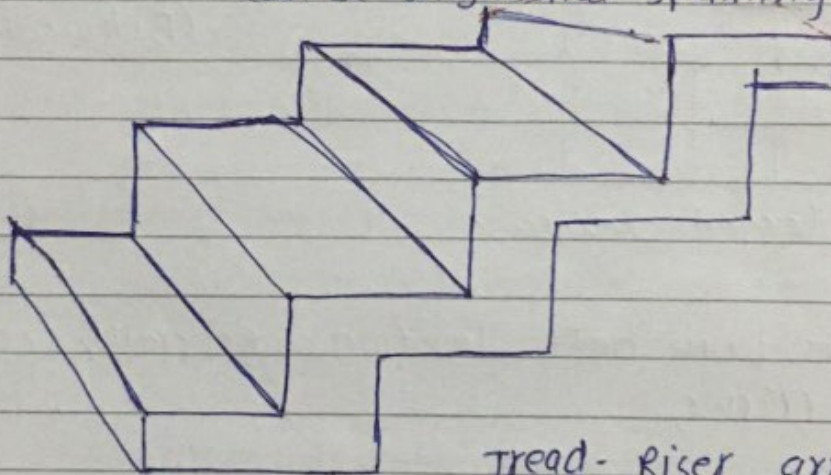
Quarter landing (90°)

Steps in staircase :-

- (i) With waist slab
- (ii) Tread - Riser arrangement
- (iii) Isolated tread

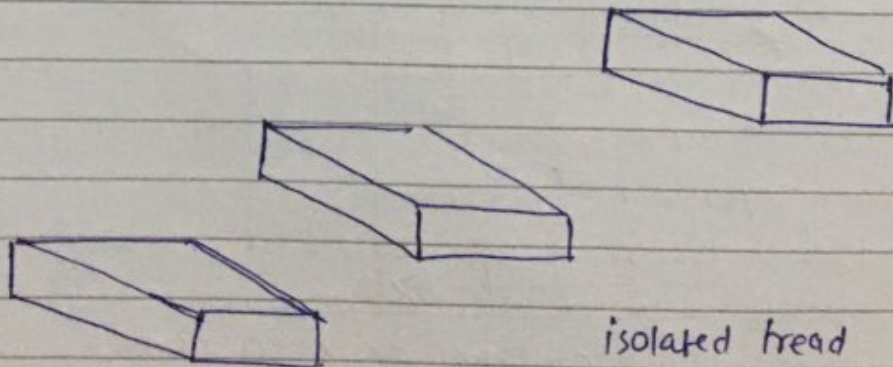


can be longitudinal spanning or transverse



Tread - Riser arrangement

can span longitudinally or transverse



isolated tread

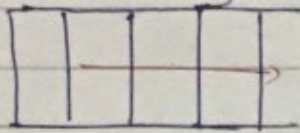
can span only, in the transverse
spanning.

Types of staircases :-

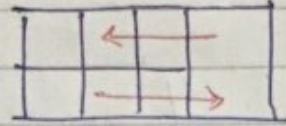
i) Based on Geometrical

ii) Based on Structural Configuration

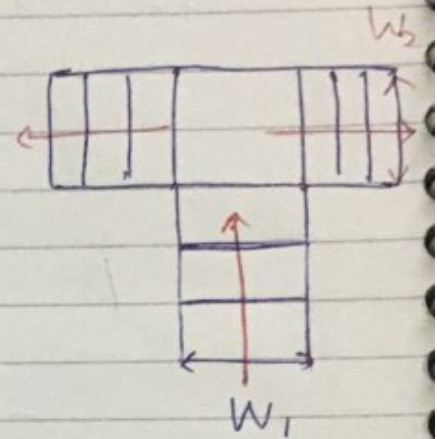
(i) Based on Geometrical



single flight

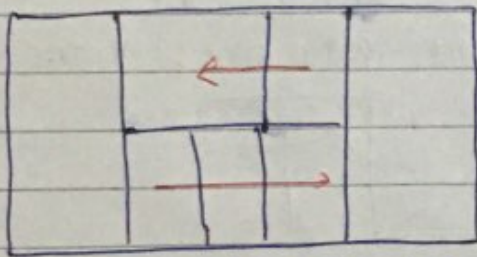


Double flight



$W_1 > W_2$

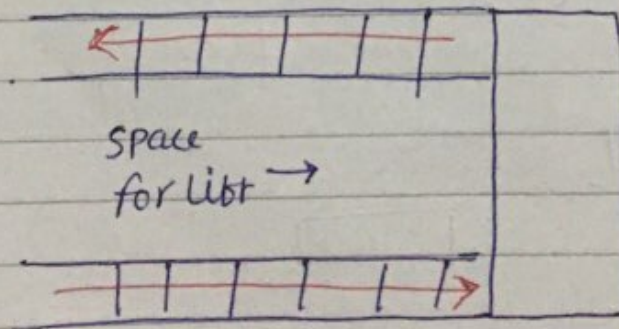
(Bifurcated staircase)



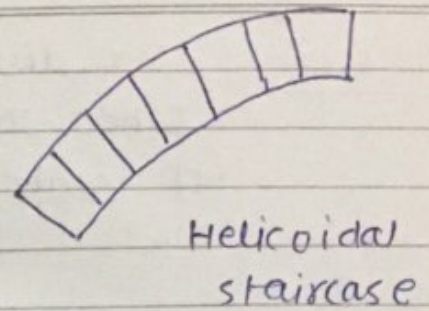
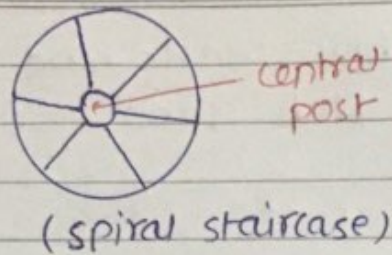
Dog-Legged staircase

→ Two flights with half landing, generally less space b/w two flights

→ Useful when space available is less and generally provided in residential building



open well or open newel



(ii) Based on structural configuration.

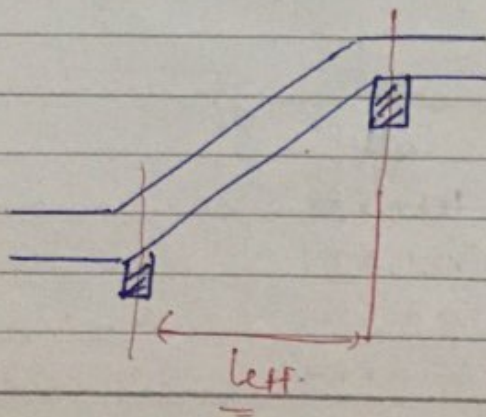
- longitudinal spanning (along inclined)
- Transverse spanning (along width)

Longitudinal spanning:-

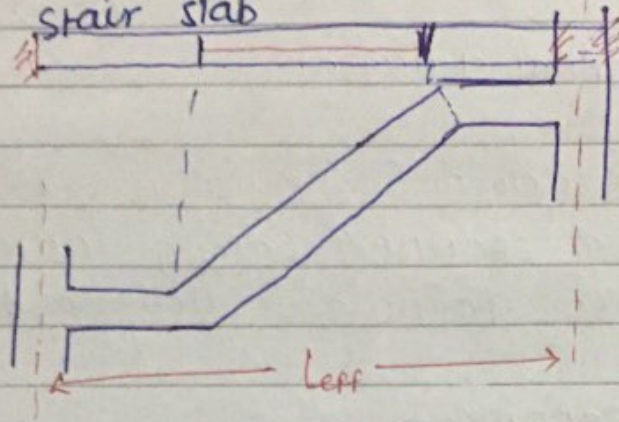
- When two or more supports are provided parallel to the riser at two or more locations causing the slab to bend longitudinally b/w the supports
(Maybe waist slab type or thread riser arrangement)
- The thickness depends on effective span and effective span shall be calculated as follows per IS: (code 456)

Case-(i)

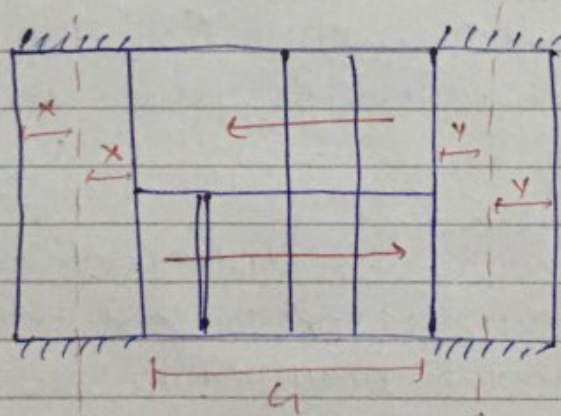
The Horizontal c-c distance of beams when slab is supported at top and bottom riser by beams spanning parallel with the riser.



case-(ii) C-c distance b/w walls and beam support
When loading slab span in the same dirⁿ
of stair slab



case-(iii) The horizontal distance equal to the going
of stair + at each end either half the
width of landing, 1 meter whichever is less
→ When the stair slab is spanning onto the edge
of landing slab which span parallel to the riser

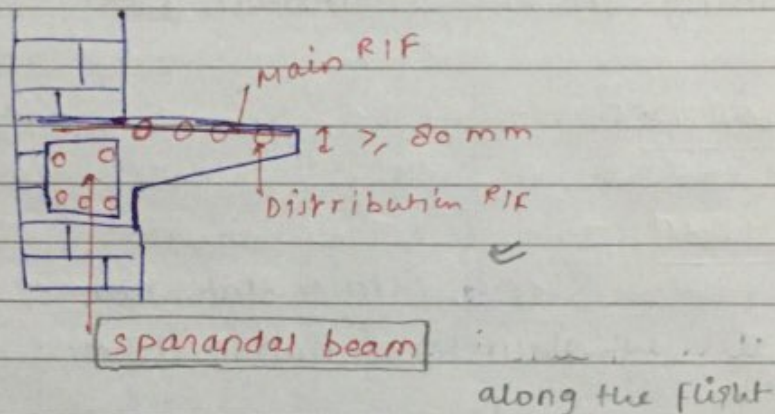


X	Y	Leff
< 1	< 1	$L + x + y$
< 1	> 1	$L + x + 1$
> 1	> 1	$L + 1 + 1$
> 1	< 1	$L + 1 + y$

B) Transverse spanning:-

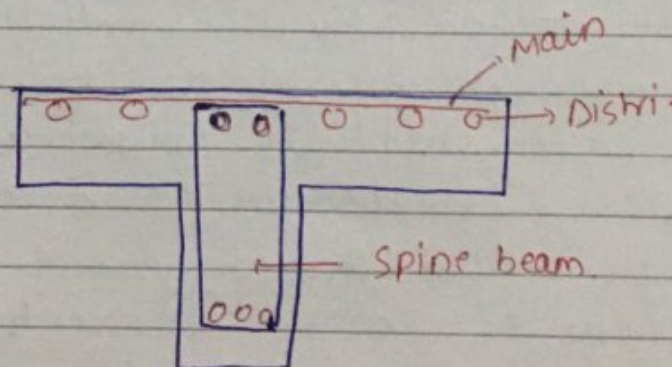
either the waist slab or the slab component or isolated tread slab or tread riser unit are supported on their sides or are cantilever along the width dirⁿ.

(i) case-(i) Slab cantilever from a RCC wall or spandrel beam

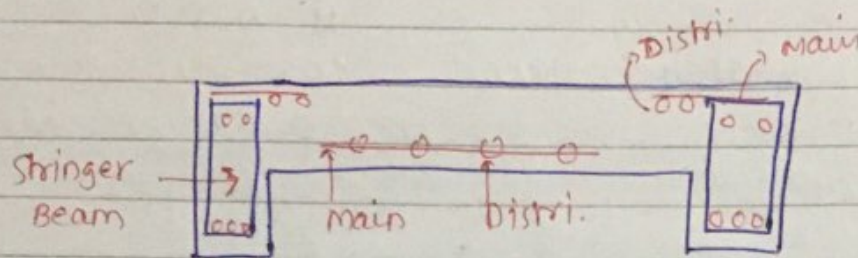


Spandrel beam is placed above a wall. all peripheral beams in a building can be called as spandrel beam.

(ii) case-(ii) Doubly cantilever from a central beam (spine beam)



Case (iii) slab supported b/w two stringer beam (or RCC Wall + one stringer beam)



NOTE: In all the above cases the beams are spanning in the longitudinal dirⁿ

Load component :-

(i) Dead load :-

steps, stair slab, beams, finishes, balusters, handrail etc.

(ii) Live load :-

- 5 kN/m^2 on the horizontal projection that is going and landing in general

- 3 kN/m^2 when overcrowding is unlikely and the floor live load does not exceed 2 kN/m^2 - Generally for residential building

- for structurally independent cantilever slab (Isolated tread) a concentrated load of 1.3 kN at the free end.

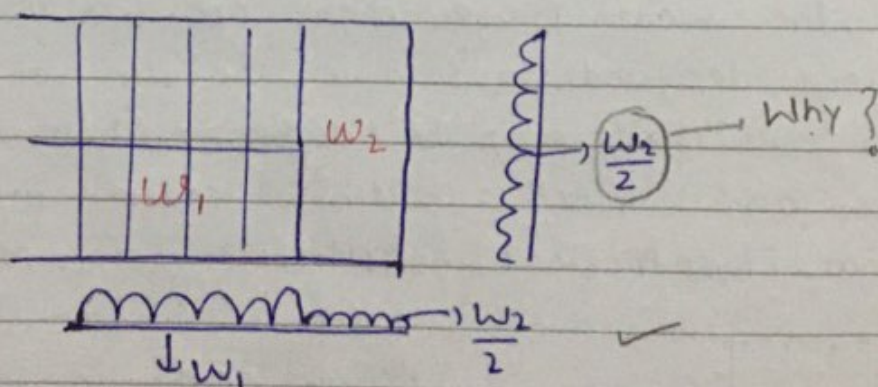
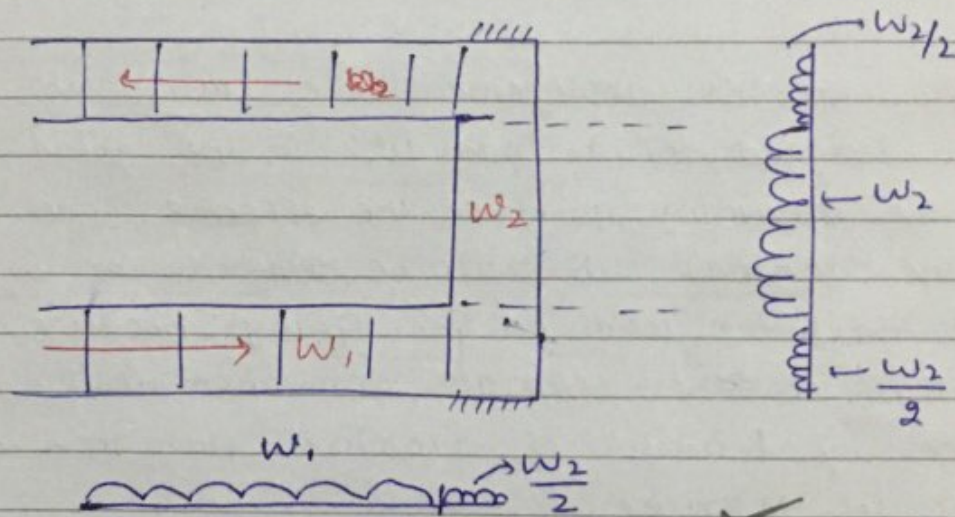
Max. BM due to UDL or concentrated load shall be considered for design in case of Isolated tread.

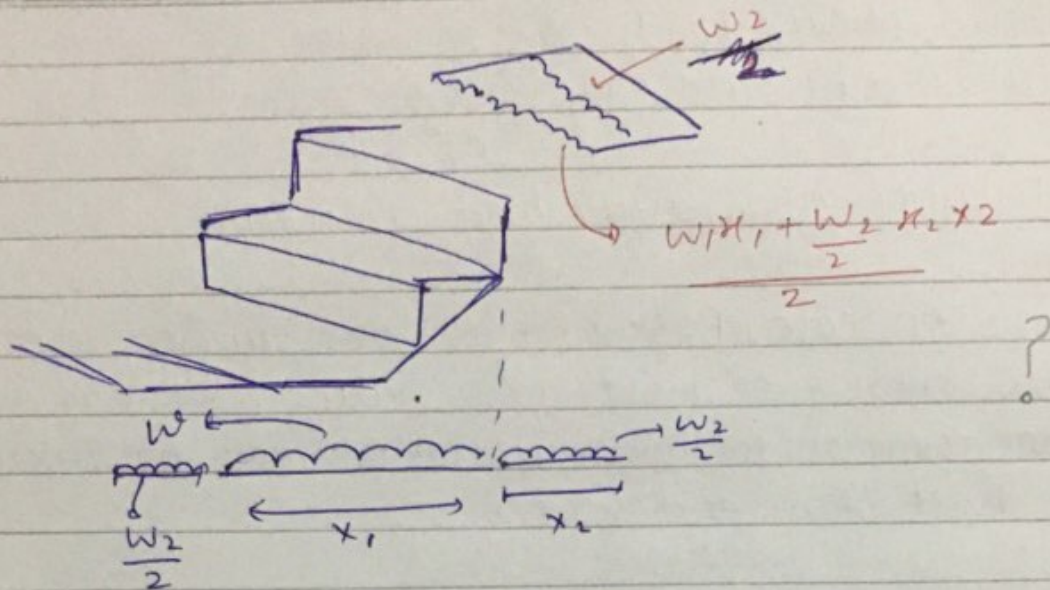
- All the above loads are characteristic load and shall be multiplied by load factor.

Distribution of Gravity load on special cases:-

Case-1

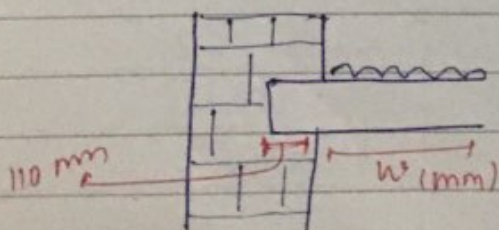
In case of stair with open well or when span partly crossing at right angle occurs, the load on areas common to any such span may be taken as one half in each dirⁿ.





NOTE In this arrangement the stair slab shall be designed for a load w_1 in the going $+ \frac{w_2}{2}$ in the remaining part of the effective span. The landing slab shall be designed for w_2 load $+ \frac{1}{2}$ the load on the going because the entire loading on the staircase is transmitted to the supporting edges by bending of landing slab in a dirⁿ parallel to riser

Case-2 When flights or landing are embaded into the Wall for a length not less than 110 mm and are designed to span in the dirⁿ of flight, a 150 mm strip may be deducted from the loaded area and effective width of section increased by 75 mm (hypothetical consideration) for design purpose.



Actual load $\rightarrow$ in W mm

load assume $\rightarrow$ in $(W-150)$ mm

Width assume $\rightarrow (W+75)$ mm

Load effect on different arrangement :-

① In Isolated plate slab :- can be designed as cantilever, simply supported, or fixed.

② Load effect on Waist slab :- span can be transverse or longitudinal.

- steps are Generally non-structural (Brick or pcc) However some nominal R/F provided to protect cracking of nosing.

a) For Transverse span :- Normal load component ($W \cos \theta$), cause the slab to bend in transverse plane and R/F provided at Top or Bottom depending on the support condition.

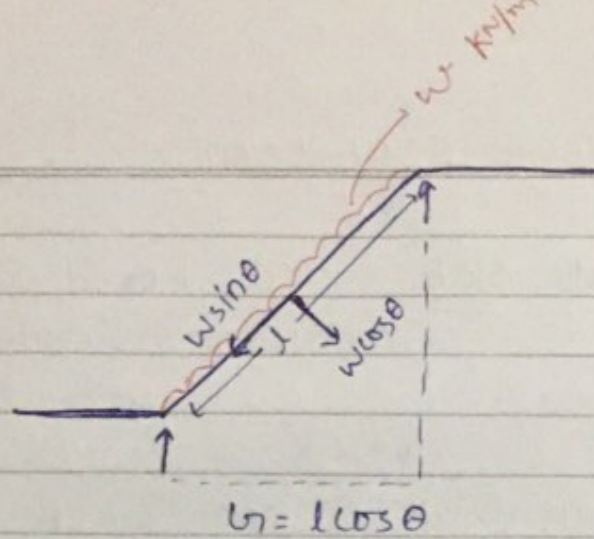
Tangential load ($W \sin \theta$), causes the slab to bend on its own plane but due to higher MOI, very small stresses will be induced and we don't generally design it.

b) For spanning longitudinally :-

Calculate the thickness of slab from deflection criteria (can be calculated by L/d criteria)

The Normal load component will cause bending in vertical plane containing the span direction. (parallel to longitudinal axis of slab)

- The $W \sin \theta$ component will try to compress the slab
- Also the BM can be calculated by considering the entire vertical load acting on the projected horizontal length. (m)



$$\text{B.M considering 'l'} = \frac{w \cos \theta l^2}{8}$$

$$\text{Total load} = wl$$

$$\text{projected span } l_{\text{proj}} = \frac{wl}{l} = \frac{wl}{l \cos \theta} = \frac{w}{\cos \theta}$$

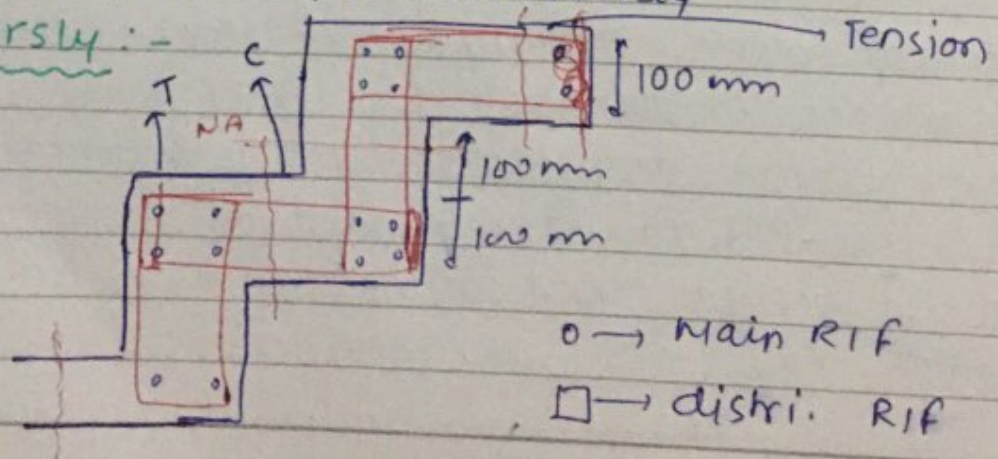
$$\text{B.M considering 'l' } = \frac{\frac{w}{\cos \theta} \cdot l^2}{8} = \frac{w l^2 \cos^2 \theta}{8 \cos \theta}$$

$$\text{B.M} = \frac{w \cos \theta l^2}{8}$$

③ Load effects on trade-Riser Arrangement :-

- The slab is repeatedly folded and analysis is difficult
- can span longitudinally or Transversely

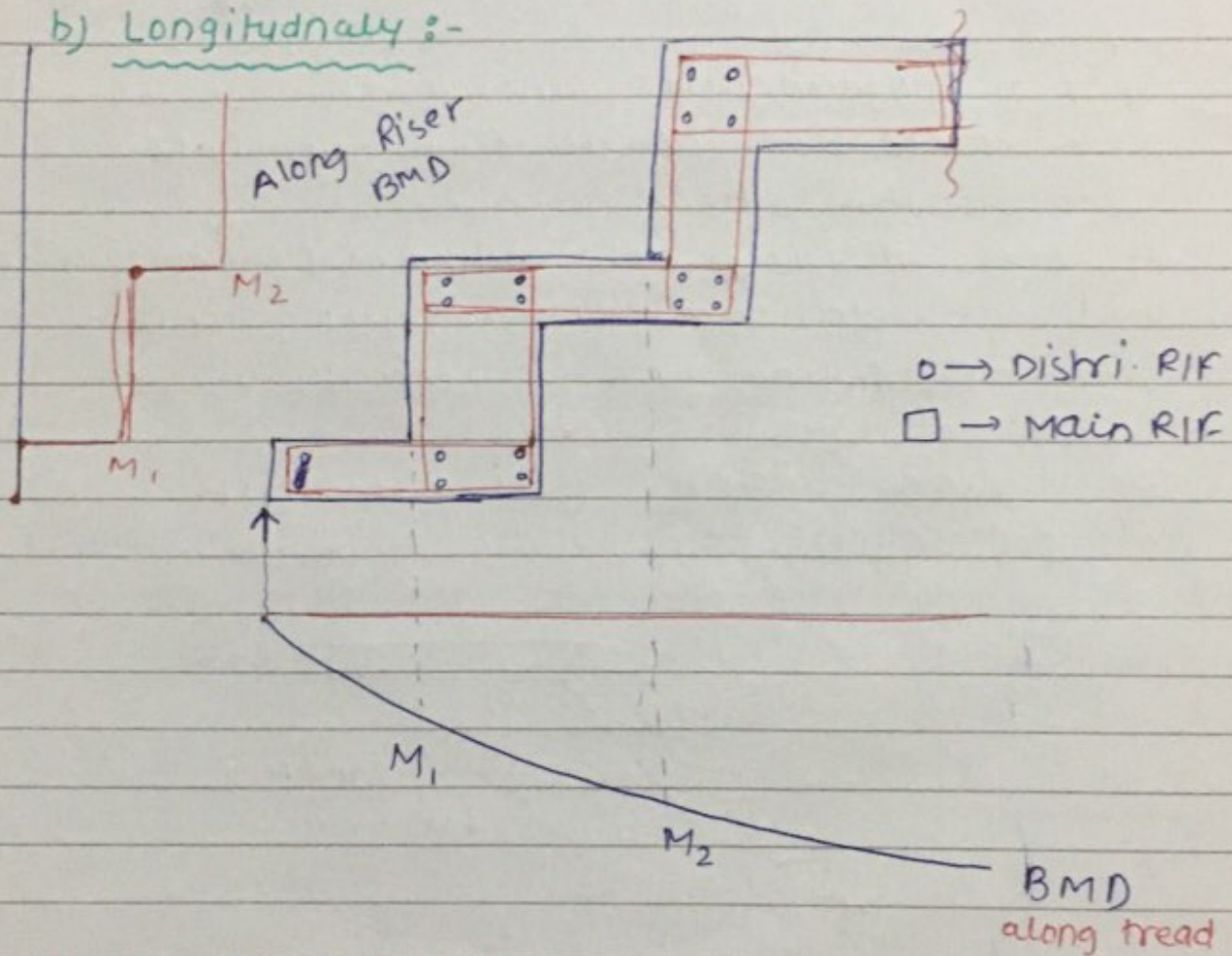
a) Transversely :-



- can be designed as a Flange Beam (Identical units)
However we design it like a Rectangular Beam

- In this arrangement $\frac{1}{2}$ of every tread is in Compression and the ^{other} half is in Tension

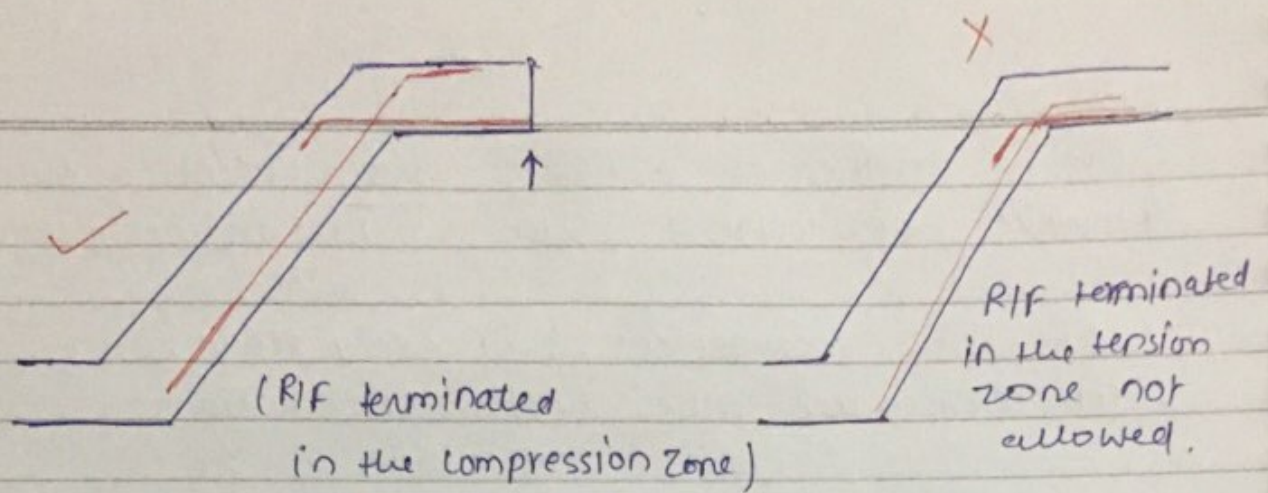
b) Longitudinally :-



Typical RIF Detailing :-

All the criteria regarding RIF in staircase are same as that in solid slab

NOTE: Staircase are prone to failure due to EQ loading and generally inclined beams are provided with the staircase slab, the beams shall be connected to Main structure.



Que. For a longitudinally spanning staircase draw the service load diagram from given Data

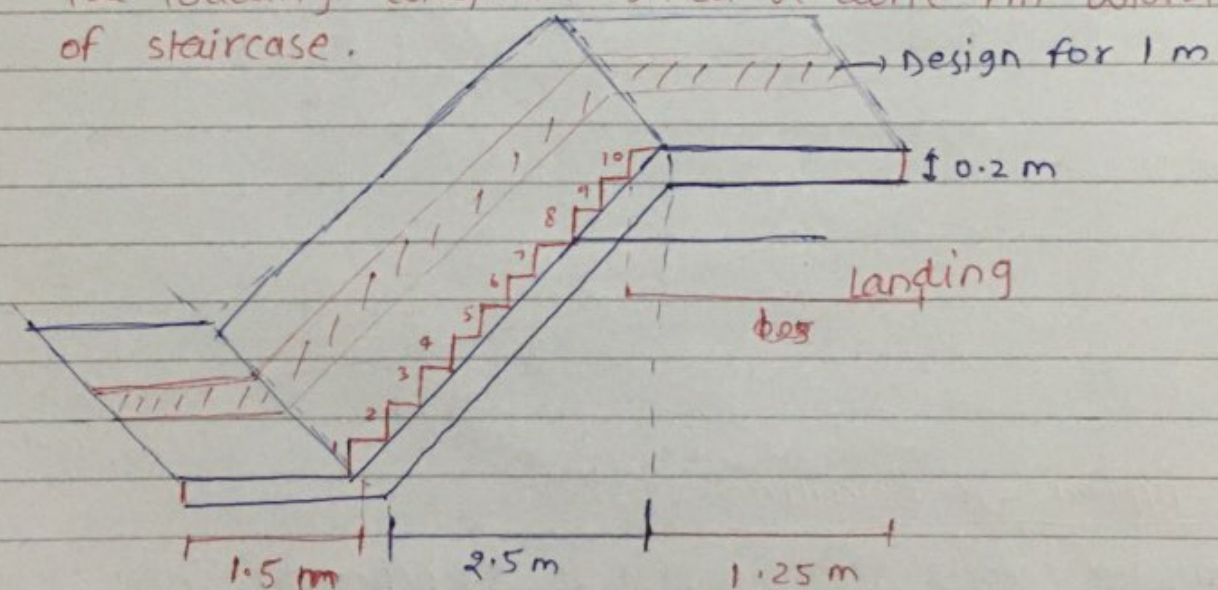
Riser = 150 mm, tread = 250 mm

thickness of waist slab = thickness of landing = 0.2 m

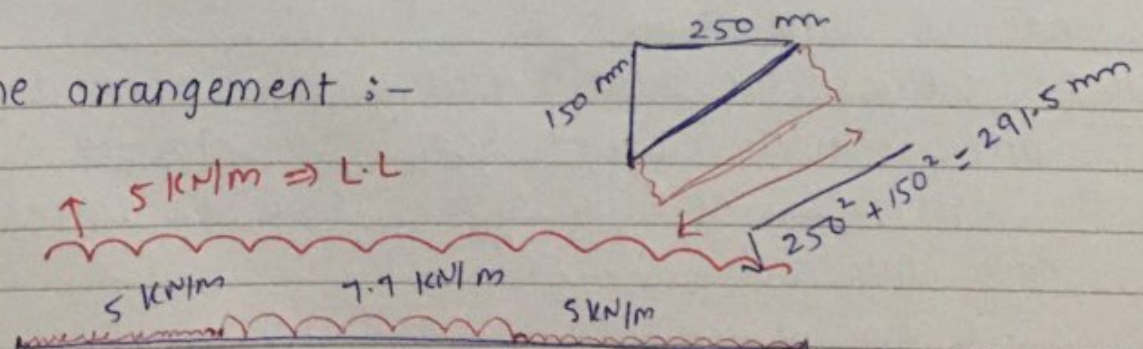
width of staircase = 1.2 m, LL = 5 kN/m²

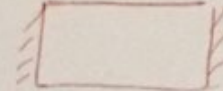
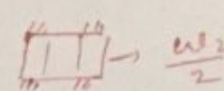
width of landing = 1.5 m. Height = 1.5 m.

The loading diagram shall be done 1 m width of staircase.



one arrangement :-

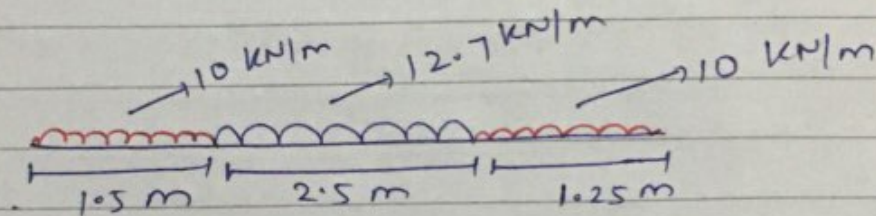


}  $\Rightarrow$ Don't divide w_2 by 2.
 $\frac{w_2}{2}$

DL of one arrangement = $\left[\frac{1}{2} \times 0.25 \times 0.15 \times 1 + 0.2 \times 0.2915 \times 1 \right] \times 25 = 1.926$ KN
 (Assuming 1m width) of staircase

projected load Along the going. = $\frac{1.926 \times 10}{2.5}$
 $= \frac{1.926 \times 10}{0.25 \times 10} = 7.7 \text{ KN/m}$

load on landing = $0.2 \times 1 \times 25 = 5 \text{ KN/m}$



Based on the above service load diagram We shall calculate the factored load and corresponding M_u & V_u .
 RIF shall be designed considering 1m width.

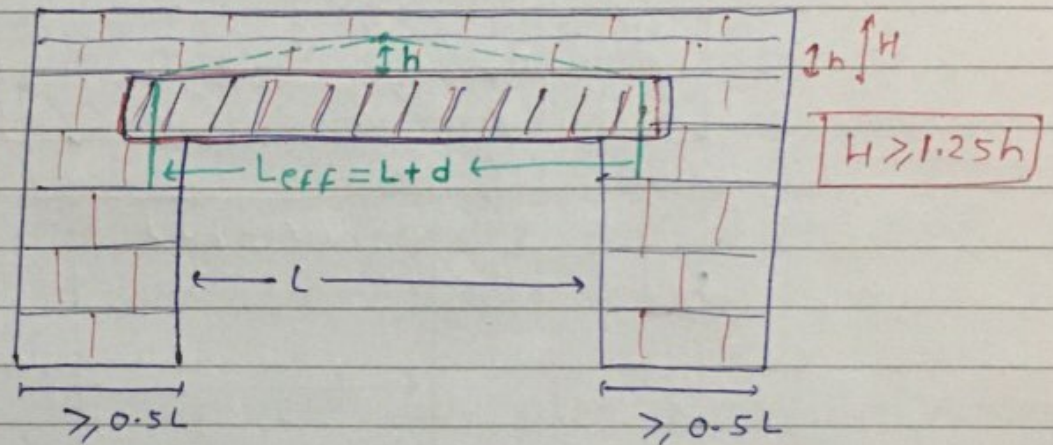
LINTEL

Lintels are the beams provided over opening in walls for doors and windows.

The load over the lintel will be that of brick masonry + load from Roof or slab (depending on the type) + load of ~~sunshade~~ if any.
Sunshade

Load Transfer Mechanism:-

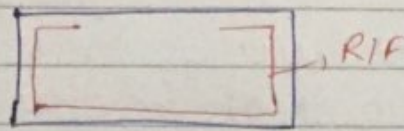
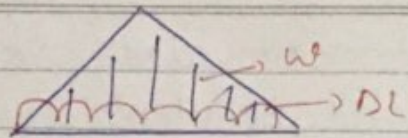
Case-1.



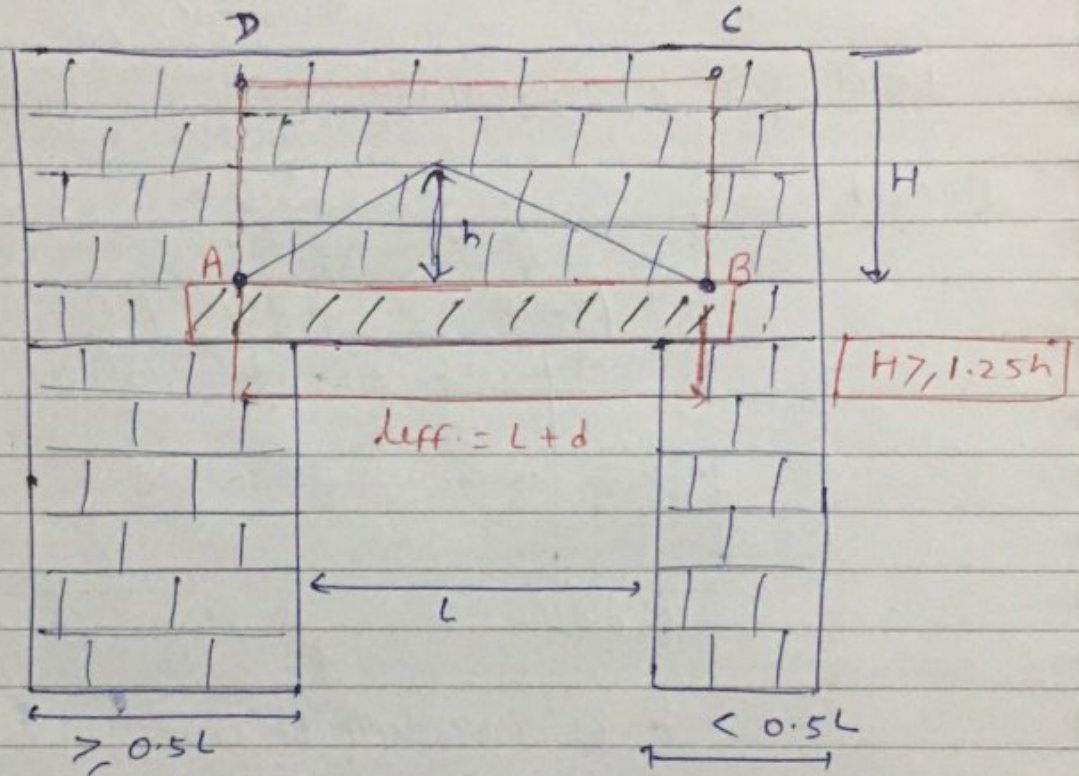
d = effective depth of Lintel.

- for proper arching action we need sufficient side width of walls and also adequate height above the Lintel.
- for good Masonry, Better Arch action and hence take $\theta = 45^\circ$
- for poor masonry, $\theta = 60^\circ$

In the above case we need to design the Lintel for the Triangular load + DL of Lintel.

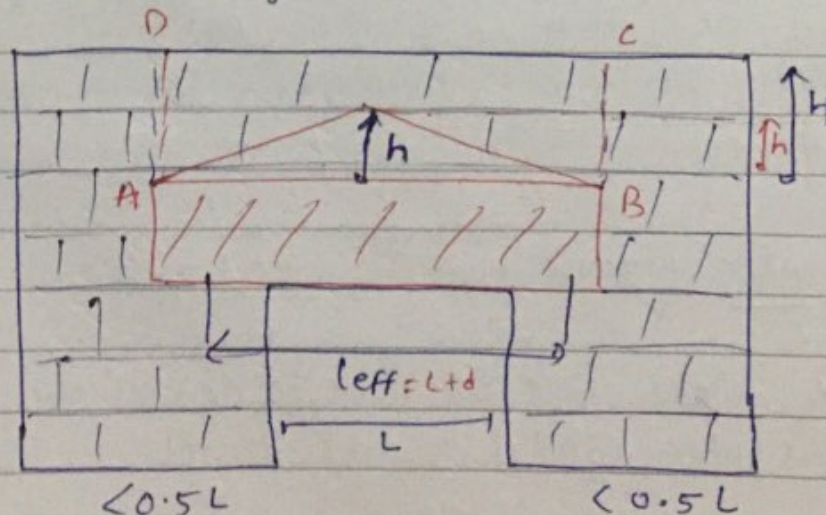


Case-2



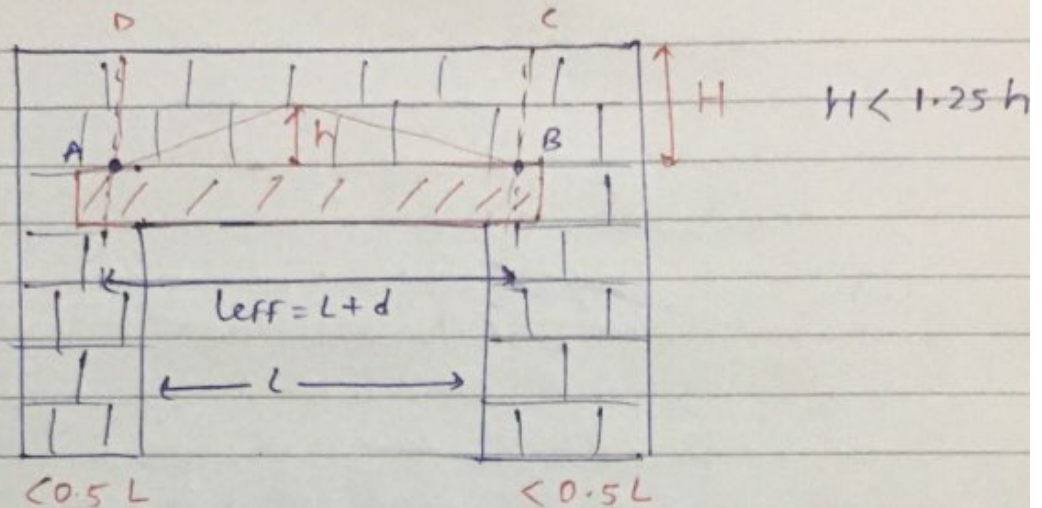
partial arch action, consider load of square
~~square~~ $ABCD$ + DL for design.

Case-3



No arch action and Lintel shall be designed for rectangular Load ABCD + Load from roof + DL

Case (iv)



No Arch action, consider load of rectangular ABCD + Load from roof + DL

NOTE

- In case 2 if $L_{eff} \geq H$, We need to consider it like case-3, i.e. design lintel for Rectangular load + Roof load + DL
- In case of frame structure load from roof/will be transferred to the beam and not to the infill^{slab} walls.

WATER TANK

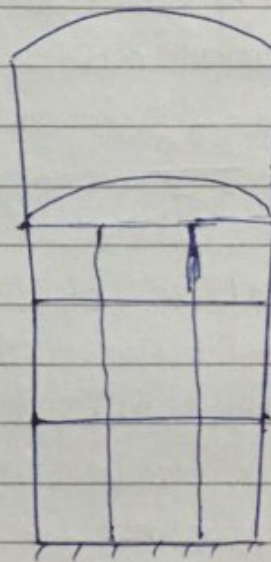
Classification :-

Based on location :-

- (a) Underground
- (b) Resting on Ground
- (c) Elevated

Based on shape

- ① circular $\rightarrow$ structurally most efficient
- ② Rectangular
- ③ square
- ④ Intze tank



Design Methods (IS 3370:2009) $\rightarrow$ code for liquid retaining structure

- ① can be designed by using both WSM & LCM

However analysis that is calculation of stress for a given force or BM shall be done with WSM

Limit state design :-

Design shall be done as per

IS : 456

- crack width $\nless 0.2$ mm for the outside face and theoretically No crack is allowed on the inner face that is Water retained face.
- The crack width criteria is considered to be satisfy if the stress in steel under service load condition does not exceed 115 MPa $\rightarrow$ MS
130 MPa $\rightarrow$ HYSD

WSM :- It is Based on Adequate resistance to cracking and Adequate strength.

- Take $m = \frac{280}{3\sigma_{cbc}}$
- In calculation of stress for flexure, Axial Tension or combination of both related to resistance to cracking, the whole c/s of concrete (that is including the cover) shall be considered provided the stress in concrete is within the permissible limit.
- In strength calculation, the Tensile strength of concrete is ignored.

permissible Tensile stress in concrete

Grade	Direct Tension (σ_{ct}) MPa	Bending Tension (σ_{cbt})
M25	1.3	1.8
M30	1.5	2.0

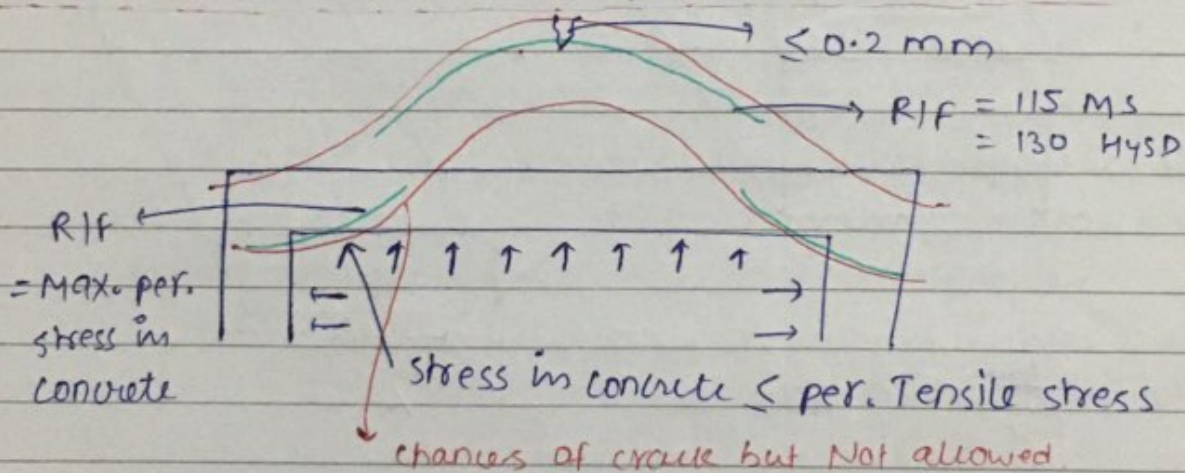
permissible compressive stress in concrete

Grade	Direct σ_{cc}	Bending σ_{cbc}
M25	6	8.5
M30	8	10

permissible stress in steel

- ① In strength calculation 115 MPa for MS
130 MPa for HYSD [415, 500]
Both

- ② In resistance to cracking i.e for NO crack condition,
 permi. stress in steel = $m \times$ permi. stress in concrete



Analysis :-

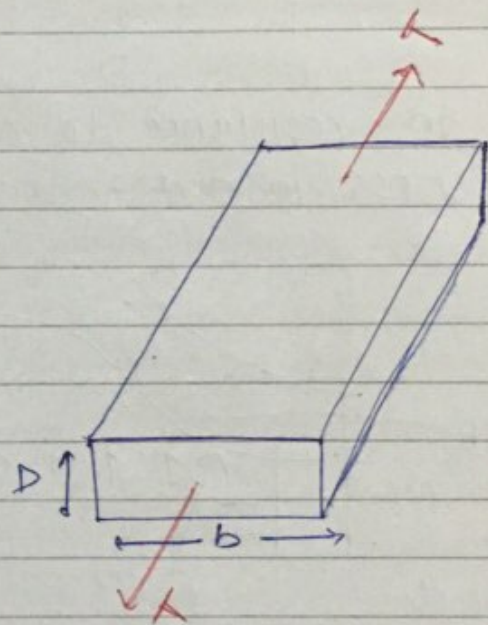
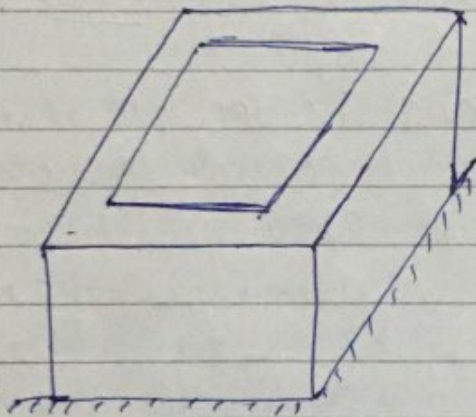
Member can be subjected to

- Axial Tension
- Bm
- Bm + Axial Tension

① Axial Tension :-

The Rf provided is such that All Tension is taken by Rf only

- for uncracked secⁿ, the stress in concrete shall be less than the permissible stress.



$$A_{st} = \frac{T}{\sigma_{st}}$$

$$\sigma_{ct} \geq \frac{T}{bD + (m-1)A_{st}}$$

Que. A Tensile force of 250 kN/m is acting on a Member of water tank, take M30 and fe 415

$$\sigma_{ct} = 1.5 \text{ MPa}, \sigma_{ctc} = 10 \text{ MPa}, \sigma_{st} = 130 \text{ MPa}$$

- calculate A_{st} required
- calculate D req. for no crack
- calculate stress in steel for uncracked secⁿ
- calculate stress in steel for cracked secⁿ

$$T = 250 \text{ kN}, b = 1 \text{ m}$$

$$A_{st} = \frac{T}{\sigma_{st}} = \frac{250 \times 10^3}{130} = 1923.07 \text{ mm}^2$$

$$(i) \quad 16 \phi \rightarrow 10 \text{ bars} = 2010 \text{ mm}^2$$

$$(ii) \quad \sigma_{ct} \geq \frac{T}{bD + (m-1)A_{st}} \leq 1.5 \quad \frac{250 \times 10^3}{1000D + \left(\frac{280}{30} - 1\right)2010}$$

$$D \geq 149.9 \text{ mm} \\ = 150 \text{ mm}$$

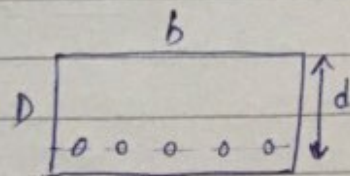
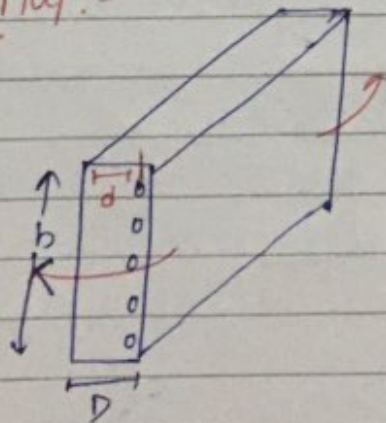
(iii) the stress in steel = $m \times$ stress in concrete

We need to calculate the Direct Tensile stress in concrete if the depth provided is more than depth calculated

$$\text{stress in stress} = 9.33 \times 1.5 \\ = 14 \text{ MPa}$$

$$(iv) \quad \text{stress in steel} = \frac{T}{A_{st, \text{provided}}} = \frac{250 \times 10^3}{2010} = 124.38 \text{ N/mm}^2$$

(ii) BM only:-



$$\frac{M}{I} = \frac{\sigma}{y} \rightarrow f_{cr}$$

$$\sigma = \frac{M}{Z} = \frac{GM}{bD^2}$$

$$\Rightarrow D = \sqrt{\frac{6M}{b\sigma}}$$

The section shall be designed as a cracked or uncracked section depending on water retained face or outside face.

- The section Dimension can be calculated
Assuming a pcc section and checking the section with provided RIF for M_{cr}
finally for uncracked BM shall be less than equal to M_{cr}

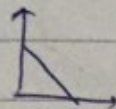
③ Member subjected to Axial Tension + BM

can be cracked or uncracked section. Water face must be design as uncracked section.

(i) uncracked section :-

$$\frac{\sigma_{ct}'}{\sigma_{ct}} + \frac{\sigma_{bt}'}{\sigma_{bt}} \leq 1$$

$$\frac{n}{a} + \frac{y}{b} \leq 1$$



$$\sigma_{ct}' = \frac{T}{bD + (m-1)A_{st}}$$

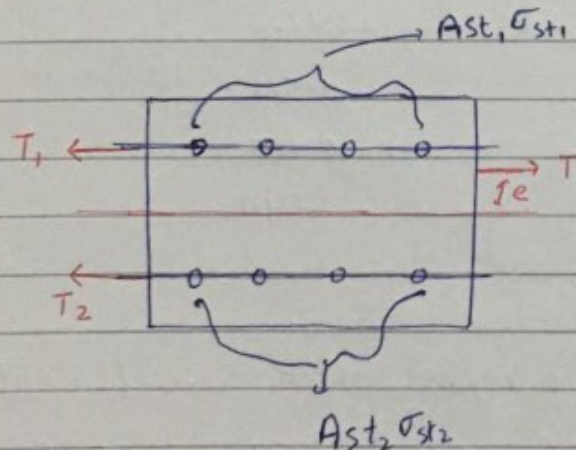
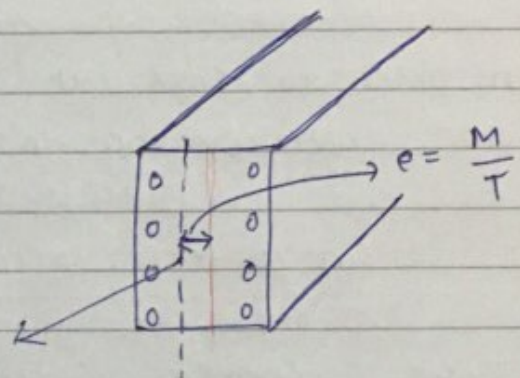
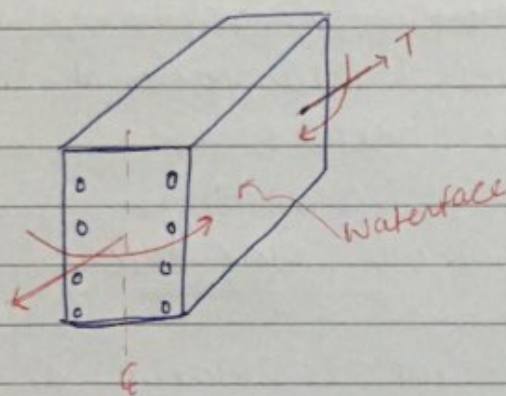
$$\sigma_{bt}' = \frac{M \cdot y}{I} \rightarrow \text{uncracked}$$

σ_{ct}' = actual direct Tensile stress in Concrete neglecting bending moment

σ_{bt}' = actual Bending Tensile stress in concrete Neglecting Axial force.

(ii) cracked section :-

Case-a. When eccentricity is small i.e Tensile force is large.



$$T = \sigma_{st1} A_{st1} + \sigma_{st2} A_{st2}$$

$$T(d_2 + e) = T_1(d_1 + d_2)$$

- In this case the line of action of force lies within the two layer of RIF and it can be assumed that the concrete is in Tension. The strength of concrete can be Neglected when it is in Tension
- The stress in steel can be calculated from force & Moment equilibrium.

• The stress in adjoining concrete to steel = $\frac{\sigma_{st}}{m}$

Case-b: When eccentricity is large i.e. Force lies outside the C/S (can lie inside also)

In this case some concrete will be in compression and we can't neglect the strength of concrete in compression

In order to find out stress in the material Trial and error procedure has to be adopted

Minimum cement, Min. Grade, Max. W/c :-

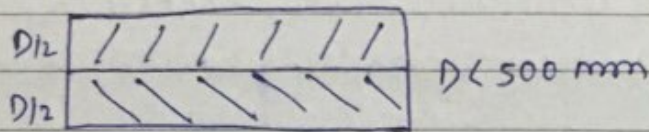
Type	Min. Grade	Minimum cement (kg/m ³)	Maximum W/c
PCC	M20	250	0.50
RCC	M30	320	0.45
PSC (pre stressed).	M40	360	0.40

- Max. cement concrete $\nless 400 \text{ kg/m}^3$ of concrete (IS 3370)
- For small capacity water tank upto 50 m^3 @ location where there is difficulty in making M30, the Min. Grade can be taken as M25
- However this exception shall not be applicable for coastal areas.

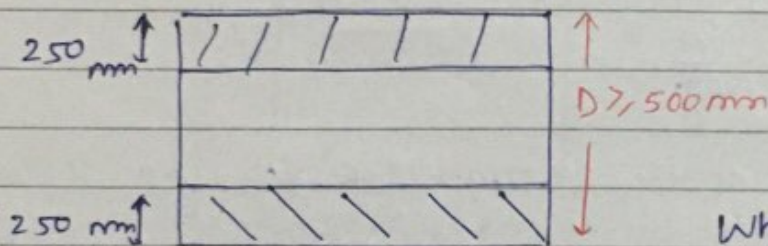
Minimum RIF:-

Min. RIF in each of the two dirⁿ at right angle within each surface zone should be as follows:-

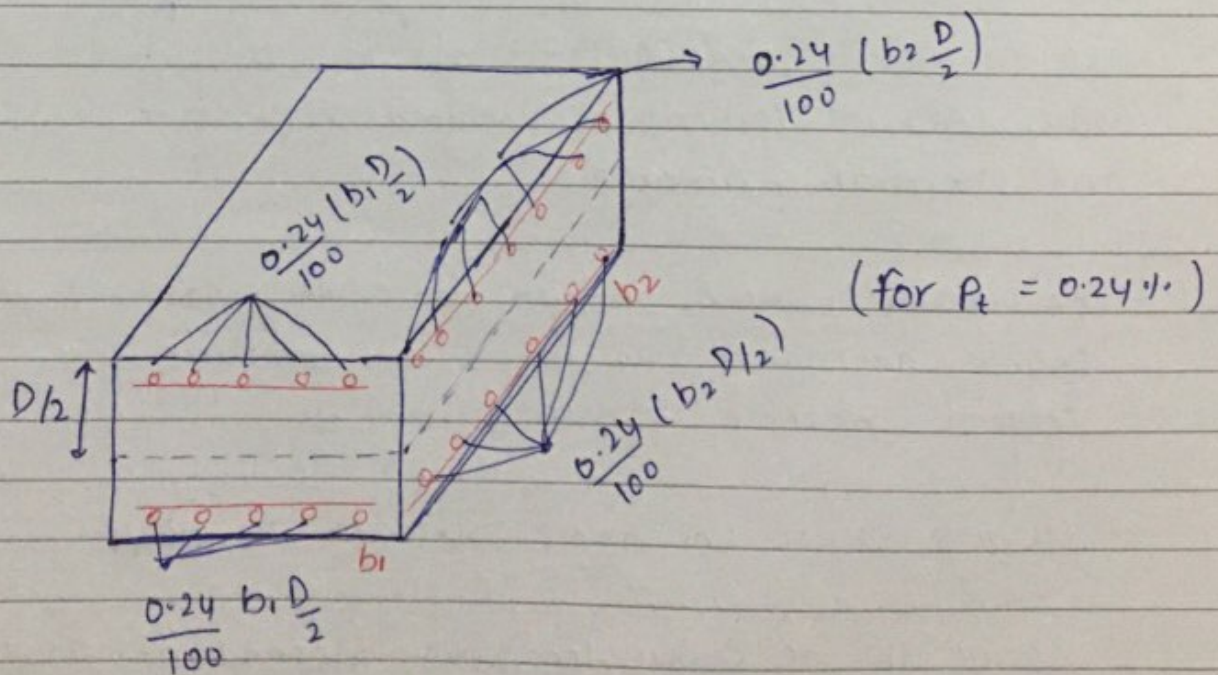
HYS D	MS	(In General)
0.35 %	0.64 %	
0.24 %	0.40 %	(When all dimensions of tank ≤ 15 m.)



When $D < 500$ mm
each surface zone depth = $D/2$



When $D > 500$ mm
each surface zone depth = 250 mm



In wall of thickness < 200 mm the calculated RIF may be placed in one face only.

for liquid faces or parts of the Member enclosing the liquid, exposure cond. is severe and cover to all RIF taken as 45 mm

- The spacing should generally not exceed 300 mm or thickness of the Member whichever is less
- The Min. RIF criteria is Based on effective distribution of crack and critical steel ratio.

$$\rho_{\text{critical}} = \frac{f_{ct}}{f_y}$$

f_{ct} = direct Tensile strength of immature concrete

Crack Control :-

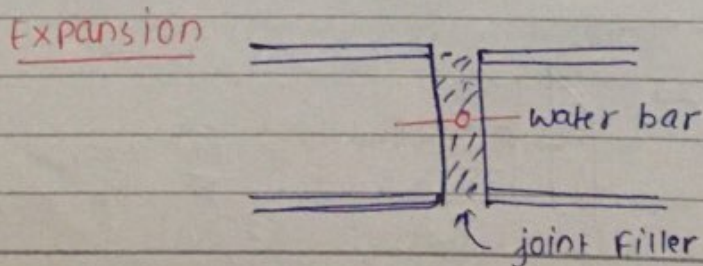
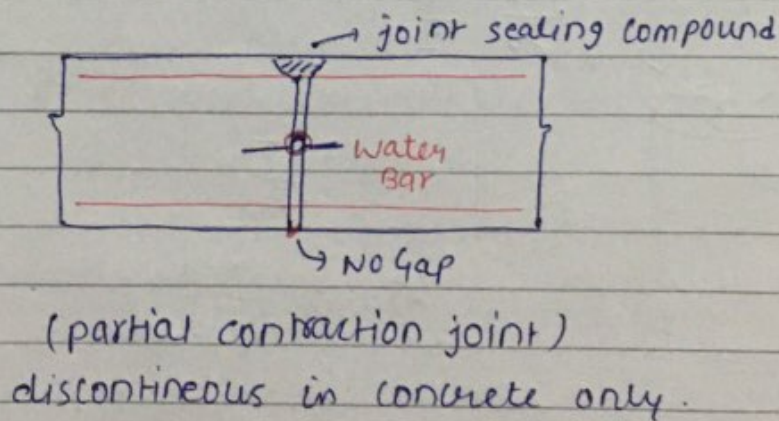
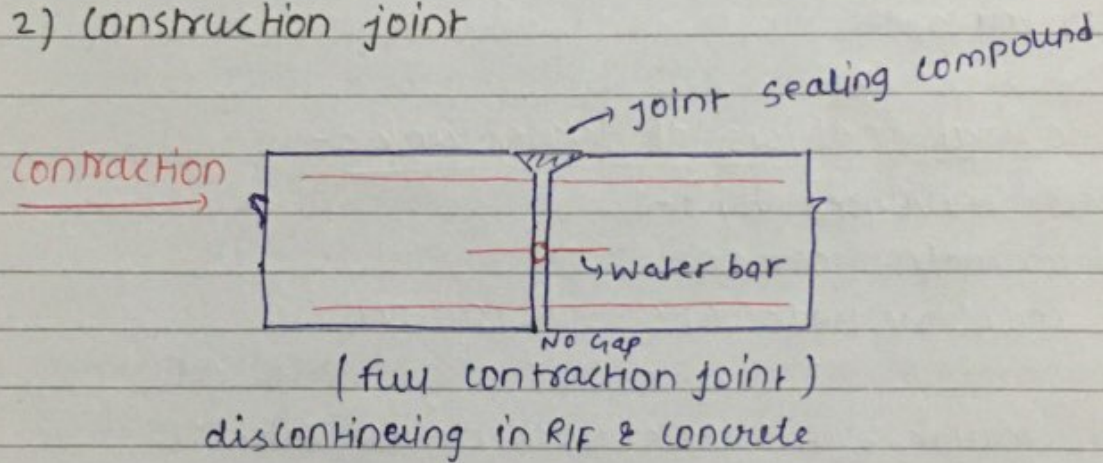
- use of porous aggregate shall be avoided
- Max. cement content is limited to reduce drying and Thermal shrinkage.
- The concrete Mix should have the largest practical coarse aggregate as this will reduce the cement content
- Curing shall be done atleast 14 days
- Large No. of small dia bars instead of small no. of large dia bars.

- slow filling on the tank for the first time
- Risk of cracking shall be Minimised by reducing the restrain on the free expansion or contraction of the structure by providing joints.

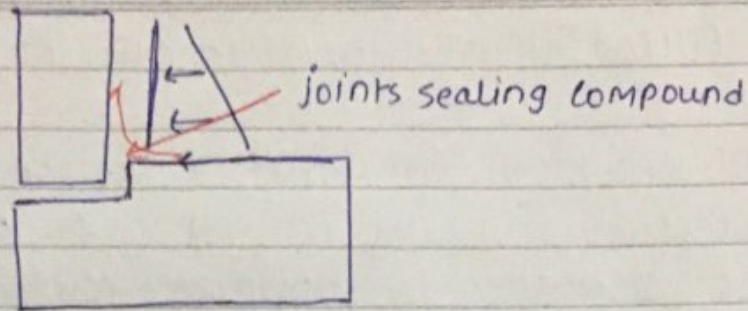
Type of joints:-

1) Movement joint $\begin{cases} \text{contraction joint} \\ \text{Expansion joint} \\ \text{sliding joint} \end{cases}$

2) Construction joint



sliding



Discontinuity in both concrete and RIF

Water tank with sliding joint b/w the floor slab and wall is called flexible base Water tank.

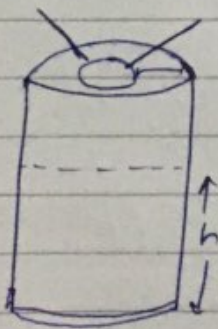
Structural Analysis of Water Tank :-

can be Analysed by :-

- 1) Approximate Methods
- 2) IS code Methods (IS 3370: Part-IV)

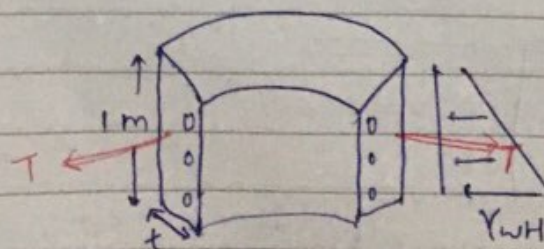
For a circular water tank with flexible base

$$\text{Hoop Tension} = \frac{PD}{2}$$



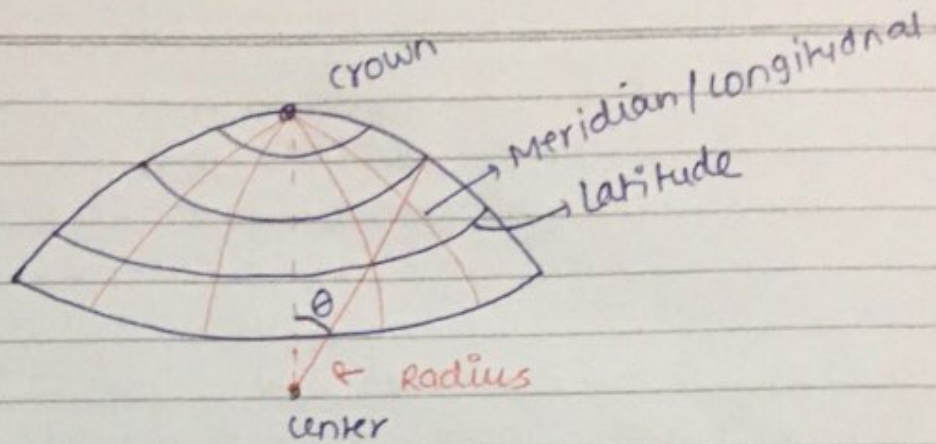
$$A_{st} = \frac{T}{\sigma_{st}}$$

$$\sigma_{cr} \geq \frac{T}{1 \times t + (m-1) A_{st}}$$



$$T = \frac{PD}{2} \quad (P = \gamma_w h, \quad D = D_{ia})$$

Domes :-



- At crown $\theta = 0$

- Meridional thrust = $\frac{WR}{1 + \cos \theta}$ $\rightarrow$ Always compressive

- Hoop stress = $\frac{WR(\cos^2 \theta + \cos \theta - 1)}{(1 + \cos \theta)}$

$W \rightarrow$ UDL above the Dome.

- Along the latitude the stress can be compressive or Tensile (Hoop stress) it will be Tensile after a θ value of $51^\circ 49'$

- At crown Both Hoop and meridional thrust = $\frac{WR}{2}$

